11. exercise sheet: Quantum Field Theory

Aufgabe 24:

(a) Use the Feynman rules of QED to compute the scattering amplitude for a 2-to-2 process of distinguishable fermions to lowest order; verify that this is given by a one photon exchange.

(b) Consider the non-relativistic limit and compute the effective one-body potential between the fermions (analogously to the Yukawa potential discussed in the lectures).

(c) Verify that the fermion-fermion as well as the anti-fermion–anti-fermion potential is repulsive, whereas the fermion–anti-fermion potential is attractive.

(d) By noting that the sign change between repulsion and attraction arises from a factor of $-g_{00} = -1$ in the vector boson propagator, discuss the sign of the gravitational force between fermions/anti-fermions mediated by a one-graviton exchange with a propagator being proportional to $\sim \frac{1}{2}((-g_{\mu\rho})(-g_{\nu\sigma}) + (-g_{\mu\sigma})(-g_{\nu\rho}))$. What is your prediction for the ALPHA experiment at CERN, trying to measure the gravitational force between matter and anti-matter?

Aufgabe 25:

(a) Use the defining property of the Dirac algebra,

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu},$$

to show that $\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$ anti-commutes with all γ^μ, i.e., $\{\gamma_5, \gamma^\mu\} = 0$, independently of the representation of the Dirac matrices.

(b) Show also that $\gamma_5^2 = \mathbb{1}$ holds independently of the representation.

(c) γ-trace technology: Show that

$$\mathrm{tr}[\gamma_\mu\gamma_\nu] = 4g_{\mu\nu}, \qquad \mathrm{tr}[\gamma_5] = 0,$$
$$\mathrm{tr}[\gamma_\mu\gamma_\nu\gamma_5] = 0, \qquad \mathrm{tr}[(\text{odd \# of } \gamma_\mu\text{'s})\gamma_5] = 0.$$