# 6. Exercise sheet: Quantum Field Theory

**Aufgabe 13:**

Consider a real scalar field that interacts with a classical source $J(x)$,

$$H = H_0 + \int d^d x J(t, \mathbf{x}) \phi_S(\mathbf{x}).$$

(a) Convince yourself that the probability for no particle to be produced by the source is given by ($D = d + 1$)

$$P(0) = \left| \langle 0 | T \left[ \exp \left( -i \int d^D x J(x) \phi(x) \right) \right] | 0 \rangle \right|^2 ,$$

where $\phi(x)$ is the field operator in the interaction picture.

(b) Use Wick's theorem to show

$$P(0) = \exp(-\lambda), \quad \text{mit } \lambda = \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\mathbf{p}}} |J(\bar{p})|^2,$$

where $J(\bar{p})$ is the Fourier transform of $J(x)$, and $\bar{p}^\mu = (E_{\mathbf{p}}, \mathbf{p})$.

(c) Now show that the probability for $n$ particles to be created by the source is given by

$$P(n) = \frac{\lambda^n}{n!} \exp(-\lambda) \quad \text{(Poisson distribution)}.$$

(Hint: note that the particles are indistinguishable.)

(d) Verify that the mean number of produced particles is given by $\langle N \rangle = \lambda$.

**Aufgabe 14:**

Consider a real scalar field with interaction Hamiltonian

$$\mathcal{H}_I = \frac{g}{3!} \phi^3,$$

where $g$ denotes the coupling. Compute the $S$ matrix of this theory to order $g^2$ using Wick's theorem in terms of normal ordered products and Wick contractions. Draw the corresponding Feynman diagrams.