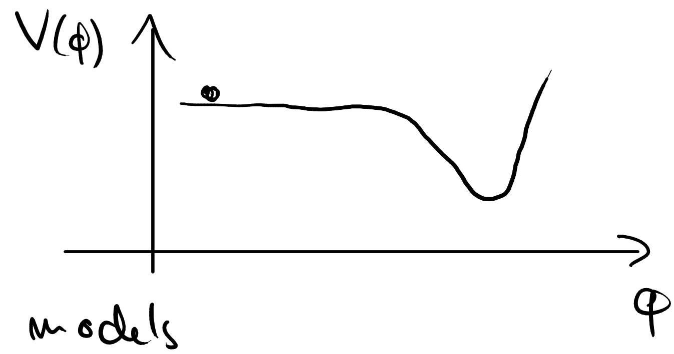
14 (Inflationarily) Many Models of Inflation

As already emphasized, inflation should be considered as a paradigm or a set of ideas in the context of which on can address some naturalness problems of the cosmological standard model, rather than a solid building block of cosmology (as, e.g., BBN or recombination).

On the one hand, the idea of inflation is too unspecific to single out a unique model, on the other hand, the observational data is not able to falsify many models.

While inflation solves some naturalness problems, it also creates its own puzzles. Hence, many models are constructed such that these intrinsic problems are mitigated or such that other questions of fundamental physics can be simultaneously addressed (e.g. the strong CP-problem, baryogenesis or the electroweak hierarchy problem).

An obvious problem of inflation is the arbitrariness of the potential. Of course, it is easy to draw or parametrize a potential that gives rise to a sufficient amount of inflation with a slow-roll regime. However



the scalar field in such models has to couple to matter (which is quantum) and to gravity (which may be quantum). Therefore quantum fluctuations will contribute to the shape of the potential and thus also to the slow-roll parameters. The details depend on the model, but a typical quantum correction to $\eta_V = M_p^2 \frac{\partial^2 V/\partial\phi^2}{V}$ reads

$$\Delta \eta_V \sim c \frac{M_p^2}{\Lambda^2} \qquad (14.1)$$

where Λ is a high energy scale up to which the quantized theory holds. Since gravity is expected to set the highest scale up to which a quantum field theory can be trusted, one typically expects $\Lambda < M_p$. In (14.1) c is a constant predicted by the model but is assumed to be $\mathcal{O}(1)$ as a generic case. Hence we conclude $\Delta \eta_V > 1$ (14.2)

such that inflation wouldn't last long enough for a generic model. This is the so-called "η problem" of inflation which any fundamental model of inflation has to address.

It thus seems that we have replaced the naturalness problems of the hot big bang by the problem of getting naturally a suitable potential.

However, there exist ideas to solve the η problem: One idea is to impose a shift symmetry $\phi \to \phi + \text{const.}$ which requires the potential to be flat exactly, and then mildly break this symmetry either explicitly or spontaneously in such a way that the η - problem is avoided.

Shift-symmetric theories can easily be formulated by allowing only derivative couplings, e.g.

$$S = \int dt\, d^3x \left(\frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) + g\left[(\partial_\mu \varphi)(\partial^\mu \varphi)\right]^2 + \ldots \right] \tag{14.3}$$

(as appears naturally for Goldstone bosons)

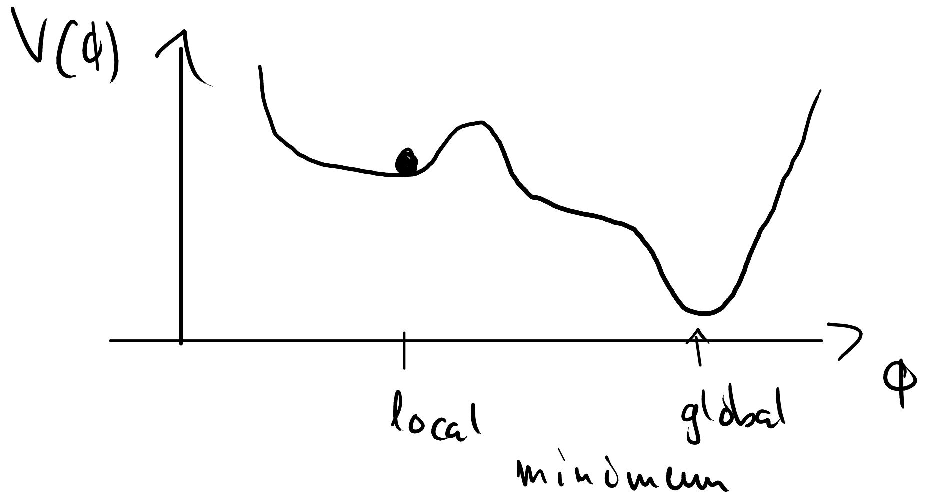
Another option is to impose a symmetry, which makes c in (14.2) naturally small. An example is given by supersymmetric theories where bosonic an fermionic fluctuation contributions can cancel (to some degree).

Of course, the η problem is also solvable, if Λ can be chosen larger than M_p. However, this is the regime of quantum gravity where little is known about. Though inflation is constructed in such a way that no quantum gravity is needed for the validity of the basic equations, inflation seems to be sensitive to UV (ultraviolet = extremely high energy) properties of the models. This has also been visible in quadratic inflation, where we needed trans-Planckian field amplitudes.

Another issue is the following: we introduced inflation to solve the problem of unnatural initial conditions (flatness problem, super-horizon correlations). However, what provides the initial conditions for inflation? E.g. how are the initial conditions for the scalar field chosen? Why should ϕ start in a suitable region where inflation is possible? Why should it start at sufficiently small $\dot\phi(t)$?

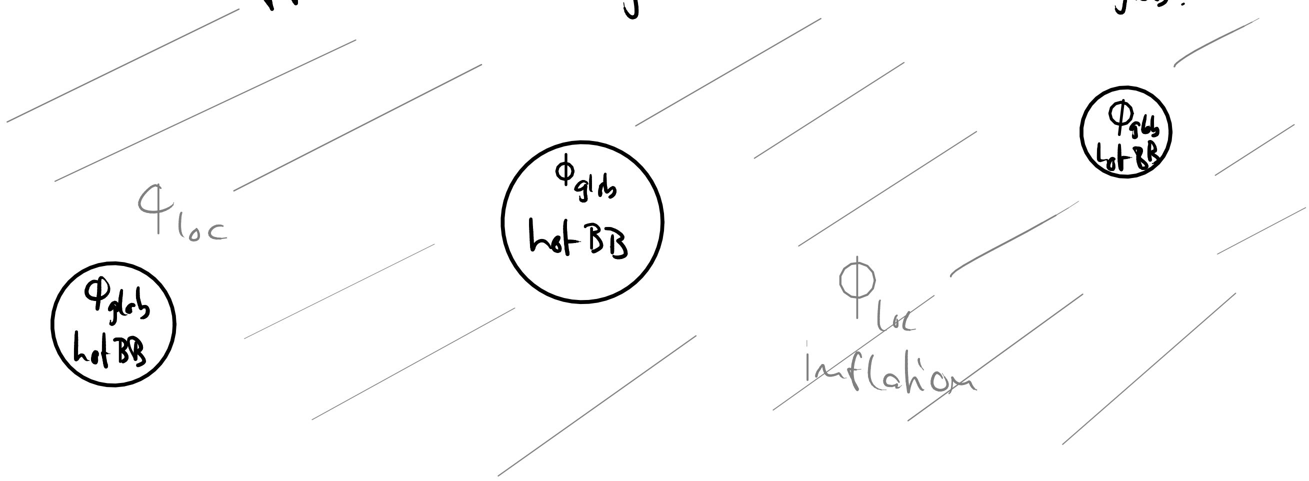
In this case an (to my taste unsatisfactory) anthropic answer can be given: let $\phi(t,\vec{x})$ be allowed to start anywhere in the potential landscape; for some values of $\vec{x}$, it may already be in the minimum, for others, it may be in a suitable slow-roll region. Then, those regions, where the conditions are suitable, will start to inflate and space expands. After inflation those regions will have the largest volume and dominate the possible scenarios which we may be able to observe. (Also, patches of the universe where no reheating took place may not have formed nuclei and human beings who could do measurements).

A particularly fashionable variant of such ideas is <u>eternal inflation</u>. There are various ways of realizing eternal inflation, let's consider the following one: think of a potential with a local and a global minimum, e.g.



If the amplitude is initially in the basin of attraction of the local minimum it will eventually sit in that local minimum with $\dot{\phi} = 0$ and thus $w_\phi = -1$. This is an inflationary phase that can last for a long time. Classically, the amplitude would remain there for ever. Quantum mechanically, the amplitude may tunnel eventually towards the global minimum. Such tunneling will occur locally in certain regions (bubbles) of space forming a pocket in which a hot big bang cosmology is kicked off. If the tunneling rate is Γ, the probability that the amplitude stays in the local inflationary minimum is $\sim e^{-\Gamma t}$. At the same time the volume of the inflationary region increases with $a^3 \sim e^{-3Ht}$.

If the rate of exponential expansion is larger than the decay rate, then the creation of space in the inflationary minimum will globally never end. Inflation ends only inside the pockets where ϕ approaches the global minimum ϕ_{glob}.



One of these pockets would be our universe whereas others would form disconnected further universes. Together this would be a multitude of universes, a "multiverse". (NB: this is a misnomer, since universe means already the 'total', i.e. everything). Depending on the model, such pockets or bubbles could be created at relative speeds implying that bubble collisions could be possible. Signatures of such collisions have been and are searched for, but have not been found.

The idea of eternal inflation could be realized in string theory which appears to have a <u>very</u> large number of metastable vacua (local potential minima). In addition to the picture of eternal inflation sketched above, the physics in each of these vacua would be different (different particles, masses, couplings, etc.).

The number of metastable vacua is, in fact, so long that any kind of naturalness problem, most prominently the cosmological constant problem, can be declared to be resolved by anthropic reasoning: Λ has the chance to have this particular value in one of the universes, so by chance (or suitable parameter conspiracy) we happen to live in this one, where we can observe it at this value. In other universes we would observe a different value (or would not exist and thus not be able to observe a different value).

In summary, it is clear that such ideas rely on a great deal of speculation. Scientifically, it is a severe problem that most of these ideas cannot be falsified as a matter of principle - not by lack of experimental capabilities. While some of these ideas resonate well with the contemporary pop culture, some people consider this line of "research" as the end of physics as an accomplishment of the scientific method.

For sure, research in this direction is in a crisis as it lacks relevant experimental or observational input.

Hence, let us end this section with one of the earliest models of inflation which has so far stood the test of time (and data) and can be linked to modern field theory approaches to quantum gravity: the <u>Starobinsky</u> <u>model</u>.

Let us first note that Einstein's equation follows from an action principle based on the Einstein Hilbert action

$$S[g] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R \;\equiv\; \frac{M_P^2}{2}\int d^4x \sqrt{g}\, R \qquad (14.4)$$

where $d^4x\sqrt{-g}$ is the invariant volume element (invariant under a change of coordinates) with $g = \det g_{\mu\nu}$ and R being the Ricci scalar (we have ignored here a possible cosmological constant).

R is indeed the simplest conceivable invariant curvature scalar. In a quantum version of the theory, one expects (14.4) to be a leading order form with higher order quantum corrections yielding further invariant terms in the action.
Hence, a more general action could be

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{g}\; f(R) \qquad (14.5)$$

where $f(R)$ may have an expansion in curvature powers:

$$f(R) = R + \frac{1}{6M^2} R^2 + \ldots \qquad (14.6)$$

where M is a mass scale guaranteeing that the coefficient has the right mass dimension. Of course, one could go to higher orders and also include more complicated terms, but (14.6) is the next-to-simplest choice, which has been Starobinsky's choice.

Now consider the following action involving also a scalar field χ:

$$S[g,\chi] = \frac{M_P^2}{2} \int d^4x \sqrt{g} \left(f(\chi) + (R-\chi) \frac{\partial f}{\partial \chi} \right) \tag{14.7}$$

The equation of motion for χ follows from the action principle:

$$\delta_\chi S = 0 \quad \Rightarrow \quad \cancel{\frac{\partial f}{\partial \chi}} + (R-\chi) \frac{\partial^2 f}{\partial \chi^2} - \cancel{\frac{\partial f}{\partial \chi}} = 0 \tag{14.8}$$

For a general f, (14.8) is solved by

$$\chi = R \tag{14.9}$$

Inserting (14.9) into (14.7) precisely leads back to the general action (14.5) which includes Starobinsky's choice (14.6) for a suitable

$$f(\chi) = f(R).$$

Now, let us confine ourselves to (14.6)

$$f(\chi) = \chi + \frac{\chi^2}{6M^2} \tag{14.10}$$

and define a new scalar field by

$$\varphi := \frac{\partial f}{\partial \chi} = 1 + \frac{\chi}{3M^2} \tag{14.11}$$

$$\Rightarrow \quad \chi = 3M^2(\varphi - 1).$$

The action now reads

$$S[g,\varphi] = \frac{M_P^2}{2}\int d^4x\sqrt{g}\,\left(f(\chi(\varphi)) + (R - \chi(\varphi))\,\varphi\right)$$

$$= \frac{M_P^2}{2}\int d^4x\sqrt{g}\left(\cancel{\chi} + \frac{\chi^2}{6M^2} - \cancel{\chi} - \frac{\chi^2}{3M^2} + R\varphi\right)$$

$$= \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R\varphi - \frac{M_P^2}{12M^2}\chi^2\right)$$

$$= \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R\varphi - \frac{3}{4}M_P^2M^2(\varphi-1)^2\right)$$

(14.12)

For the next step, we make use of a conformal transformation of the metric

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad (14.13)$$

where Ω is some suitable function. We also take over the result from differential geometry that the Ricci scalar transforms under (14.13) as $R \to \tilde{R}$, where the two are related by

$$R = \Omega^2 \left(\tilde{R} + 6 \tilde{\Box} \omega - 6 \tilde{g}^{\mu\nu} \partial_\mu \omega \partial_\nu \omega \right) \qquad (14.14)$$

where $\omega = \ln \Omega$ and $\tilde{\Box}$ is the d'Alembertian with respect to $\tilde{g}_{\mu\nu}$.

Since $\sqrt{-g} = \sqrt{-\det g_{\mu\nu}}$, we have that

$$\sqrt{-g} = \sqrt{-(\det \tilde{g}) \cdot \Omega^{-8}} = \Omega^{-4} \sqrt{-\tilde{g}} \qquad (14.15)$$

Choosing $\Omega^2 = \varphi$, we get

$$\sqrt{-g}\, \varphi R \to \Omega^{-4} \sqrt{-\tilde{g}}\, \Omega^2 \Omega^2 R + \ldots = \underline{\underline{\sqrt{-\tilde{g}}\, \tilde{R} + \ldots}} \qquad (14.16)$$

which is the Einstein Hilbert term.

This choice transforms the last term in (14.14) into

$$\frac{M_p^2}{2} \cancel{\Omega^{-4}} \sqrt{-\tilde{g}}\, \cancel{\Omega^2}\, \cancel{\Omega^2} \left(6 \tilde{g}^{\mu\nu} \partial_\mu \omega \partial_\nu \omega \right) \qquad (14.17)$$

which suggests to define

$$\phi = -\sqrt{6} M_p \omega = -\sqrt{6} M_p \ln \Omega = -\sqrt{\tfrac{3}{2}} M_p \ln \Omega^2$$

$$= -\sqrt{\tfrac{3}{2}} M_p \ln \varphi \qquad (14.18)$$

$$\overset{(14.17)}{\Longrightarrow} \qquad \sqrt{-\tilde{g}} \ \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \qquad (14.19)$$

(The 2nd term in (14.14) is a surface term and can be ignored.)

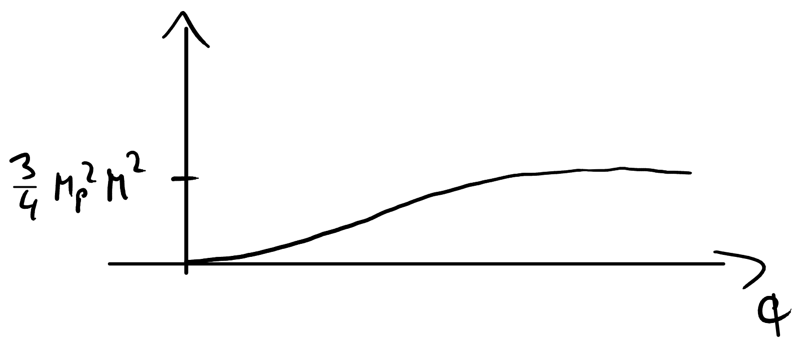
Resolving (14.18) for φ,

$$\varphi = e^{-\sqrt{\frac{2}{3}} \frac{1}{M_p} \phi}, \qquad (14.20)$$

the action becomes

$$\underline{\underline{S[g,\phi]}} = \int d^4x \sqrt{-\tilde{g}} \left(\frac{M_P^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \underline{\underline{\frac{3}{4} M_P^2 M^2 \left(e^{-\sqrt{\frac{2}{3}} \frac{1}{M_P} \phi} - 1 \right)^2}} \right) \tag{14.21}$$

which is a theory of gravity including a scalar field with a rather flat potential



that can give rise to a slow-roll inflationary phase with suitable properties.

For a suitable value of $M \simeq 10^{-5} M_P$, the model is compatible with CMB data (in particular it predicts the correct "spectral tilt".)