11. Some Naturalness Problems of Big Bang Cosmology

Naturalness problems are open and puzzling questions that search for an understanding of certain values of physical constants or specific initial conditions required to parametrize the observed phenomena in nature. Often these problems go along with arguments based on established concepts that make the observed parameter values appear implausible, very special or fine-tuned – and thus unnatural. We have already seen such examples in cosmology for the case of the "cosmic coincidence problem" (i.e. the fact that accelerated expansion has set in rather recently), and most prominently for the "cosmological constant problem" (i.e. the fact that dark-energy has a value ~ 120 orders of magnitude smaller than a value naively suggested by QFT considerations).

Big bang cosmology has further naturalness problems that concern the initial conditions at the big bang. Before we take a closer look, it is important to stress that naturalness problems do not constitute consistency problems. That is, the theory works well and there is no issue of ill-posedness that needs to be solved for getting a predictive and consistent theory. In this strict sense, naturalness problems are no real problems but merely features of the theory that appear unsatisfying from a certain educated perspective.

From an epistemological viewpoint, it is thus not even clear whether such problems can be solved or do require a solution at all.

Still, in many cases, such problems have served as a source of creativity for constructing more fundamental underlying models or mechanisms that "explain" or at least parametrize the unnatural parameters in a plausible way, making them appear more natural.

In this way, naturalness problems can mark a fruitful cornerstone for the construction of superior models — but can also misguide a generation of researches (the ancient Greek theory of epicycles may be an interesting example in this case).

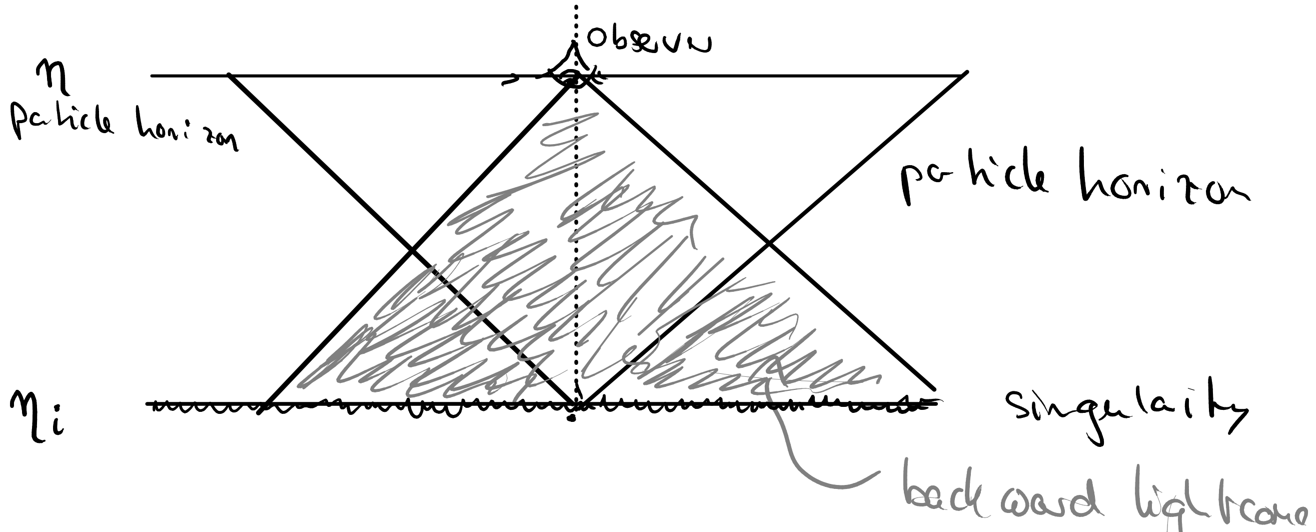
1) The horizon problem

The horizon problem arises from the observation of homogeneity in the universe and the implicit assumption that all regions that look the same should have been in causal contact at some early point in the cosmic history. Some mechanism has to "tell" the different regions that they should look the same, or dynamically establish equal properties.

In order to discuss causal connections, let us define the particle horizon d_H as the greatest comoving distance from which an observer at time t will be able to receive signals (traveling at the speed of light). This is best studied using conformal time and comoving coordinates where light geodesics corresponds to straight lines and the conformal time difference $\Delta\eta$ measures the distance between two points on such lines. With the big bang occuring at $t_i = 0$, the particle horizon is defined as

$$d_H(\eta) = \eta - \eta_i = \int_{t_i}^{t} \frac{dt}{a(t)} \tag{11.1}$$

In conventional big bang cosmology studied so far, we can choose $\eta_i = 0$ and $d_H(\eta) = \eta$



Since the observer can only be influenced by events in his/her backward light cone, the particle horizon corresponds to the intersection of that backward cone with the initial spacelike singularity surface η_i.

Eq. (11.1) can be rewritten as

$$d_H(\eta) = \int_{t_i}^{t} \frac{dt}{a(t)} = \int_{t_i}^{t} \underbrace{\frac{da}{a}}_{d\ln a} \underbrace{\left(\frac{da}{dt}\right)^{-1}}_{\frac{1}{aH}} = \int_{\ln a_i}^{\ln a} \frac{1}{aH} \, d\ln a, \tag{11.2}$$

where $a_i = 0$, $\ln a_i \to -\infty$, corresponds to the big bang singularity.

It is useful to think of the integrand $\frac{1}{aH} = \frac{1}{\dot{a}}$ as a comoving Hubble radius.

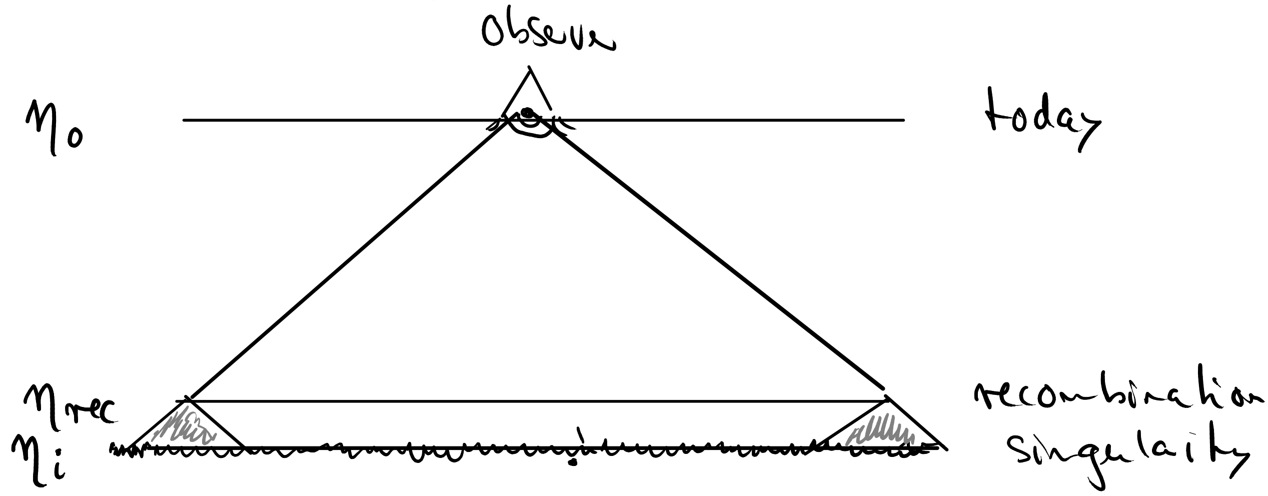
For ordinary radiation or matter, we have

$$\begin{aligned} &\text{radiation:} \quad a \sim \sqrt{t}\,, \quad \dot{a} \sim \frac{1}{\sqrt{t}}\,, \quad \frac{1}{aH} = \frac{1}{\dot{a}} \sim \sqrt{t} \\ &\text{matter:} \quad a \sim t^{2/3}\,, \quad \dot{a} \sim \frac{1}{t^{1/3}}\,, \quad \frac{1}{aH} = \frac{1}{\dot{a}} \sim t^{1/3}\,, \end{aligned} \tag{11.3}$$

i.e. the comoving Hubble radius increases monotonically as a function of time (and of the scale factor). Since the measure in (11.2) is logarithmic, the integral is dominated by its maximum which is at late times.

This leads sometimes to the statement that $d_H \sim \frac{1}{aH}$, identifying the Hubble radius with a horizon.

In any case, also in conformal time, we have that the distance between big bang and the formation of CMB, say the period of recombination, is much smaller than the distance big bang - today, $\eta_{rec} \ll \eta_0$.



Now, consider photons arriving at an observer from opposite directions. The photons were created during recombination at a time, when the particle horizon was much smaller than it is today $d_H(\eta_{rec}) = \eta_{rec} \ll \eta_0 = d_H(\eta_0)$.

Hence the backward light cones of the events creating the photons have no overlap and hence are causally disconnected.

How can these photons "know" that they should have the same temperature?

Phrased differently: if the universe was initiated with different temperatures at different patches of the initial singularity, there would not have been sufficient time to equilibrate these temperatures such that we see an almost perfect Planckian spectrum in all directions now.

If there wasn't enough time to communicate among different regions, why do they look so similar. This is the <u>horizon problem</u>.

Let us make this more quantitative: consider a universe with matter and radiation:

Friedmann equation: $$H^2 = H_0^2 \left[\Omega_m a^{-3} + \Omega_r a^{-4} \right], \quad \Omega_r = a_{eq} \Omega_m \tag{11.4}$$

The comoving Hubble radius then reads

$$\frac{1}{aH} = \frac{1}{H_0} \left[\Omega_m a^{-1} + \Omega_r a^{-2} \right]^{-1/2}$$

$$= \frac{1}{\sqrt{\Omega_m}} \frac{1}{H_0} \frac{1}{\sqrt{\frac{1}{a} + \frac{a_{eq}}{a^2}}} = \frac{1}{\sqrt{\Omega_m}} \frac{1}{H_0} \frac{a}{\sqrt{a + a_{eq}}} . \tag{11.5}$$

This shows that the Hubble radius indeed grows with a.

The particle horizon then reads

$$\eta = \int \frac{d\ln a}{aH} = \frac{1}{\sqrt{\Omega_m}} \frac{1}{H_0} \int_0^a \frac{da}{\sqrt{a+a_{eq}}}$$

$$= \frac{2}{\sqrt{\Omega_m}} \frac{1}{H_0} \left(\sqrt{a+a_{eq}} - \sqrt{a_{eq}} \right) \qquad (11.6)$$

Using that matter-radiation equilibrium occurred at $z_{eq} \simeq 3400$, and recombination/decoupling at $z_{rec} \simeq 1100$ (and $1+z = \frac{a_0}{a}$), we have

$$\eta_{rec} = \frac{2}{\sqrt{\Omega_m}} \frac{1}{H_0} \left[\sqrt{\frac{1}{1100} + \frac{1}{3400}} - \sqrt{\frac{1}{3400}} \right]$$

$$\simeq 0.0175 \frac{2}{\sqrt{\Omega_m}} \frac{1}{H_0} = 0.0175\, \eta_0$$

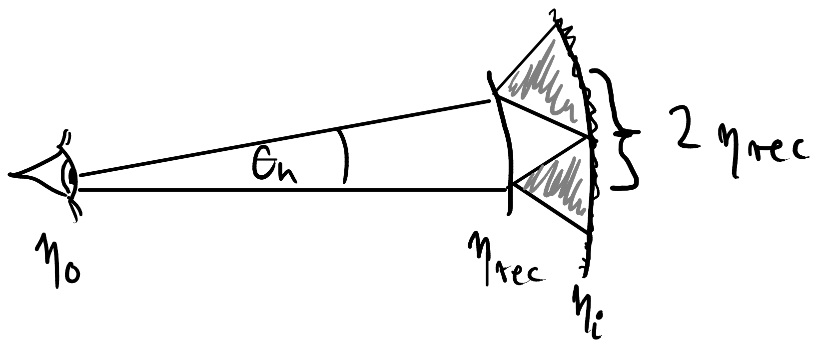
$$\eta_0 \simeq \frac{2}{\sqrt{\Omega_m}} \frac{1}{H_0} \qquad (11.7)$$

In terms of a length scale, we have

$$\eta_0 \simeq 15.1 \text{ Gpc}$$

$$\eta_{rec} \simeq 265 \text{ Gpc} \qquad (11.8)$$

Now we can ask how far apart two patches of sky must be such that they have not been in causal contact



According to the sketch, we have

$$\underline{\underline{\theta_h}} = \frac{2\, d_H(\eta_{rec})}{d_H(\eta_0 - \eta_{rec})} = \frac{2\,\eta_{rec}}{\eta_0 - \eta_{rec}}$$

$$= \frac{2 \cdot 0.0175}{1 - 0.0175} = 0.036\,(\text{rad}) \simeq \underline{\underline{2^\circ}} \tag{11.9}$$

(NB: accounting for the presence of dark energy, we would get 2.3°.

However, correcting for the fact that information in the primordial plasma does not propogate at the speed of light, but at the speed of sound, which is $c_s = \frac{c}{\sqrt{3}}$, the sound horizon is at $\theta_s = \frac{\theta_h}{\sqrt{3}} = 1.3°$) (11.10)

As a consequence, the CMB consists of very many patches that have not been in causal contact (in conventional big bang cosmology) but still shine at the same temperature.

The flatness problem

A closely related naturalness problem is the flatness problem. In order to understand it, let us define the time-dependent curvature parameter

$$\Omega_k(t) = \frac{(a_0 H_0)^2}{(a H)^2} \underset{\substack{\uparrow \\ \text{today}}}{\Omega_k} \qquad (11.11)$$

where we use that the curvature scales with $\sim \frac{1}{a^2}$ in the Friedmann equation.

In sect. 5, we discussed that Ω_k is observationally bound by $|\Omega_k| < 0.005$.

Since the Hubble radius $\frac{1}{aH}$ is growing in conventional big bang cosmology, (11.11) tells us that $\Omega_k(t)$ was even smaller in the past.

Using (11.5), we have (ignoring dark energy)

$$\Omega_k(t) = \frac{\Omega_k}{\Omega_m} \frac{a^2}{a + a_{eq}} \qquad (11.12)$$

(using the normalization $a_0 = 1$).

For instance at matter-radiation equality, where $a_{eq} \simeq \frac{1}{3400}$, we have

$$|\Omega_k(t_{eq})| = \frac{|\Omega_k|}{\Omega_m} \frac{a_{eq}}{2} < 10^{-6}. \qquad (11.13)$$

Going further backwards into the radiation dominated epoch, we find

$$|\Omega_k(t_{BBN})| < 10^{-16} \qquad (11.14)$$

at the time of big bang nucleosynthesis (a period we have data from).

This shows that the curvature must

have been finely adjusted initially to be compatible with observation nowadays.

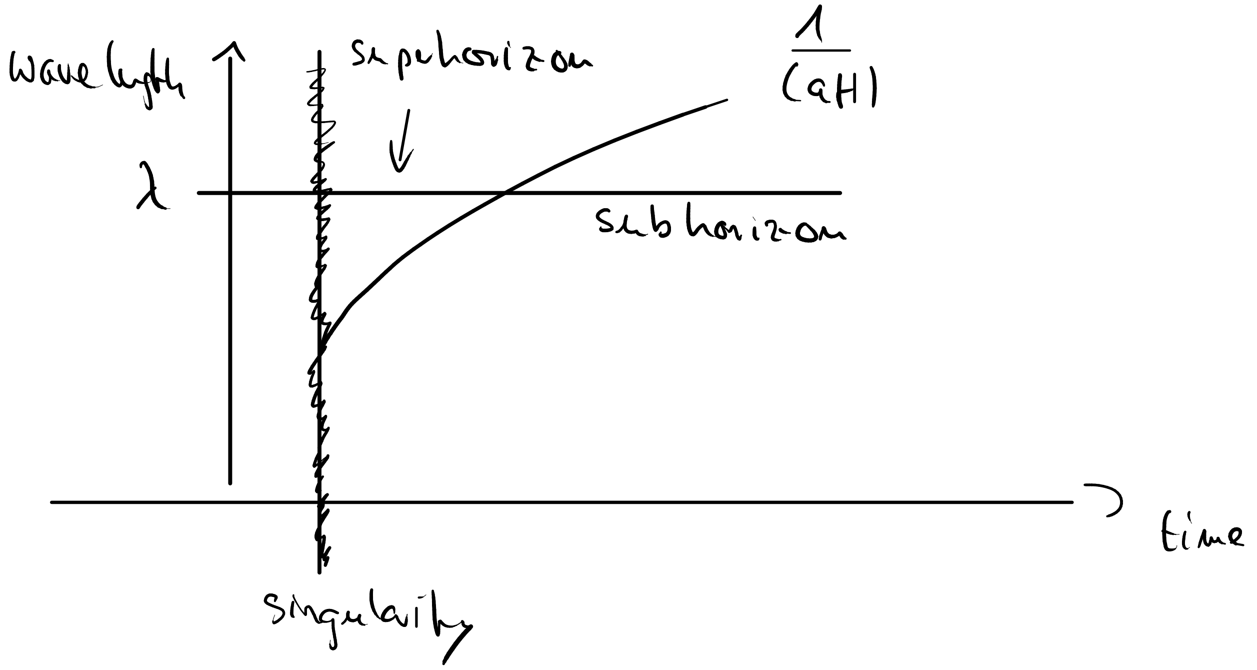
In Newtonial cosmology, the case of zero curvature requires a perfect balance of kinetic and potential energies. This implies that also the velocity distribution of all particles must have been highly fine-tuned to allow for such small curvature (in each of the causally disconnected patches).

Superhorizon Correlations

While one might be tempted to say that a flat and perfectly homogeneous universe might be singled out as a simple, symmetric and "most natural" option for the initial conditions, there is yet another naturalness problem associated with correlations of fluctuations as seen, for instance, as deviations from the perfect Planckian spectrum in the CMB.

Consider a wavelength λ characterizing a fluctuation (visible e.g. as a characteristic distance between hotter and colder spots in the CMB.

In comoving coordinates, the wavelength remains fixed, while the Hubble radius increases.



This implies that any wavelength which is within the horizon today ("subhorizon") has been outside at sufficiently early times ("super-horizon").

Now, the particle horizon at recombination / decoupling is about 265 Mpc, cf. (11.8).

Still, the CMB exhibits correlations between fluctuations on scales larger than this at recombination.

This rules out to simply postulate homogeneous and flat initial conditions for the universe, but would require to choose initial conditions that are correlated across causally disconnected regions.

Historically, another similar problem has motivated the development of scenarios that "solve" these naturalness problems: grand unified theories (GUTs) predict the existence of heavy magnetic monopoles as solitonic solutions of the field equations. In a cosmological phase transition, they would have been copiously produced yielding a finite and observable density today (which clearly has not been observed). This "monopole problem" has been a strong incentive for inflationary scenarios discussed in the next section.

Let us end this section with another remark:

In our considerations, we implicitly assumed that our equations hold down to arbitrarily small scale factors, i.e. arbitrarily close to the singularity. There is no justification for such an assumption. Rather, we expect that a quantum theory of gravity is needed to describe the period close to the singularity. Consider, e.g., the particle horizon

$$\eta = \int_0^t \frac{dt'}{a(t')} \qquad (11.15)$$

Let us introduce a time δt beyond which the classical GR description holds:

$$\eta = \underbrace{\int_0^{\delta t} \frac{dt'}{a(t')}}_{\text{Quantum gravity regime}} + \underbrace{\int_{\delta t}^{t} \frac{dt'}{a(t')}}_{\text{GR holds}} \qquad (11.16)$$

Now, it could well be that all naturalness problems discussed above are ultimately resolved by the correct theory of quantum gravity, i.e. the period between 0 and δt.

If so, we may stop with our lecture course here and wait with the further discussions until we have found this theory.

However, there are models that are consistent in the legitimate regime beyond δt that can "solve" the naturalness problem purely in the classical GR regime.

Hence, I think they are worthwhile studying, and so we continue with this lecture course.