8 Recombination and Photon Decoupling

Let us take a first look at the period when atoms formed (recombination) and the universe became transparent to photons (photon decoupling). At temperatures above 1 eV, the universe was in a plasma phase of free electrons and light nuclei (mostly protons). Electrons and protons interacted via Coulomb scattering and photons were strongly coupled to electrons via Compton scattering.
Below $T \simeq 0.3\,\text{eV}$, e^- and nuclei combined to form neutral atoms such that the density of free electrons decreased substantially.
Correspondingly, the photon mean free path grew rapidly and at some point became larger than the horizon distance ("visible universe" at the time)
Below $T \simeq 0.25\,\text{eV}$, the photons decoupled from matter, the universe became transparent, and we can observe these photons nowadays as CMB.

While we could safely neglect the chemical potential during the periods where relativistic matter dominates, it becomes useful now for the description of reaction processes.

Consider a generic particle reaction of particles p_1, p_2, p_3, and p_4 of the type

$$p_1 + p_2 \Longleftrightarrow p_3 + p_4 \qquad (8.1)$$

(for instance, in the plasma phase, we have

$$e^- + p^+ \Longleftrightarrow H + \gamma \text{, or } e^- + \gamma \Leftrightarrow e^- + \gamma \text{)}$$

To each species, we can ascribe a chemical potential μ_i, denoting the energy needed (or gained) to add one such particle to the plasma. In chemical equilibrium, we have

$$\mu_1 + \mu_2 = \mu_3 + \mu_4 \qquad (8.2)$$

For the present case, we note that the chemical potential for photons vanishes $\mu_\gamma = 0$.

Two photons can be superimposed without any cost.

(More formally, chemical potentials are associated with conserved charges, but photon number is not a conserved charge.)

This implies that the chemical potential for a particle and that for its antiparticle must be equal and opposite in sign, e.g.:

$$\mu_{e^-} + \mu_{e^+} = \mu_\gamma + \mu_\gamma = 0 , \qquad (8.3)$$

as is clear from the annihilation to two photons.

Let us apply this concept of chemical equilibrium to Hydrogen recombination (the formation of Hydrogen atoms in the universe).

NB: recombination, of course, also includes the formation of He and Li atoms. However, their recombination was completed before that of hydrogen (higher binding energy); moreover more than 90% of all protons contribute to hydrogen, hence neglecting the other elements here is justified.

$$e^- + p^+ \longleftrightarrow H + \gamma \qquad (8.4)$$

At about $T \sim 1\,eV$, the reaction keeps the particles in equilibrium. Since $T < m_e, m_p, m_H$, all matter particles are non relativistic, and their densities are given by (6.14):

$$n_{i,eq.} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{\frac{1}{T}(\mu_i - m_i)} \tag{8.5}$$

where we have included the dependence on μ_i as arising from the distribution function, and $i = e^-, p^+, H$. Since $\mu_\gamma = 0$, the chemical equilibrium reads

$$\mu_{e^-} + \mu_{p^+} = \mu_H \tag{8.6}$$

Using (8.5), we can form a ratio of densities, such that the chemical potentials drop out by (8.6):

$$\left(\frac{n_H}{n_e n_p}\right)_{eq} = \frac{g_H}{g_e g_p} \left(\frac{m_H}{m_e m_p} \frac{2\pi}{T}\right)^{3/2} e^{\frac{1}{T}(m_p + m_e - m_H)} \tag{8.7}$$

In the prefactor, we approximate $m_H \approx m_p$, but the difference is relevant in the exponent, since

$$m_p + m_e - m_H = E_I = 13.6\,\mathrm{eV} \tag{8.8}$$

is precisely the ionization energy of hydrogen. Since e^- and p^+ are non-relativistic, they feature

$$g_e = g_p = 2 \tag{8.9}$$

spin states as internal degrees of freedom.

These spins add up in hydrogen according to the angular momentum representations

$$\underline{2} + \underline{2} = \underline{1} + \underline{3} \qquad (8.10)$$

$$\uparrow \; + \; \uparrow \; = \; \uparrow\downarrow \; + \; \uparrow\uparrow$$

such that $g_H = 4$.

Since the universe is electrically neutral (at least to a very good accuracy), we have $n_p = n_e$,

$$\Rightarrow \left(\frac{n_H}{n_e^2}\right)_{eq} = \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{\frac{E_I}{T}} \qquad (8.11)$$

It is useful to describe recombination in terms of the fraction of free electrons,

$$X_e := \frac{n_e}{n_p + n_H} = \frac{n_e}{n_e + n_H} \qquad (8.12)$$

For a fully ionized universe, we have $n_H = 0$ and thus $X_e = 1$. If there are only neutral atoms, we have $X_e = 0$.

If we neglect the small number of He and Li atoms, the denominator in (8.12) can be approximated by the baryon density:

$$n_e + n_H \simeq n_b = \eta\, n_\gamma \overset{(6.10)}{=} \eta\, \frac{2\zeta(3)}{\pi^2} T^3 , \qquad (8.13)$$

where $\eta \simeq 6 \cdot 10^{-10}$ is the baryon-to-photon ratio, cf. Eq. (5.11). It is useful to consider

$$\left(\frac{1-X_e}{X_e^2}\right)_{eq} = \frac{1 - \frac{n_e}{n_e+n_H}}{\frac{n_e^2}{(n_e+n_H)^2}} = \frac{\frac{n_H}{n_e+n_H}}{\frac{n_e^2}{(n_e+n_H)^2}}$$

$$= \underbrace{\frac{n_H}{n_e^2}}_{(8.11)} \underbrace{n_b}_{(8.13)}$$

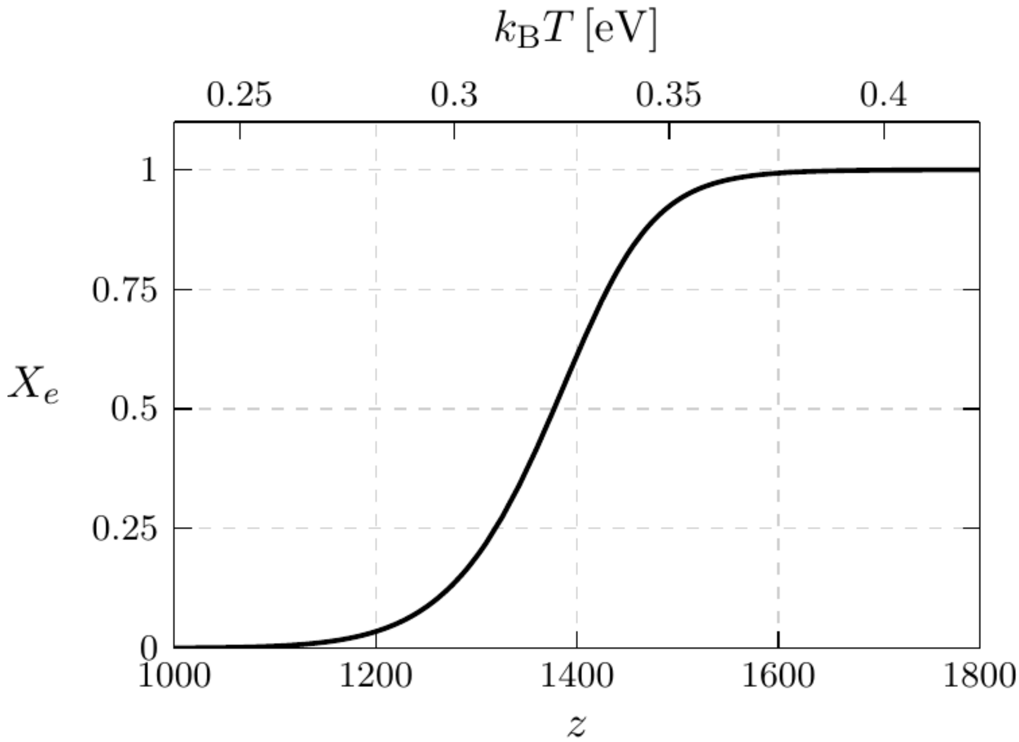
$$= \frac{2\zeta(3)}{\pi^2}\, \eta \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{\frac{E_I}{T}} \qquad (8.14)$$

This is the Saha equation.

It's solution can be written as

$$X_e = \frac{-1 + \sqrt{1+f}}{2f} , \quad f(T,\eta) = \frac{2\zeta(3)}{\pi^2}\, \eta \left(\frac{2\pi T}{m_e}\right)^{3/2} e^{\frac{E_I}{T}}$$

$$(8.15)$$



(The solution can be plotted vs. temperature, or – equivalently – vs. redshift using

$$T = T_0 (1 + z) \tag{8.16}$$

since the temperature is linked to a characteristic wavelength, say the peak of Planck's formula, which is subject to cosmological redshift as in (8.16))

As a convention, the recombination temperature T_{rec} is defined by

$$X_e(T_{rec}) = 0.5 , \tag{8.17}$$

which, using (8.15), yields

$$T_{rec}\Big|_{Saha} \simeq 0.32 \text{ eV} \simeq 3760 \text{ K} . \qquad (8.18)$$

Using (8.16), this corresponds to redshift

$$z_{rec}\Big|_{Saha} \simeq 1380 . \qquad (8.19)$$

It turns out that the Saha approximation assuming chemical equilibrium is not fully accurate. The details are rather complex and lead to a slightly delayed recombination at

$$z_{rec} \simeq 1270, \; T_{rec} \simeq 0.3 \text{eV}. \; (8.20)$$

Since recombination occurred in the matter-dominated era (matter-radiation equality was at $z_{eq} \simeq 3400$), the scale factor expanded as $a(t) = \left(\frac{t}{t_0}\right)^{2/3}$ (ignoring dark energy), which yields

$$t_{rec} = \frac{t_0}{(1+z_{rec})^{3/2}} \simeq 290\,000 \text{ years} \qquad (8.21)$$

for the time of recombination.

As is visible from the figure, the electron density $\hat{=}$ ionization fraction did not drop instantaneously. The transition corresponds to a time span of several ten thousand years.

One may wonder that the recombination temperature $T \simeq 0.3\,eV$ is substantially lower than the ionization energy $E_I = 13.6\,eV$. The reason is that Planck's black-body formula is only exponentially damped towards high frequencies $\hat{=}$ photon energies, and that the baryon to photon ratio is $\eta = 6 \cdot 10^{-10}$, i.e. for each proton there are $\sim 10^9$ photons, a few of which still have sufficiently high energy to ionize the atom above $T = 0.3\,eV$.

Let us now turn to the phenomenon of photon decoupling. Even though closely linked to recombination, the free electron ratio per se is not a good measure for photon decoupling. The relevant process coupling photons to the primordial plasma is

Compton scattering (or Thomson scattering as its low energy limit):

$$e^- + \gamma \leftrightarrow e^- + \gamma \tag{8.22}$$

With photons moving at the speed of light, the interaction rate for this process is

$$\Gamma_\gamma = n_e \sigma_T \quad , \quad \text{where} \quad \sigma_T = \frac{8\pi}{3} \frac{\alpha^2}{m^2} \tag{8.23}$$

is the Thomson cross section,

$$\sigma_T \simeq 1.7 \cdot 10^{-3}\ \mathrm{MeV}^{-2} . \tag{8.24}$$

Photon decoupling is defined by the moment when the interaction rate drops below the expansion rate of the universe

$$\Gamma_\gamma (T_{dec}) = H (T_{dec}) . \tag{8.25}$$

Since $\Gamma_\gamma \sim n_e$, the interaction rate decreases during decoupling:

$$\begin{aligned} \Gamma_\gamma (T) &= n_e \sigma_T = n_b X_e \sigma_T \\ &= \frac{2\zeta(3)}{\pi^2} \eta \sigma_T X_e(T)\ T^3 . \end{aligned} \tag{8.26}$$

The expansion rate in that period follows the matter-dominated case. We use here the Friedmann eq. in the form

$$\frac{H^2}{H_0^2} = \frac{\Omega_m}{a^3} \tag{8.27}$$

and combine it with $a \sim \frac{1}{T}$

$$\Rightarrow \quad H = H_0 \sqrt{\Omega_m} \left(\frac{T}{T_0} \right)^{3/2} = H(T) \tag{8.28}$$

to write the Hubble parameter as a function of temperature. For (8.26) = (8.28), we find for the decoupling temperature

$$X_e(T_{dec})\, T_{dec}^{3/2} \simeq \frac{\pi^2}{2\zeta(3)} \frac{H_0 \sqrt{\Omega_m}}{\eta \sigma_T T_0^{3/2}} \tag{8.29}$$

Using the Saha equation for $X_e(T_{dec})$ and the known parameters on the right-hand side, we find

$$T_{dec}\big|_{Saha} \simeq 0.27\ \text{eV} \tag{8.30}$$

Again, the Saha approximation yields a

slightly too high temperature. A more complete treatment leads to

$$T_{dec} \simeq 0.25\,eV \simeq 2940\,K, \qquad (8.31)$$

and the corresponding redshift and time of decoupling

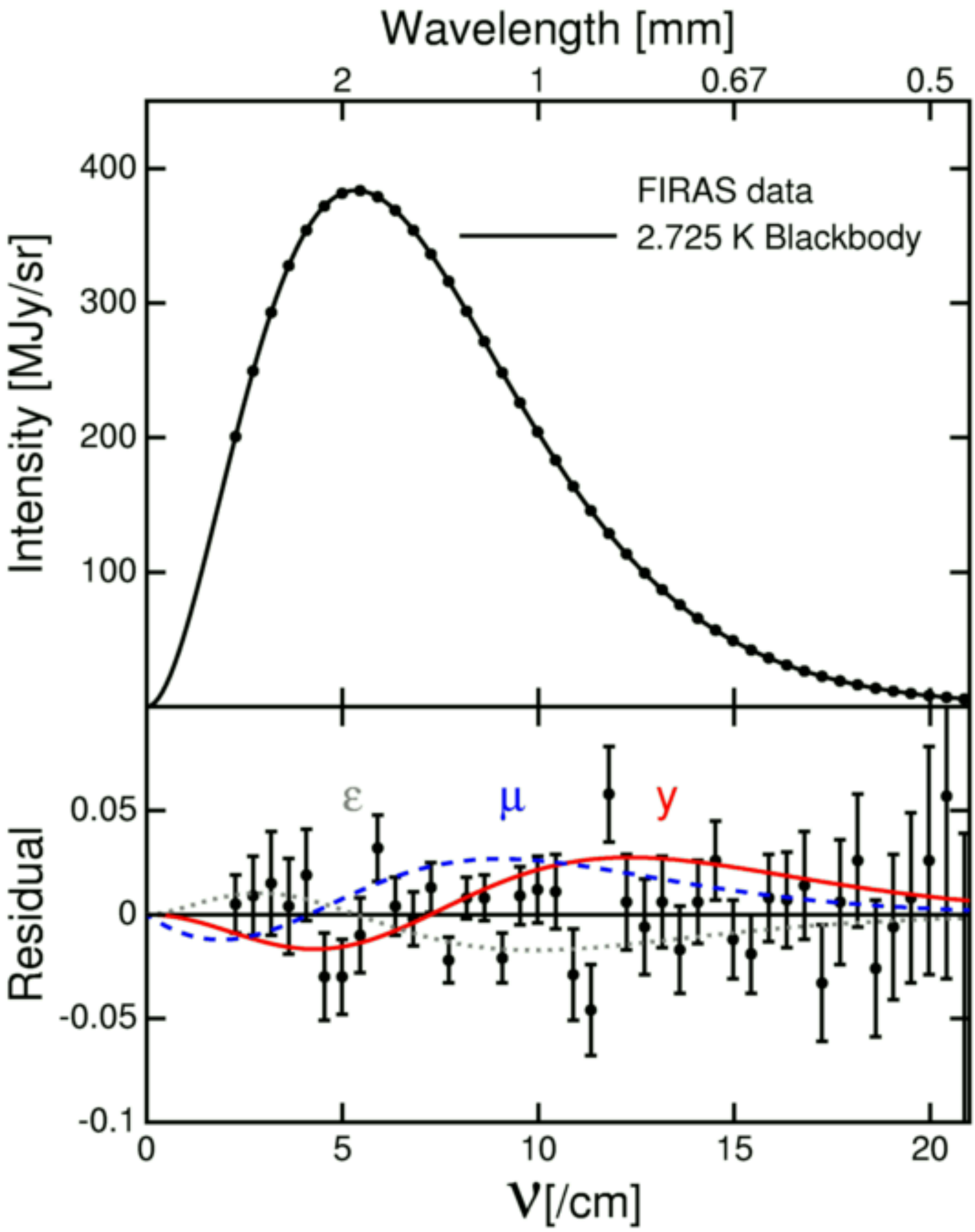
$$\begin{aligned} z_{dec} &\simeq 1100 \\ t_{dec} &\simeq 380\,000 \text{ years} \end{aligned} \qquad (8.32)$$

Even though T_{rec} and T_{dec} appear to be rather close to one another, the ionization fraction decreases significantly between recombination and decoupling:

$$X_e(T_{rec}) = 0.5, \quad X_e(T_{dec}) \simeq 0.01. \qquad (8.33)$$

This demonstrates that a large degree of charge neutrality with electrons being bound in atoms was necessary to render the universe transparent for photons.

The basic assumptions of a plasma in equilibrium leading to a period of recombination and a decoupling of the photons can be verified: if this scenario is true, the CMB must exhibit a black-body spectrum (a photon gas in equilibrium). Here is the data from the COBE satellite (using the FIRAS camera):



(almost) perfect black-body spectrum

error bars far too small to be visible in the upper plot.

Since the particles in the recombining plasma obeyed some statistical distribution, also the precise moment when a CMB photon scattered for the last time is statistically distributed. Given the scattering rate at time t, $\Gamma_\gamma(t)$, the probability for a photon to have scattered between t and today t_0 is

$$p(t) = \int_t^{t_0} \Gamma_\gamma(t)\, dt \qquad (8.34)$$

Looking backwards from today, this probability can also be interpreted as an optical depth, and the moment of last scattering can be defined as $p(t_*) = 1$.

To a good approximation, we have $t_* = t_{dec}$. However, the precise moment t_* depends on the details of the electron density and its fluctuations by the time of decoupling which is not well captured by the present equilibrium treatment.

But this consideration makes it clear that not all photons scattered at precisely the same time for the last time. Rather, this last-scattering moment will follow some distribution. For the same reason, the "surface of last scattering" should be thought of as having some thickness out of which the CMB has by now arrived in our detectors