5. Our Universe

Before we study the thermal history of the universe and the evolution of structure formation, let us briefly summarize what we actually know about the composition of the (homogeneous) universe.
For a glance at the bigger picture, let me simply list the facts which arise from many detailed and partly very precise observations without going into the observational details – and without explaining the detailed connection between observations and the underlying theory. Part of the latter is subject to the subsequent sections.

Milestones of precision cosmology are the measurements of the cosmic microwave background (CMB) by COBE (1989–1993), WMAP (2001–2010) and Planck (2009–2013, last precision data release in 2018). The CMB constitutes an almost perfect black-body spectrum characterized by a temperature ("today")

$$T_0 = 2.7255 \pm 0.0006 \text{ K}. \qquad (5.1)$$

(In fact, it is the "best" black-body spectrum ever observed.)

Thermodynamics of ideal bosonic quantum gases relates the temperature (5.1) to the photon-number density:

$$n_{\gamma,0} = 2 \, \frac{\zeta(3)}{\pi^2} \left(\frac{k_B T_0}{\hbar c} \right)^3 \simeq 410 \, \frac{\text{photons}}{\text{cm}^3} \qquad (5.2)$$

which makes the CMB one of the most intense radiation sources in astronomy. From thermodynamics, we also get the energy density:

$$u_{\gamma,0} = \rho_{\gamma,0} \, c^2 = 2 \cdot \frac{\pi^2}{30} \, \frac{(k_B T_0)^4}{(\hbar c)^3} \simeq 0.26 \, \frac{\text{eV}}{\text{cm}^3} \qquad (5.3)$$

$$\simeq 4.6 \cdot 10^{-31} \, \frac{\text{kg}}{\text{m}^3}$$

In terms of the critical density, this gives

$$\underline{\underline{\Omega_\gamma}} \simeq \underline{\underline{5.35 \cdot 10^{-5}}} \, . \qquad (5.4)$$

Since neutrinos decoupled earlier, and the photon gas was heated longer through $e^+ + e^-$ annihilation, the cosmic neutrino background ($C\nu B$) is expected to have a lower temperature, $T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0}$

(see my TD lecture notes for the details of the corresponding entropic reasoning.)

For massless neutrinos, the corresponding contribution to the energy density would follow analogously from (5.3) (more precisely, from the corresponding formula for a fermionic quantum gas).

However, neutrinos are massive. The precise mass values are not yet known, we only know the mass differences between the different species ν_e, ν_μ, ν_τ, suggesting masses on the $\mathcal{O}(1-100\,\mathrm{meV})$ scale.

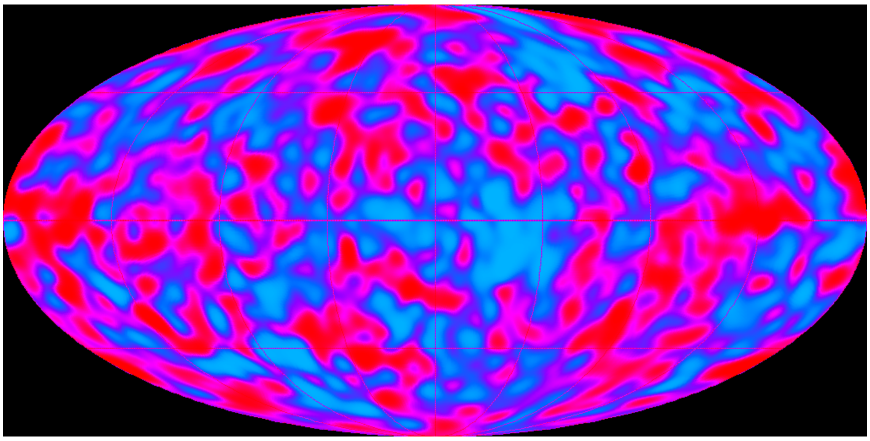
This leads to a contribution of

$$\underbrace{0.0012 <}_{\text{(finite masses)}} \Omega_\nu \underbrace{< 0.003}_{\text{cosmology (C}\nu\text{B)}} \qquad (5.5)$$

to the energy density (neutrinos are non-relativistic today and thus their energy density has decreased only with $\sim \frac{1}{a^3}$ in the recent past).

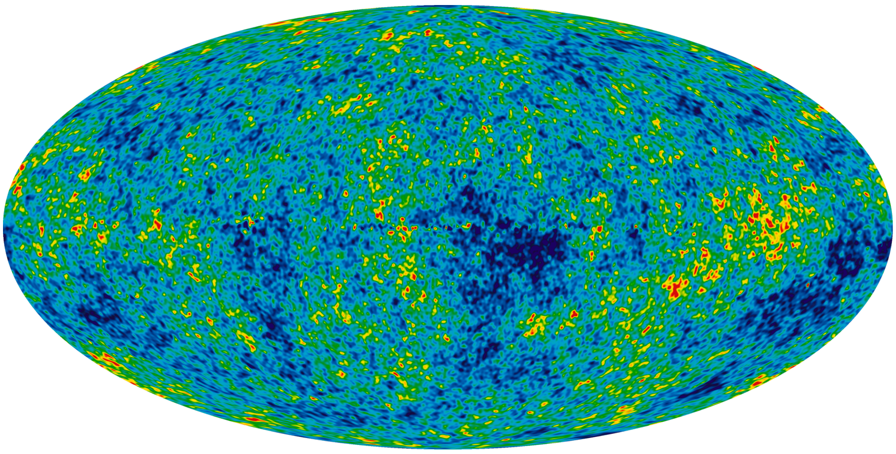
The CMB measurements also have measured with increasing accuracy that the spectrum is not perfectly Planckian, but is superimposed by fluctuations on the scale of

$\frac{\Delta T}{T} \sim 10^{-5}$. The properties of these fluctuations contain an enormous amount of information about the composition of our universe.



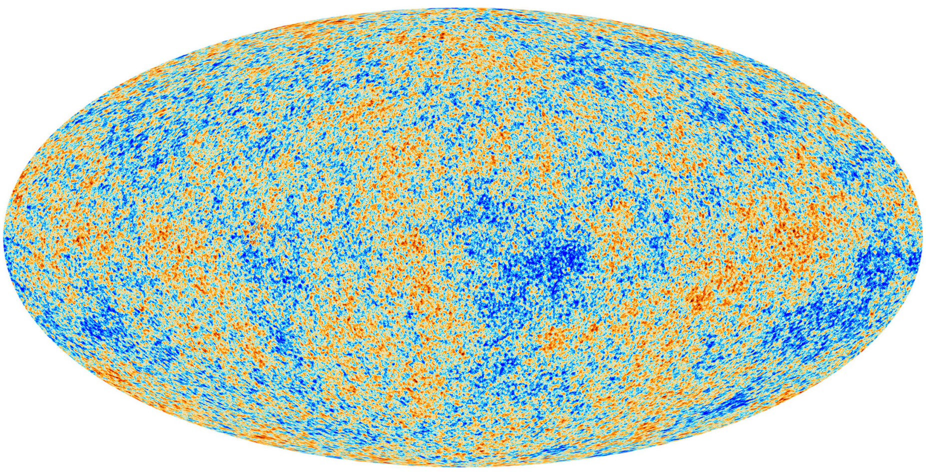
COBE

1989 – 1993



WMAP

2001 – 2010

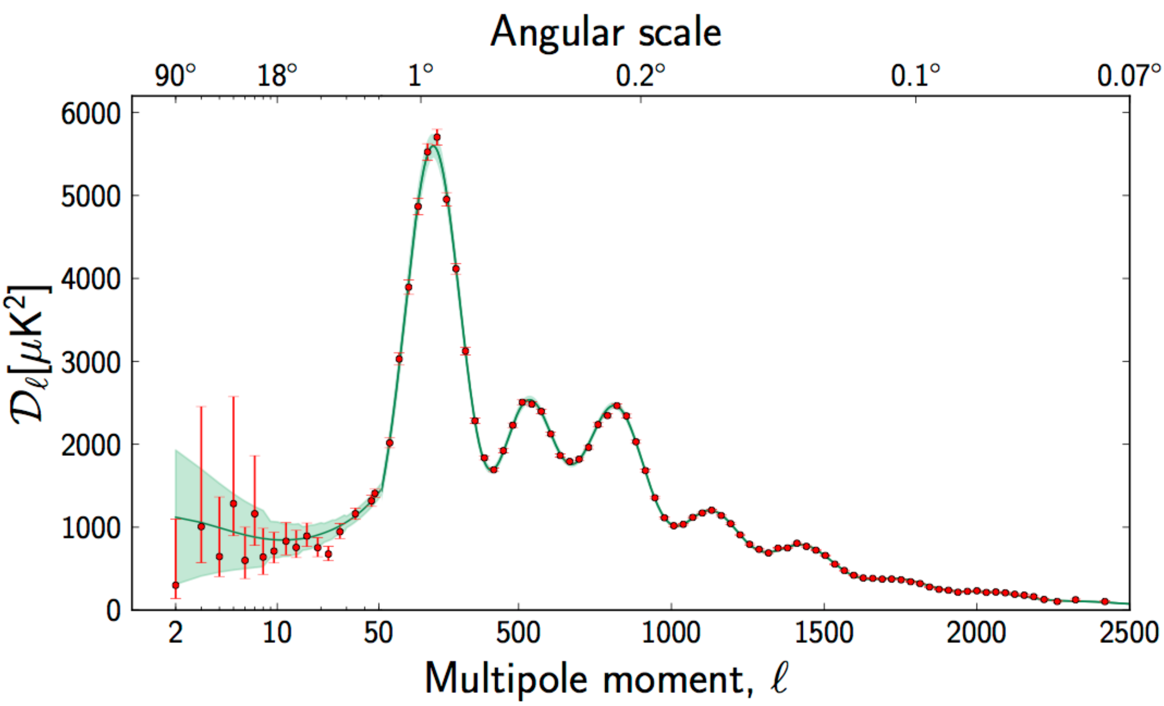


Planck

2009 – 2012

These satellite missions have charted the CMB with increasing precision, identifying the distribution of spots that are slightly hotter or cooler than (5.1).

A multipole expansion of these fluctuations allows to determine the fluctuation correlation function and the power contained in the corresponding multipoles; we will analyze the underlying physics in more detail in later sections, let me here just show the plot:



CMB power spectrum from Planck data

The peak positions and heights depend on the details of the cosmological model. Hence, a fit of the data can fix the model parameters rather accurately.

For instance, the peak positions depend on the curvature of the FLRW model. This is because the curvature changes the angular diameter distance between events at different times in the evolution of the universe. A detailed analysis suggest an upper bound on the curvature

$$|\Omega_k| < 0.005 \,. \tag{5.6}$$

Using the definition (4.14),

$$\Omega_k = - \frac{kc^2}{(R_0 H_0)^2} \tag{5.7}$$

we conclude

$$R_0 > 14\, c\, H_0^{-1} \simeq 4.2 \cdot 10^4 \, \frac{1}{h} \, \mathrm{Mpc} \,. \tag{5.8}$$

(5.6) tells us that curvature makes up less than 1% of the cosmic energy budget today. Since matter $\sim \frac{1}{a^3}$, radiation $\sim \frac{1}{a^4}$, but curvature $\sim \frac{1}{a^2}$, curvature has been even much less relevant in the past.

The density of baryons can not only be extracted from a CMB fit but is independently known from the primordial phase in which first nuclei have been synthesized (big bang nucleo-synthesis BBN). The observation of the abundances of the light elements gives thus a direct estimate for the baryon density. The most precise number, however comes from the CMB analysis:

$$\Omega_b h^2 = 0{,}02242 \pm 0{,}00014 \tag{5.9}$$

Together with the data for the Hubble constant $h = 0.6766$, we get

$$\Omega_b \simeq 0.049, \tag{5.10}$$

i.e. about 5% of the critical density. This corresponds to about 0.3 protons (H atoms) per cubic meter, $n_{b,0} \simeq 0.3\ (m^{-3})$

Comparing this to the photon number density in (5.2), we obtain the baryon-to-photon ratio,

$$\underline{\underline{\eta}} := \frac{n_b}{n_\gamma} \simeq \underline{\underline{6 \cdot 10^{-10}}}, \tag{5.11}$$

a dimensionless ratio that occurs frequently in cosmology.

Also the cold dark matter density leaves its imprints in the CMB; the Planck '18 data infers

$$\Omega_c h^2 = 0.11933 \pm 0.00091 \qquad (5.12a)$$

$$\Rightarrow \Omega_c \simeq 0.26 \qquad (5.12b)$$

which is about a quarter of the critical energy density and more than five times as much as visible (known) matter in the universe.

The total matter density thus is

$$\Omega_m \simeq 0.31 \qquad (5.13)$$

Since $\rho_m \sim \frac{\Omega_m}{a^3}$ and $\rho_r \sim \frac{\Omega_r}{a^4}$, we can determine the size of the scale factor by the time of matter radiation equilibrium:

$$a_{eq} = \frac{\Omega_r}{\Omega_m} = \frac{\Omega_\gamma \left(1 + \left(\frac{4}{11}\right)^{1/3}\right)}{\Omega_m}$$

$$\simeq 2.9 \cdot 10^{-4}. \qquad (5.14)$$

The corresponding redshift of matter-radiation equality is

$$z_{eq} = 3400. \qquad (5.15)$$

This is a bit before the universe has become transparent for photons when the CMB has been released.

From the constraint (4.19):

$$1 = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k \tag{5.16}$$

with Ω_r and Ω_k being very small and $\Omega_m \simeq 0.31$, we can infer that most of the energy density of the universe today must be in the form of dark energy $\Rightarrow \Omega_\Lambda \simeq 0.69$. In fact the CMB fit of Planck (2018) yields

$$\Omega_\Lambda = 0.6889 \pm 0.0056 \tag{5.17}$$

This is also in agreement with estimates based on Supernova Typ Ia data (luminosity distance as function of redshift).

Given the fact that we have so far understood the nature of matter and radiation to some extent, most of the energy density of the universe with $\Omega_\Lambda + \Omega_c \simeq 95\%$ is made from so far little understood substances.

Using $\Omega_m \sim \frac{1}{a^3}$ and $\Omega_\Lambda \simeq \text{const.}$, we can determine the scale factor at matter-dark energy equality:

$$a_{m\Lambda eq.} = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \simeq 0.77 \qquad (5.18)$$

which corresponds to a redshift of about

$$\underline{\underline{z_{m\Lambda eq}}} = \frac{1}{a_{m\Lambda eq}} - 1 \simeq \underline{\underline{0.3}} \qquad (5.19)$$

Now, knowing the composition of the universe, we can integrate the Friedmann equation (4.18):

$$\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

$$= \frac{1}{H_0^2} \frac{1}{a^2} (\dot{a})^2$$

$$\Rightarrow H_0 \int_0^t dt = \int_0^{a(t)} \frac{da}{\sqrt{\Omega_r a^{-2} + \Omega_m a^{-1} + \Omega_k + \Omega_\Lambda a^2}} \qquad (5.20)$$

Integrating to today $t = t_0$, where $a = a_0 = 1$, we can deduce the age of the universe t_0.

The result from (5.20), $t_0 \simeq 13.8$ billion years, matches well with the Planck result:

$$t_0 = 13.787 \pm 0.020 \text{ billion years} \quad (5.21)$$

A little surprise is found when integrating the Friedmann equation to the time of matter-dark energy equality (5.18) or matter-radiation equality (5.14).
We find

$$t_{m\Lambda\, eq} \simeq 10.2 \text{ billion years} \quad (5.22a)$$

$$t_{eq} \simeq 50\,000 \text{ years} \quad (5.22b)$$

Whereas matter-radiation equality occured comparatively early in the cosmic evolution 50 000 years after the big bang, matter-dark energy equality with dark-energy becoming dominant happened fairly recently, 2.6 billion years ago, i.e. approx. when our sun and planet earth were about 2 billion years old.
This observation - if not puzzle - why dark energy started to dominate rather close

to the present time, and thus Ω_Λ and Ω_m being still of the same order of magnitude is called the <u>cosmic coincidence problem</u>.

As many "problems" in fundamental physics, the coincidence problem is <u>not</u> a consistency problem of the theory, but rather a <u>naturalness</u> problem, i.e. it occurs unnatural to us that we are living in this period where Ω_Λ and Ω_m are similar in magnitude.