0.1 Units

Cosmological equations exhibit natural constants from those theories that are relevant building blocks:

Newton's constant	G	(gravity)
Planck's constant	$\hbar$	(quantum mechanics)
Speed of light	c	(relativity)
Boltzmann's constant	k_B	(thermodynamics)

It is therefore natural to choose our units such that these constants acquire the value

$$G = \hbar = c = k_B = 1 . \qquad (0.1)$$

For instance, $k_B = 1$ means that temperatures are measured in energy units. $c = 1$ implies that velocities are measured in units of the speed of light. $\hbar = 1$ means that actions or angular momenta are measured in units of $\hbar$; also, with photon energies satisfying $E = \hbar\omega$, we see that frequencies are also measured in energy units. Consequently, time and distances are measured in inverse energy units. With $E = mc^2 \overset{c=1}{=} m$, also masses are measured in energy units.

A typical useful energy unit familiar from particle physics is 1 GeV which approximately corresponds to the proton mass (for $c = 1$).

For $\hbar = c = 1$, Newton's constant itself carries units of $(\text{energy})^{-2}$. Hence, it can be used to define a natural energy or mass scale. Units that refer to this scale are called Planckian units.

For instance, the Planck mass is given by

$$m_P = \left(\frac{\hbar c}{G}\right)^{1/2} \simeq 2{,}177 \cdot 10^{-8}\,\text{kg} \overset{c=1}{\simeq} 1{,}221 \cdot 10^{19}\,\text{GeV} \qquad (0.2)$$

where the right-hand sides exhibit the relation to human-made units. Correspondingly, we can derive a Planck length, time and temperature:

$$\begin{aligned} l_P &= \left(\frac{G\hbar}{c^3}\right)^{1/2} \simeq 1{,}616 \cdot 10^{-35}\,\text{m} \\ t_P &= \frac{l_P}{c} \simeq 5{,}391 \cdot 10^{-44}\,\text{s} \\ T_P &= \frac{m_P c^2}{k_B} \simeq 1{,}416 \cdot 10^{32}\,\text{K} \end{aligned} \qquad (0.2)$$

A comparison to extreme human-made scales, e.g. the energy scale of the world's most powerful collider LHC at CERN $E_{LHC} = 14\,\text{TeV} = 1{,}4 \cdot 10^3\,\text{GeV}$, reveals that Planckian units correspond to ultra-high energy scales (ultra-short time/distance scales).

While Planckian units are most useful for the fundamental considerations, they may be less useful for describing specific astronomical quantities. For instance, distances in astronomy are usually measured in parsecs or Mega parsecs,

$$1\,\text{pc} = 3.26 \text{ light years} \simeq 3{,}086 \cdot 10^{16}\,\text{m}$$
$$1\,\text{Mpc} = 10^6\,\text{pc} \qquad (0.3)$$

Here is a list of typical length scales

	km	Mpc
Earth	6371	$2.1 \cdot 10^{-16}$
Sun	696340	$2.3 \cdot 10^{-14}$
Solar System	$1.5 \cdot 10^{8}$	$4.9 \cdot 10^{-11}$
Milky Way	$1.0 \cdot 10^{18}$	0,032
Local Group	10^{20}	3
Local Supercluster	$5 \cdot 10^{21}$	150
(visible) Universe	$5 \cdot 10^{23}$	14000

Masses of astronomical objects (stars, black holes, galaxies, etc.) are typically expressed in terms of the mass of the sun

$$M_\odot \simeq 1{,}989 \cdot 10^{30}\,\text{kg} \qquad (0.4)$$

Finally, when we couplings between light and charged matter, we use the Heaviside-Lorentz unit system in which the Coulomb force law between two electrons of charge e reads

$$\vec{F} = \frac{\alpha}{r^2} \hat{e}_r \tag{0.5}$$

with the fine structure constant $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$.