Pair production of phonons in Bose-Einstein condensates with curved and expanding acoustic metric

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Team & publications


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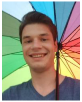
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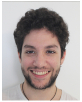
Álvaro Parra-López


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Tobias Haas


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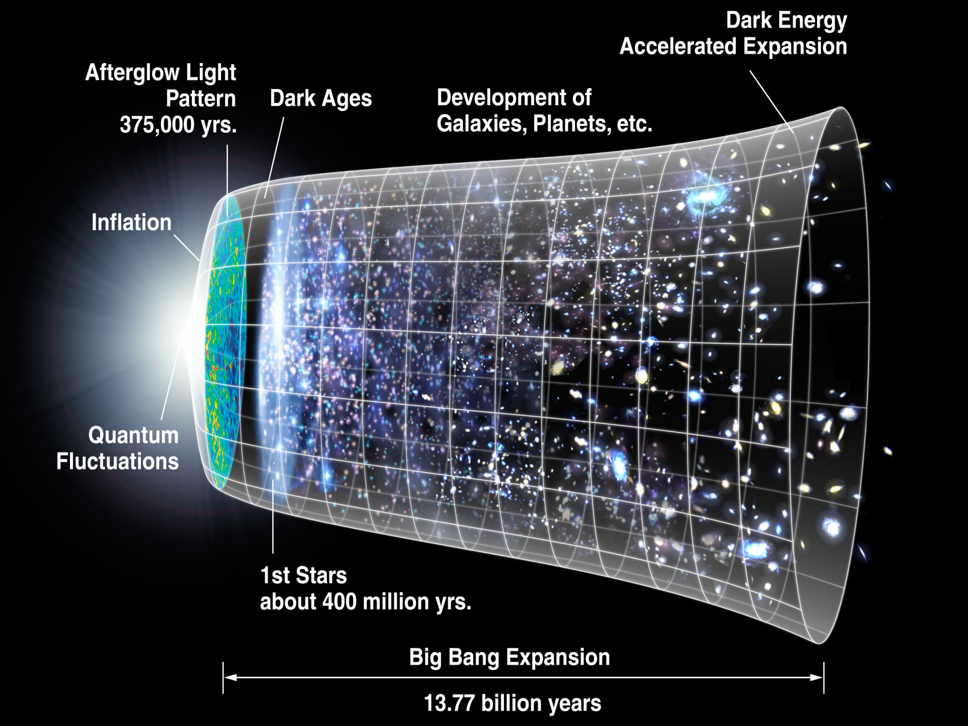
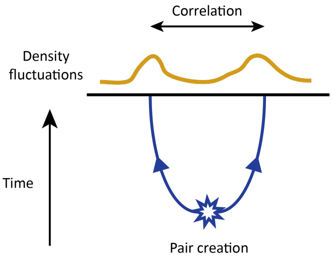
- Celia Viermann, Marius Sparn, Nikolas Liebster, Maurus Hans, Elinor Kath, Álvaro Parra-López, Mireia Tolosa-Simeón, Natalia Sánchez-Kuntz, Tobias Haas, Helmut Strobel, Stefan Floerchinger, Markus K. Oberthaler, *Quantum field simulator for dynamics in curved spacetime*, arXiv:2202.10441
- Mireia Tolosa-Simeón, Álvaro Parra-López, Natalia Sánchez-Kuntz, Tobias Haas, Celia Viermann, Marius Sparn, Nikolas Liebster, Maurus Hans, Elinor Kath, Helmut Strobel, Markus K. Oberthaler, Stefan Floerchinger, *Curved and expanding spacetime geometries in Bose-Einstein condensates*, arXiv:2202.10399
- Natalia Sánchez-Kuntz, Álvaro Parra-López, Mireia Tolosa-Simeón, Tobias Haas, Stefan Floerchinger, *Scalar quantum fields in cosmologies with 2+1 spacetime dimensions*, Phys. Rev. D 105, 105020 (2022)


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years

Quantum origin of fluctuations

- Universe was almost homogeneous at early times
- small fluctuations magnified by gravitational attraction
- primordial fluctuations most likely quantum fluctuations maginfied by inflation

  [Mukhanov & Chibisov (1981), Hawking (1982), Starobinsky (1982), Guth & Pi (1982), Bardeen, Steinhardt & Turner (1983), Fischler, Ratra & Susskind (1985)]

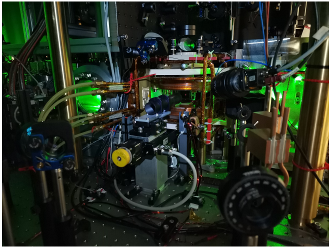


Non-relativistic quantum fields

- Bose-Einstein condensate in two dimensions
  [Gross (1961), Pitaevskii (1961)]

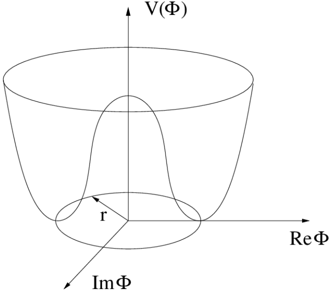
$$\Gamma[\Phi] = \int \mathrm{d}t\, \mathrm{d}^2 x \Big\{ \hbar \Phi^*(t,\mathbf{x}) \left[i\frac{\partial}{\partial t} - V(t,\mathbf{x}) \right] \Phi(t,\mathbf{x}) \\ - \frac{\hbar^2}{2m} \boldsymbol{\nabla}\Phi^*(t,\mathbf{x}) \boldsymbol{\nabla}\Phi(t,\mathbf{x}) - \frac{\lambda(t)}{2} \Phi^*(t,\mathbf{x})^2 \Phi(t,\mathbf{x})^2 \Big\}$$

- traping potential $V(t,\mathbf{x})$ and coupling strength $\lambda(t)$
- can be realized and controlled experimentally



[Oberthaler group, KIP Heidelberg]

Superfluid and small excitations



- Complex non-relativistic field can be decomposed

$$\Phi = e^{iS_0}\left(\sqrt{n_0} + \frac{1}{\sqrt{2}}\left[\phi_1 + i\phi_2\right]\right)$$

- real fields ϕ_1 and ϕ_2 describe excitations on top of the superfluid
- stationary superfluid density $n_0(\mathbf{x})$ and vanishing superfluid velocity

$$\mathbf{v} = \frac{\hbar}{m}\boldsymbol{\nabla} S_0 = 0$$

Sound waves / phonons

- small energy excitations are sound waves or phonons
- propagate with finite velocity, similar to light
- local speed of sound

$$c_S(t, \mathbf{x}) = \sqrt{\frac{\lambda(t)\, n_0(\mathbf{x})}{m}}$$

- sound waves propagate along

$$ds^2 = -dt^2 + \frac{1}{c_S(t, \mathbf{x})^2} (d\mathbf{x} - \mathbf{v}\, dt)^2 = 0$$

- acoustic metric for $\mathbf{v} = 0$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{c_S(t,\mathbf{x})^2} & 0 \\ 0 & 0 & \frac{1}{c_S(t,\mathbf{x})^2} \end{pmatrix}$$

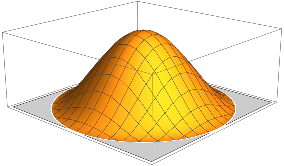
Relativistic scalar field

- low energy theory for phonons (with $\phi = \phi_2/\sqrt{2m}$)

$$\Gamma[\phi] = \int \mathrm{d}t\, \mathrm{d}^2x\, \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\}$$

- metric determinant $\sqrt{g} = \sqrt{-\det(g_{\mu\nu})}$
- acoustic metric depends on space and time like the space-time metric in Einsteins theory of general relativity !
- phonons behave like real, massless, relativistic scalar field in a curved spacetime !

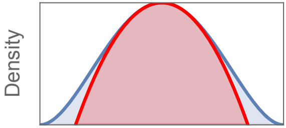
Density profiles



- assume specifically for $r = |\mathbf{x}| < R$

$$n_0(r) = \bar{n}_0 \times \left[1 - \frac{r^2}{R^2}\right]^2$$

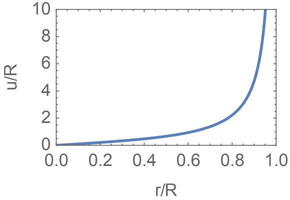
- experimental realization with optical trap and digital micromirror device
- approximate realization in harmonic trap



Acoustic spacetime geometry

- variable transform to $0 \leq u < \infty$

$$u(r) = \frac{r}{1 - \frac{r^2}{R^2}}$$



- leads to Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2\right)$$

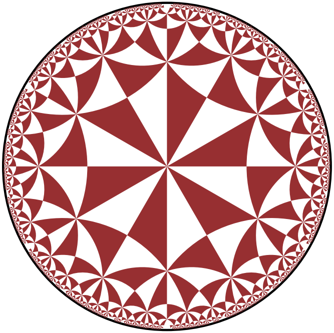
- negative spatial curvature

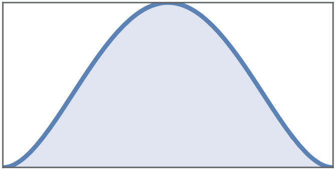
$$\kappa = -4/R^2$$

- scale factor

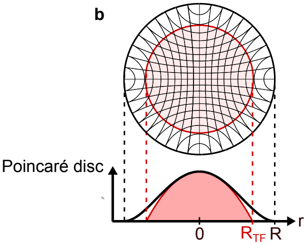
$$a(t) = \sqrt{\frac{m}{\bar{n}_0}\frac{1}{\lambda(t)}}$$

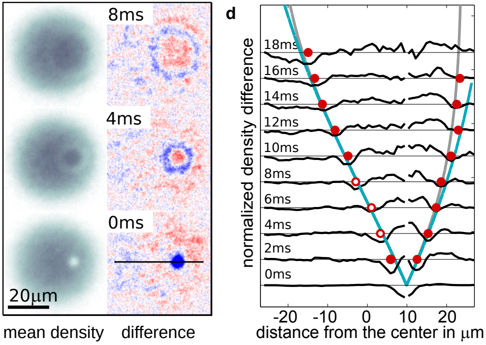
Hyperbolic geometry



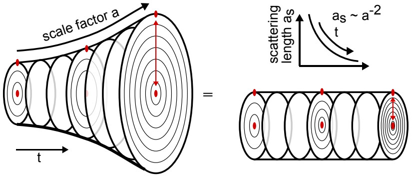


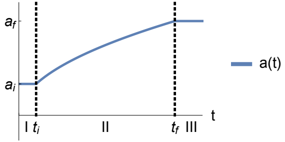
Experimental realization in a Bose-Einstein condensate





Particle production





- time-dependent scattering length induces time-dependent metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1-\kappa u^2} + u^2 d\varphi^2\right)$$

- particle concept works well in regions I and III but not in region II
- vacuum state in region I leads to state with particles in region III
- expanding space produces particles !
- analytic calculations possible for power law scale factors

$$a(t) = \text{const} \times t^\gamma$$

Mode functions and Bogoliubov transforms

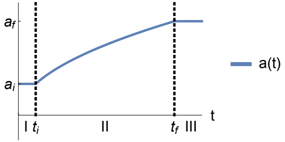
- field gets expanded in modes

$$\phi(t,u,\varphi) = \int_{k,m} \left[\hat{a}_{km}\mathcal{H}_{km}(u,\varphi)v_k(t) + \hat{a}^\dagger_{km}\mathcal{H}^*_{km}(u,\varphi)v^*_k(t)\right]$$

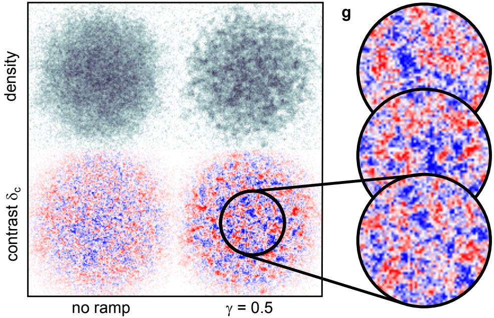
- spatial part $\mathcal{H}_{km}(u,\varphi)$ can be expressed in terms of spherical harmonics at complex angular momenta in hyperbolic geometry
- mode functions satisfy

$$\ddot{v}_k(t) + 2\frac{\dot{a}(t)}{a(t)}\dot{v}_k(t) + \frac{k^2+|\kappa|/4}{a^2(t)}v_k(t) = 0$$

- vacuum state only unique for $\dot{a}(t)=0$ where $v_k(t)\sim e^{-i\omega_k t}$
- Bogoliubov transforms between different choices of $\hat{a}_{km}$ and vacuum states



Observation of particle production

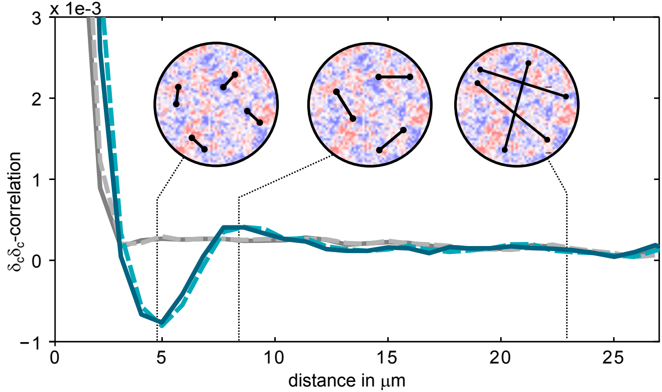


- rescaled density contrast

$$\delta_c(t,\mathbf{x}) = \sqrt{\frac{n_0(\mathbf{x})}{\bar{n}_0^3}}\,[n(t,\mathbf{x}) - n_0(\mathbf{x})] \sim \partial_t \phi(t,\mathbf{x})$$

- allows to test relativistic scalar field

Density contrast correlation function

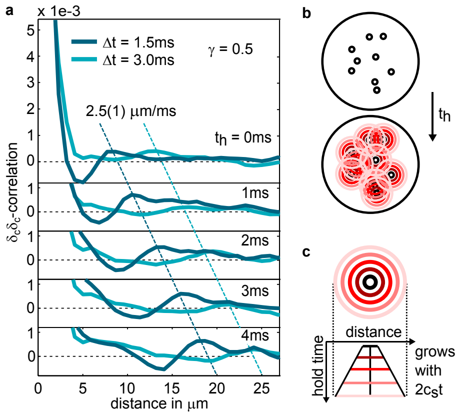


- correlation function

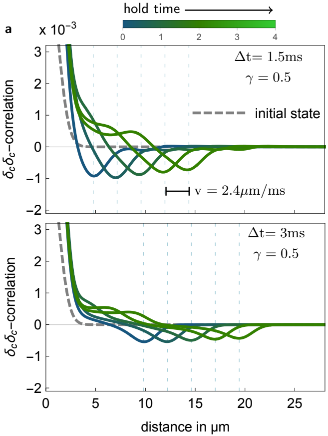
$$\langle \delta_c(\mathbf{x})\delta_c(\mathbf{y}) \rangle$$

- before and after expansion

Time dependent correlation functions after expansion



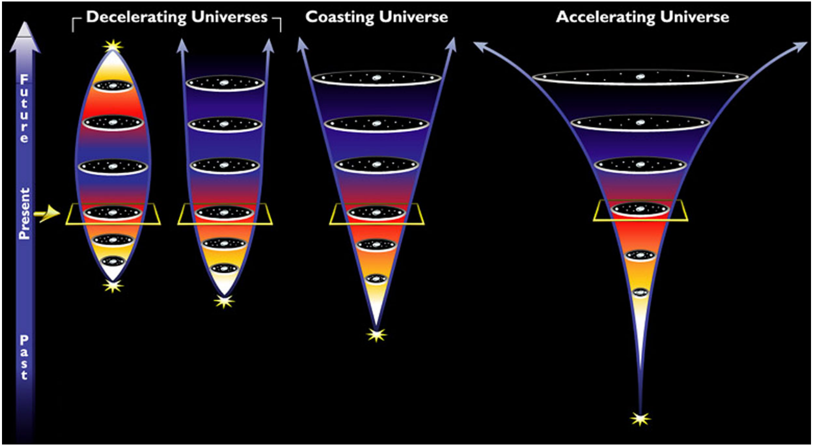
Experiment



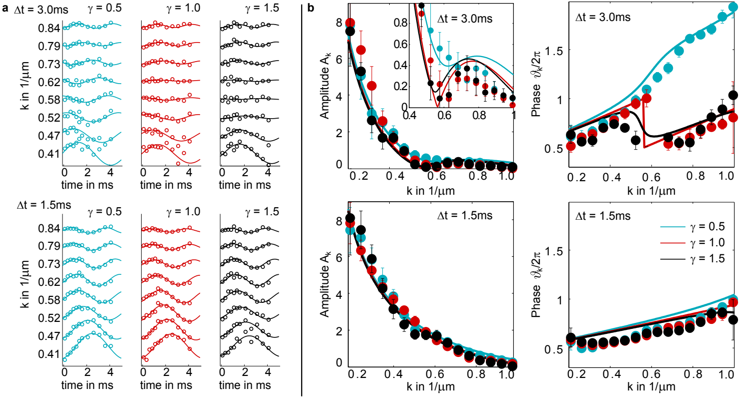
Theory

- analgous to baryon accoustic or Sakharov oscillations in cosmology
- optical resolution important for detailed shape

Expansion history



Oscillations in Fourier space



- Fourier spectrum of excitations

$$S_k(t) = \frac{1}{2} + N_k + A_k \cos(2\omega_k(t - t_\mathrm{f}) + \vartheta_k)$$

- decelerated, coasting and accelerated expansion
- good agreement with analytic theory (solid lines)

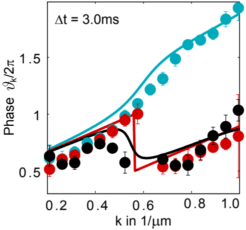
Quantum recurrences

- uniform expansion with $a(t) = Qt$ is special
- shows quantum recurrences of the incoming vacuum state at special values of wavenumber k

$$k_n = \frac{a_\mathsf{f} - a_\mathsf{i}}{\Delta t}\left[\left(\frac{n\pi}{\ln\left(a_\mathsf{f}/a_\mathsf{i}\right)}\right)^2 + \frac{1}{4}\right]^{\frac{1}{2}},$$

with integer $n = 1, 2, 3, \ldots$

- at these points one has trivial Bogoliubov coefficient $\beta_k = 0$
- can be seen experimentally as a discontinuity in the phase !

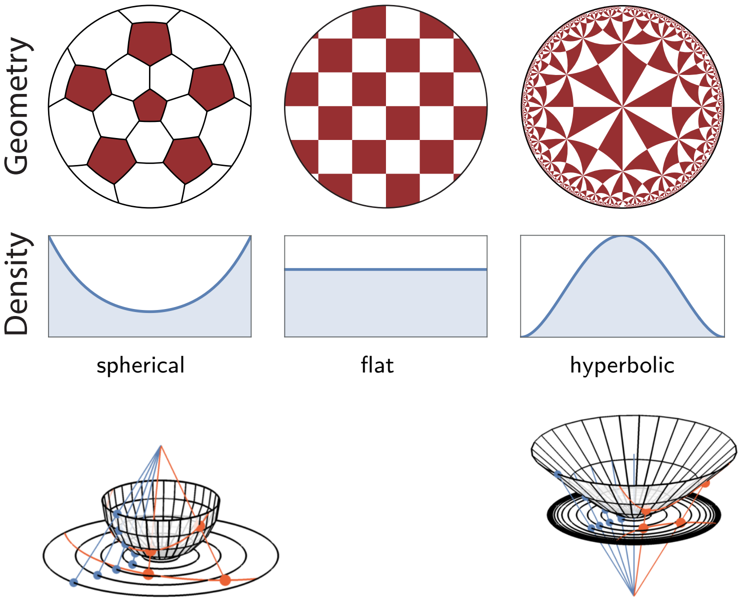


Conclusion

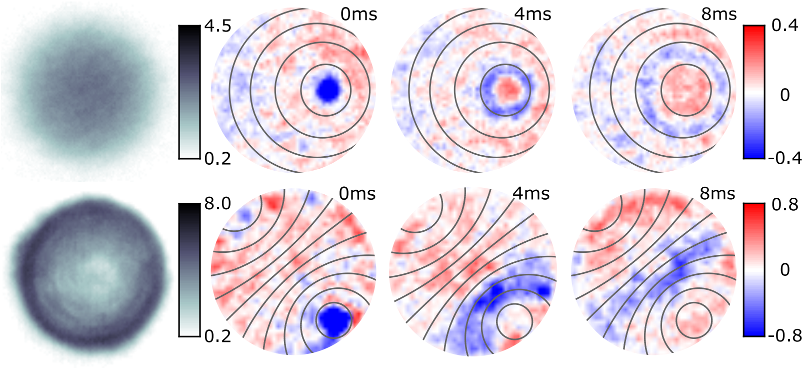
- Bose-Einstein condensates can be quantum simulators for quantum fields in curved spacetime
- Symmetric spaces with constant curvature can be realized with specific radial density profiles
- Experimental realization in two spatial dimensions
- Time-dependent coupling allows to simulate expansion
- Particle production by time-dependent scale factor
- Oscillations after expansion allow detailed investigations
- Quantum information theoretic aspects also accessible
- Extensions to three dimensions, other geometries, other field content, and more, are possible

Backup

Geometries with constant spatial curvature



Propagating sound waves



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