Low-p_T photon and di-lepton rates & electric conductivity

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Electric current

- quarks are charged and carry electric charge
- four-current composed of net charge density and current density

$$J^\mu(t, \mathbf{x}) = (\rho(t, \mathbf{x}),\ \mathbf{j}(t, \mathbf{x}))$$

- source for electro-magnetic field A_μ in Maxwell equations
- expectation value and fluctuation part

$$J^\mu(x) = \langle J^\mu(x) \rangle + \delta J^\mu(x)$$

- expectation value from net charge of quark-gluon plasma
- initial state, thermal and quantum fluctuations

Correlation and response functions in thermal equilibrium

- statistical correlation function $\Delta^{\mu\nu}_{(S)}(\omega, \mathbf{p})$ defined by

$$\frac{1}{2}\left\langle \delta J^\mu(t_1, \mathbf{x}_1)\delta J^\nu(t_2, \mathbf{x}_2) + \delta J^\nu(t_2, \mathbf{x}_2)\delta J^\mu(t_1, \mathbf{x}_1)\right\rangle$$
$$= \int_{\omega,\mathbf{p}} e^{-i\omega(t_1-t_2)+i\mathbf{p}(\mathbf{x}_1-\mathbf{x}_2)}\Delta^{\mu\nu}_{(S)}(\omega, \mathbf{p})$$

- quantifies amount of thermal and quantum fluctuations

- spectral function $\Delta^{\mu\nu}_{(\rho)}(\omega, \mathbf{p})$ defined by

$$\left\langle \delta J^\mu(t_1, \mathbf{x}_1)\delta J^\nu(t_2, \mathbf{x}_2) - \delta J^\nu(t_2, \mathbf{x}_2)\delta J^\mu(t_1, \mathbf{x}_1)\right\rangle$$
$$= \int_{\omega,\mathbf{p}} e^{-i\omega(t_1-t_2)+i\mathbf{p}(\mathbf{x}_1-\mathbf{x}_2)}\Delta^{\mu\nu}_{(\rho)}(\omega, \mathbf{p})$$

- response of current to change in electro-magnetic field $A_\mu(t_2, \mathbf{x}_2)$

- both functions depend also on temperature T
- definitions extend beyond equilibrium

The fluctuation-dissipation relation

- close to thermal equilibrium one has fluctuation-dissipation relation

$$\Delta^{\mu\nu}_{(\mathsf{S})}(\omega, \mathbf{p}) = \left[\frac{1}{2} + \frac{1}{e^{\omega/T} - 1}\right] \Delta^{\mu\nu}_{(\rho)}(\omega, \mathbf{p})$$

- statistical correlation function $\Delta^{\mu\nu}_{(\mathsf{S})}(\omega, \mathbf{p}) \to$ fluctuation
- spectral function $\Delta^{\mu\nu}_{(\rho)}(\omega, \mathbf{p}) \to$ dissipation
- contains Bose-Einstein distribution factor

$$\left[\frac{1}{2} + \frac{1}{e^{\omega/T} - 1}\right] \to \frac{T}{\omega} \quad (T \gg w)$$

- would be very interesting to test! (test of equilibration)

Electrical conductivity of the quark-gluon plasma

- for isotropic plasma and weak electric field $\mathbf{E}$
- Ohm's law for current

$$\mathbf{J} = \sigma_0 \mathbf{E}$$

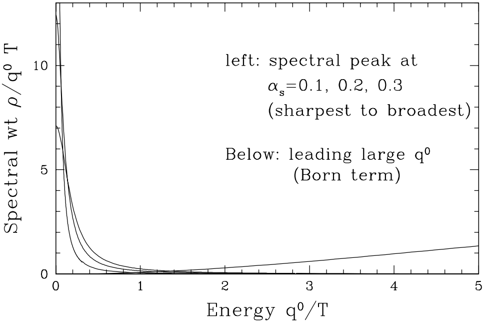
with electric conductivity σ_0

- from linear response theory

$$\sigma_0 = \frac{1}{3} \lim_{\omega/T \to 0} \frac{\Delta^{\mu}_{(\rho)\mu}(\omega, \mathbf{p} = 0)}{\omega}$$

- electric conductivity related to low frequency limit of the spectral function
- could also be probed through transport of electric charge

Transport peak in spectral function and electric conductivity



[Moore & Robert (2006)]

- spectral weight $\rho = \Delta^{\mu}_{(\rho)\mu}(p^0, \mathbf{p} = 0)$
- zero crossing of transport peak at $\omega/T \to 0$ determines conductivity

$$\sigma_0 = \frac{1}{3} \lim_{\omega/T \to 0} \frac{\Delta^{\mu}_{(\rho)\mu}(\omega, \mathbf{p} = 0)}{\omega}$$

Thermal photon and di-lepton rates

- photon rate

$$\omega \frac{dN_{\text{photons}}}{d^3p\,dtd^3x} = \frac{1}{16\pi^3}\left[\Delta^{\mu}_{(\text{S})\mu}(\omega,\mathbf{p}) - \text{vacuum expr.}\right]$$

- thermal di-lepton rate (without threshold functions)

$$\frac{dN_{\text{di-leptons}}}{d\omega d^3p\,dtd^3x} = \frac{\alpha}{24\pi^4(-\omega^2+\mathbf{p}^2)}\left[\Delta^{\mu}_{(\text{S})\mu}(\omega,\mathbf{p}) - \text{vacuum expr.}\right]$$

- allows to probe statistical current-current correlation function
- related to spectral function through fluctuation-dissipation relation

$$\Delta^{\mu\nu}_{(\text{S})}(\omega,\mathbf{p}) = \left[\frac{1}{2} + \frac{1}{e^{\omega/T}-1}\right]\Delta^{\mu\nu}_{(\rho)}(\omega,\mathbf{p})$$

Perturbative production rates at next-to-leading order

- photons: use perturbative calculation on the thermal photon rate from [Ghiglieri, Moore et al. (2013)] up to NLO with LPM resummation

$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{LO} = \left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\text{hard}} + \left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\text{soft}} + \left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\text{coll}}$$

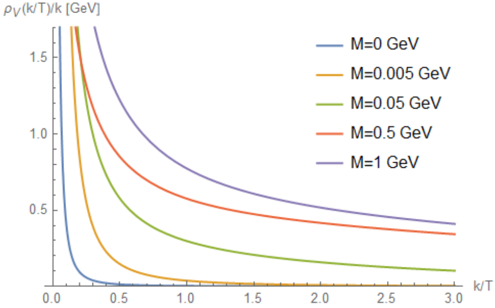
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{LO+NLO} = \left.\frac{d\Gamma_\gamma}{d^3k}\right|_{LO} + \frac{d\delta\Gamma_\gamma}{d^3k},$$

- di-leptons: use perturbative calculation of the di-lepton rate from [Laine 2014] up to NLO with LPM resummation

We use existing perturbative calculations to extract and interpolate spectral function from production rates.

Spectral function from perturbative calculations

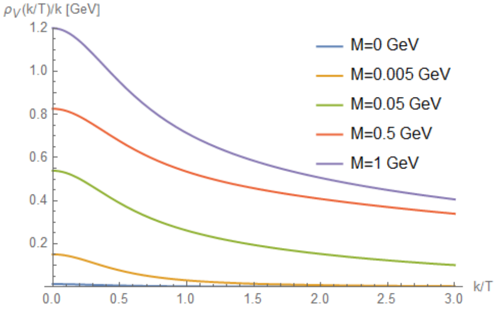
Spectral weight as function of frequency



- "infinite" spectral peak
- formally infinite conductivity, $\sigma_0 \to \infty$, from perturbative calculations

Modified spectral function

Inspired by lattice QCD calculations, we modify the perturbative result by a parameter s such that electric conductive σ_0 remains finite.



Modified spectral function fit for s=0.5

s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Values of the parameter s and the corresponding values of conductivity

Soft thermal photon rate: qualitative expectations

- in the soft limit $\omega = |\mathbf{p}| \ll T$ one has

$$\Delta^{\mu}_{(\rho)\mu}(\omega, \mathbf{p}) \to 3\omega\sigma_0, \qquad \frac{1}{e^{\omega/T} - 1} \to \frac{T}{\omega},$$

  so that

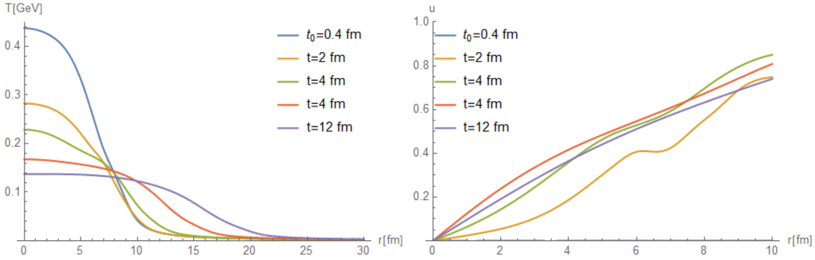
$$\omega \frac{dN_{\text{photons}}}{d^3p} \to \frac{3\sigma_0\, T}{16\pi^3} \int dt d^3x$$

- intersect of photon spectrum for small transverse moment roughly proportional to electric conductivity!
- can be made more precise by fluid dynamic calculations
- compare this to leading order soft (Low) theorem prediction for $\omega \ll 1/\tau_{\text{formation}}$

$$\omega \frac{dN_{\text{photons}}}{d^3p} \sim \frac{1}{\omega^2}$$

Fluid dynamics

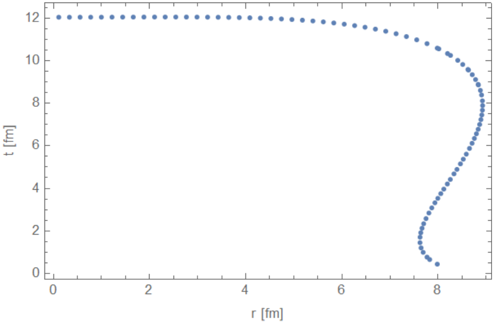
Integrate over the QGP fire ball using $T(r,t)$ and $u(r,t)$ from FluiduM [Floerchinger et al. 2019] (centrality class 0-5%)



Temperature and fluid-velocity calculated with FluiduM for selected times, for a $\sqrt{s} = 2.76TeV$ Pb-Pb-collision

Freeze-out surface from FuiduM

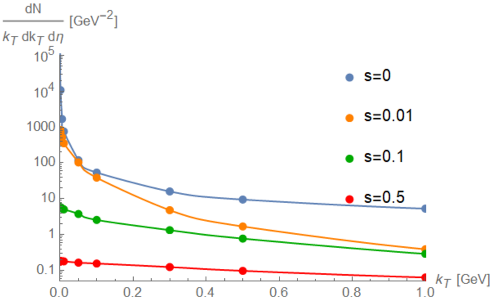
Freeze out surface: Surface in (r,t) after which particles don't interact any more (kinetical freeze out). We integrate production rate up to freeze-out surface.



Freeze-out surface for $T_{fo} = 140\, MeV$

Integrated photon rate

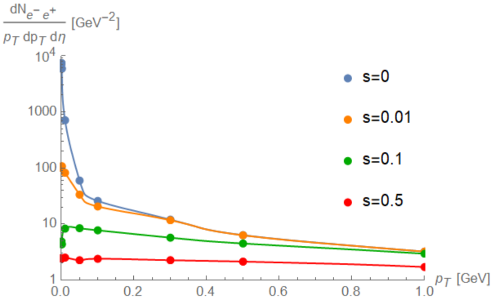
Integrated photon rate for different electrical conductivities



s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Integrated electron rate

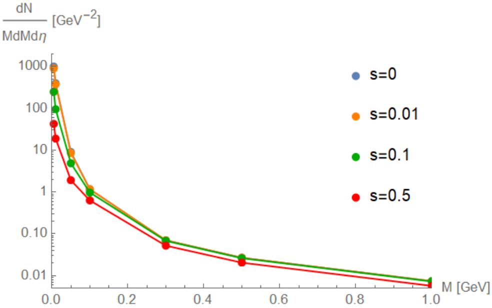
Integrated electron rate in dependence of transverse momentum p_T



s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Integrated electron rate

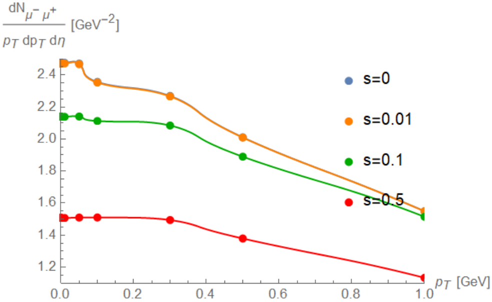
Integrated electron rate in dependence of the invariant mass M



s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Integrated myon rate

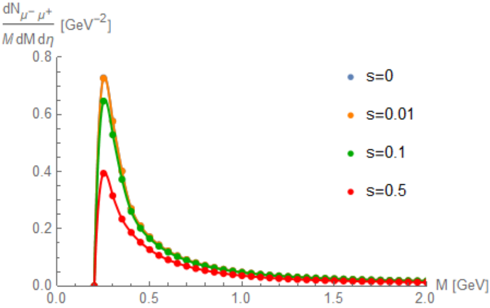
Integrated myon rate in dependence of transverse momentum p_T



s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Integrated myon rate

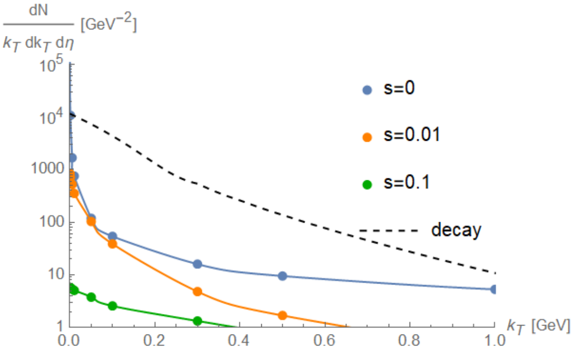
Integrated myon rate in dependence of invariant mass M



s	0.01	0.1	0.5
σ_0/T	1.81	0.013	0.0004

Photons from decays

Large contribution of photons from $\pi^0 \to \gamma\gamma$
Determined with FluiduM+FastReso



Conclusions

- soft di-leptons and photons for quark-gluon plasma allow to constrain electric conductivity
- photon and electron spectra show visible dependence on conductivity

$$\lim_{p_T \to 0} \frac{dN}{p_T dp_T d\eta d\phi} \sim \sigma_0$$

- need to reach values of $p_T < 0.1$ GeV
- only some averaged conductivity can be obtained from particle spectra, while in general it depends on temperature

$$\sigma_0 = \sigma\left(T(r,t)\right)$$

- thermal photon contribution must be disentangled from decay photons