Thermal excitation spectrum from entanglement in an expanding quantum string
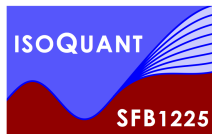
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based on

- J. Berges, S. Floerchinger & R. Venugopalan, *Thermal excitation spectrum from entanglement in an expanding QCD string* [arxiv:1707.05338]
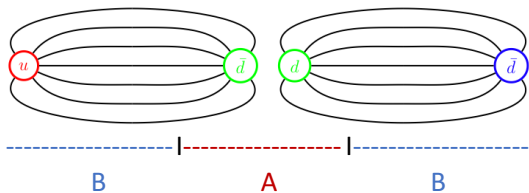- J. Berges, S. Floerchinger & R. Venugopalan [to appear]

Motivation

- Elementary particle collision experiments such as $e^+ \, e^-$ collisions show thermal-like features.
- Example: particle multiplicities
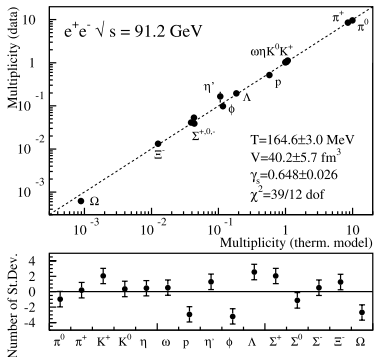
[Becattini, Casterina, Milov & Satz, EPJC 66, 377 (2010)]

- Conventional thermalization by collisions unlikely.
- Alternative explanations needed.

QCD strings

- Particle production from QCD strings.
- e. g. Lund model (Pythia).
- Different regions in a string are entangled.
- Subinterval A is described by reduced density matrix

$$\rho_A = \mathsf{Tr}_B \rho.$$

- Reduced density matrix is of mixed state form.
- Could this lead to thermal-like effects?

Microscopic model

- QCD in 1+1 dimensions described by 't Hooft model

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - ig\mathbf{A}_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{2}\text{tr}\,\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}$$

- Fermionic fields ψ_i with sums over flavor species $i = 1, \ldots, N_f$
- SU(N_c) gauge fields $\mathbf{A}_\mu$ with field strength tensor $\mathbf{F}_{\mu\nu}$
- Gluons are not dynamical in two dimensions
- Gauge coupling g has dimension of mass
- Non-trivial, interacting theory, cannot be solved exactly
- Spectrum of excitations known for $N_c \to \infty$ with $g^2 N_c$ fixed

Schwinger model

- QED in 1+1 dimension

$$\mathscr{L} = -\bar{\psi}_i \gamma^\mu (\partial_\mu - iqA_\mu)\psi_i - m_i \bar{\psi}_i \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Geometric confinement
- U(1) charge related to string tension $q = \sqrt{2\sigma}$
- For single massless fermion one can **bosonize theory** exactly

$$S = \int d^2x \sqrt{g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} M^2 \phi^2 \right\}$$

or as a Hamiltonian (cf. Luttinger liquid)

$$H = \int dx \left\{ \frac{1}{2} \Pi^2 + \frac{1}{2} \left(\frac{\partial}{\partial x} \phi \right)^2 + \frac{1}{2} M^2 \phi^2 \right\}$$

- Schwinger bosons are dipoles $\phi \sim \bar{\psi}\psi$
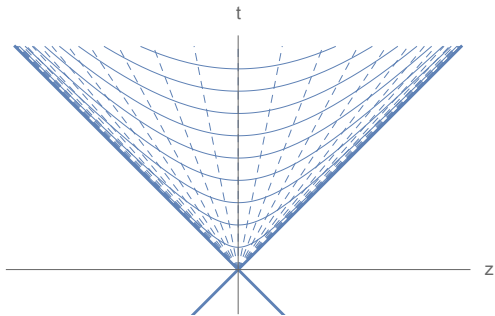- Mass is related to U(1) charge by $M = q/\sqrt{\pi}$

Transverse coordinates

- So far dynamics strictly confined to $1+1$ dimensions
- Transverse coordinates may fluctuate, can be described by Nambu-Goto action ($h_{\mu\nu} = \partial_\mu X^m \partial_\nu X_m$)

$$S_{\mathsf{NG}} = \int d^2x \sqrt{-\det h_{\mu\nu}} \left\{ -\sigma + \ldots \right\}$$
$$\approx \int d^2x \sqrt{g} \left\{ -\sigma - \frac{\sigma}{2} g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i + \ldots \right\}$$

- Two additional, massless, bosonic degrees of freedom corresponding to transverse coordinates X^i with $i = 1, 2$.

Expanding string solution 1

- Consider string formed between (external) quark-anti-quark pair on trajectories

$$z = \pm t$$

- Coordinates: Bjorken time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \text{arctanh}(z/t)$
- Metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$
- Symmetry with respect to longitudinal boosts $\eta \to \eta + \Delta\eta$

Expanding string solution 2

- Schwinger boson field depends only on τ

$$\bar{\phi} = \bar{\phi}(\tau)$$

- Equation of motion

$$\partial_\tau^2 \bar{\phi} + \frac{1}{\tau} \partial_\tau \bar{\phi} + M^2 \bar{\phi} = 0.$$

- Gauss law: electric field $E = q\phi/\sqrt{\pi}$ must approach the U(1) charge of the external quarks $E \to q_\mathsf{e}$ for $\tau \to 0_+$

$$\bar{\phi}(\tau) \to \frac{\sqrt{\pi} q_\mathsf{e}}{q} \qquad (\tau \to 0_+)$$

- Solution of equation of motion

$$\bar{\phi}(\tau) = \frac{\sqrt{\pi} q_\mathsf{e}}{q} J_0(M\tau)$$

Reduced density matrix

- Consider now physical processes such as hadron formation
- Assume that these are local processes in some space region A

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------------------|------------------|-------------------
        B                  A                  B
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- Reduced density matrix, trace over complement region B

$$\rho_A = \mathsf{Tr}_B\, \rho$$

- In general ρ_A mixed state density matrix even if ρ is pure
- Reason: entanglement between regions A and B
- Characterization by entanglement entropy

$$S_A = -\mathsf{Tr}\left\{\rho_A \ln(\rho_A)\right\}$$

Gaussian states

- Theories with quadratic action typically have Gaussian density matrix
- Fully characterized by field expectation values

$$\bar{\phi}(x) = \langle \phi(x) \rangle, \qquad \bar{\pi}(x) = \langle \pi(x) \rangle$$

and connected two-point correlation functions, e. g.

$$\langle \phi(x)\phi(y) \rangle_c = \langle \phi(x)\phi(y) \rangle - \bar{\phi}(x)\bar{\phi}(y)$$

- If ρ is Gaussian, also reduced density matrix ρ_A is Gaussian

Entanglement entropy for Gaussian state

- Entanglement entropy of Gaussian state in region A
 [Berges, Floerchinger, Venugopalan, to appear]

$$S_A = \frac{1}{2}\mathsf{Tr}_A\left\{D\ln(D^2)\right\},$$

- Operator trace over region A only
- Matrix of correlation functions

$$D(x,y) = \begin{pmatrix} -i\langle\phi(x)\pi(y)\rangle_c & i\langle\phi(x)\phi(y)\rangle_c \\ -i\langle\pi(x)\pi(y)\rangle_c & i\langle\pi(x)\phi(y)\rangle_c \end{pmatrix}.$$

- Involves connected correlation functions of field $\phi(x)$ and canonically conjugate momentum field $\pi(x)$
- Expectation value $\bar{\phi}$ does not appear explicitly
- Coherent states and vacuum have equal entanglement entropy S_A

Rapidity interval
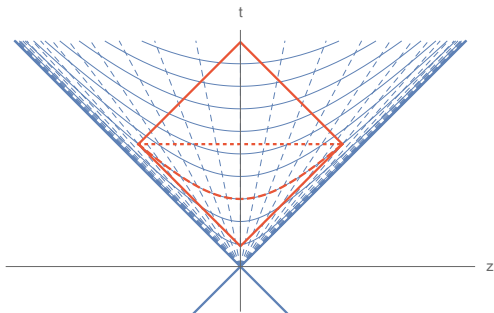
- Consider rapidity interval $(-\Delta\eta/2, \Delta\eta/2)$ at fixed Bjorken time τ
- Entanglement entropy does not change by unitary time evolution with endpoints kept fixed
- Can be evaluated equivalently in interval $\Delta z = 2\tau \sinh(\Delta\eta/2)$ at fixed time $t = \tau \cosh(\Delta\eta/2)$
- Need to solve eigenvalue problem with correct **boundary conditions**

Bosonized massless Schwinger model

- Entanglement entropy understood numerically for free massive scalars [Casini, Huerta (2009)]

- Entanglement entropy density $dS/d\Delta\eta$ for bosonized massless Schwinger model ($M = \frac{q}{\sqrt{\pi}}$)

$M\tau = 1,\ 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4}$, and 10^{-5}
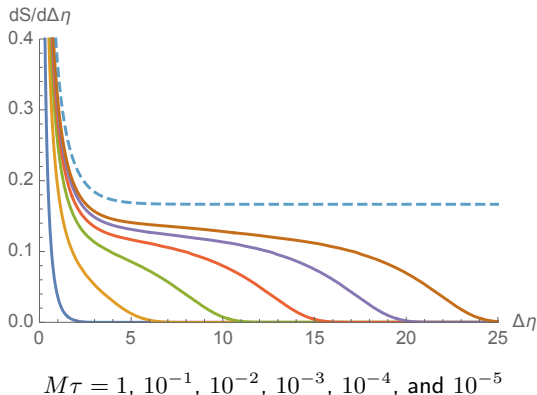
Conformal limit

- For $M\tau \to 0$ one has conformal field theory limit
  [Holzhey, Larsen, Wilczek (1994); Calabrese, Cardy (2004)]

$$S(\Delta z) = \frac{c}{3} \ln\left(\Delta z / \epsilon\right) + \text{constant}$$

  with small length ϵ acting as UV cutoff.

- Here this implies

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln\left(2\tau \sinh(\Delta\eta/2)/\epsilon\right) + \text{constant}$$
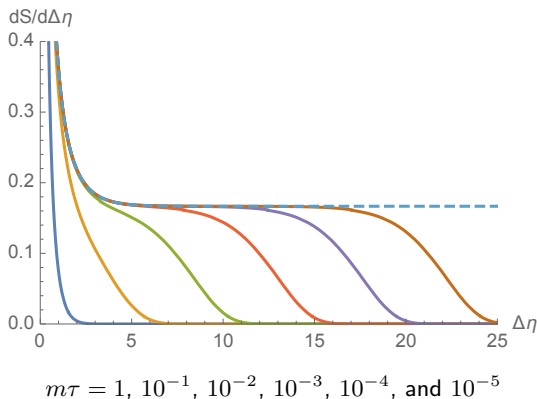
- Conformal charge $c = 1$ for free massless scalars or Dirac fermions.
- Additive constant not universal but entropy density is

$$\frac{\partial}{\partial\Delta\eta} S(\tau, \Delta\eta) = \frac{c}{6}\coth(\Delta\eta/2)$$

$$\to \frac{c}{6} \qquad (\Delta\eta \gg 1)$$

- Entropy becomes extensive in $\Delta\eta$!

Free massive fermions

- Entanglement entropy can also be calculated for free Dirac fermions of mass m

$m\tau = 1,\ 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4},$ and 10^{-5}

- Same universal plateau $c/6$ with $c = 1$ at early time
- Conformal limit corresponds to non-interacting fermions
- Consistent with or without bosonization

Universal entanglement entropy density

- For very early times "Hubble" expansion rate dominates over masses and interactions

$$H = \frac{1}{\tau} \gg M = \frac{q}{\sqrt{\pi}}, m$$

- Theory dominated by free, massless fermions
- Universal entanglement entropy density

$$\frac{dS}{d\Delta\eta} = \frac{c}{6}$$

with conformal charge c

- For QCD in 1+1 dimensions (gluons not dynamical)

$$c = N_c \times N_f$$

- From fluctuating transverse coordinates (Nambu-Goto action)

$$c = N_c \times N_f + 2 \approx 9 + 2 = 11$$

Experimental access to entanglement ?

- Could longitudinal entanglement be tested experimentally?
- Unfortunately entropy density $dS/d\eta$ not straight-forward to access.
- Measured in e^+e^- is the number of charged particles per unit rapidity $dN_{\text{ch}}/d\eta$ (rapidity defined with respect to the thrust axis)
- Around mid-rapidity logarithmic dependence on the collision energy.
- Typical values for collision energies $\sqrt{s} = 14 - 206$ GeV in the range

$$dN_{\text{ch}}/d\eta \approx 2 - 4$$

- Entropy per particle S/N can be estimated for a hadron resonance gas in thermal equilibrium $S/N_{\text{ch}} = 7.2$ would give

$$dS/d\eta \approx 14 - 28$$

- This is an upper bound: correlations beyond one-particle functions would lead to reduced entropy.

Temperature and entanglement entropy

- For conformal fields, entanglement entropy has also been calculated at non-zero temperature.

- For static interval of length l [Calabrese, Cardy (2004)]

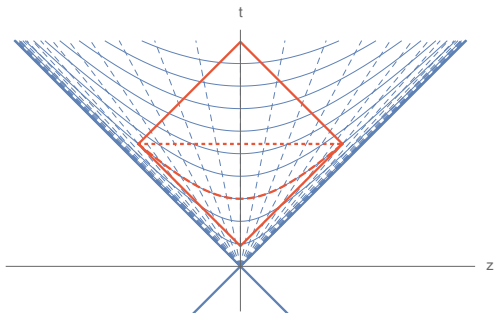$$S(T, l) = \frac{c}{3} \ln\left(\frac{1}{\pi T \epsilon} \sinh(\pi l T)\right) + \text{const}$$

- Compare this to our result in expanding geometry

$$S(\tau, \Delta\eta) = \frac{c}{3} \ln\left(\frac{2\tau}{\epsilon} \sinh(\Delta\eta/2)\right) + \text{constant}$$

- Expressions agree for $l = \tau \Delta\eta$ (with metric $ds^2 = -d\tau^2 + \tau^2 d\eta^2$) and time-dependent temperature

$$T = \frac{1}{2\pi\tau}$$

Modular or entanglement Hamiltonian 1

- Conformal field theory
- Hypersurface Σ with boundary on the intersection of two light cones
- Reduced density matrix
  [Casini, Huerta, Myers (2011), Arias, Blanco, Casini, Huerta (2017)]

$$\rho_A = \frac{1}{Z_A} e^{-K}, \qquad Z_A = \mathsf{Tr}\, e^{-K},$$

- Modular or entanglement Hamiltonian K.

Modular or entanglement Hamiltonian 2

- Modular or entanglement Hamiltonian is **local expression**

$$K = \int_{\Sigma} d\Sigma^{\mu} \, \xi^{\nu}(x) \, T_{\mu\nu}(x).$$

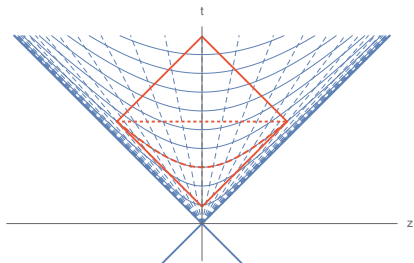- Energy-momentum tensor $T_{\mu\nu}(x)$ and $\xi^{\nu}(x)$ is a vector field

$$\xi^{\mu}(x) = \frac{2\pi}{(k-p)^2}[(k-x)^{\mu}(x-p)(k-p) + (x-p)^{\mu}$$
$$\times (k-x)(k-p) - (k-p)^{\mu}(x-p)(k-x)]$$

with end point of the future light cone k and starting point of the past light cone p.

- Inverse temperature and fluid velocity

$$\xi^{\mu}(x) = \beta^{\mu}(x) = \frac{u^{\mu}(x)}{T(x)}$$

Modular or entanglement Hamiltonian 3

- For k very far in the future $\xi^\mu(x) \to 2\pi\, x^\mu$
- Fluid velocity in τ-direction & time-dependent temperature

$$T(\tau) = \frac{\hbar}{2\pi\tau}$$

- **Entanglement between different rapidity intervals alone leads to local thermal density matrix at very early times !**
- Hawking-Unruh temperature in Rindler wedge $T(x) = \hbar c/(2\pi x)$

Testing the mechanism with cold atoms

- Lieb-Liniger model has linear dispersion at small momenta $\omega = v_s\, p$
  - at $\gamma \gg 1$ sound velocity $v_s = v_F = \pi n/m$
  - at $\gamma \ll 1$ sound velocity $v_s = \sqrt{\gamma}\, n/m = \sqrt{gnm}$
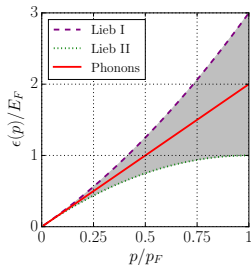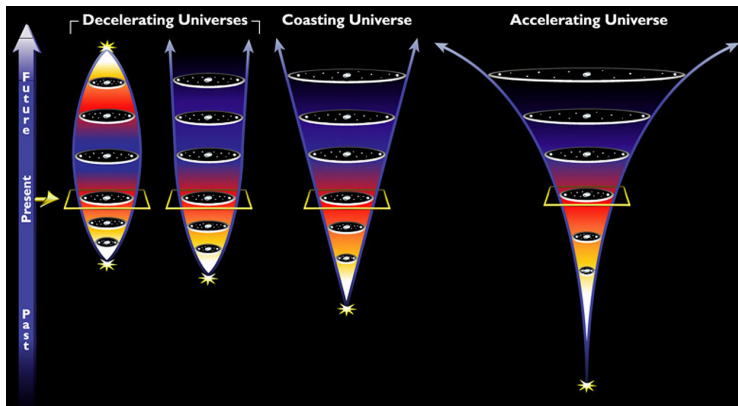

for $\gamma \gg 1$ [De Rosi et al. (2017)]

- Effective metric for phonons

$$ds^2 = -v_s^2 dt^2 + dx^2$$

Expanding geometries

- Expanding geometries can be realized by interplay of
  - longitudinal expansion
  - time dependent change of sound velocity $v_s(t)$
  - time dependent gap or mass $M^2(t)$

Conclusions

- Rapidity intervals in an expanding string are entangled
- Entanglement comes in via boundary terms
- At very early times theory effectively conformal

$$\frac{1}{\tau} \gg m, g$$

- Entanglement entropy extensive in rapidity $\frac{dS}{d\Delta\eta} = \frac{c}{6}$
- Determined by conformal charge $c = N_c \times N_f + 2$
- Reduced density matrix for conformal field theory is of locally thermal form with temperature

$$T = \frac{\hbar}{2\pi\tau}$$

- Entanglement could be important ingredient to understand apparent "thermal effects" in e^+e^- and other collider experiments
- Mechanism could be tested with cold atoms