Fluctuations in the quark-gluon plasma and in the cosmological fluid

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CERN, The big bang and the little bangs, 08/2016.

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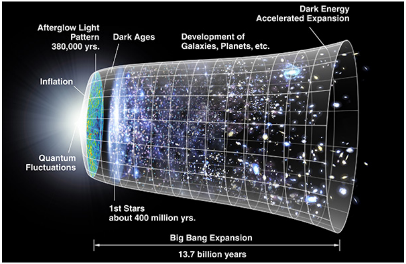
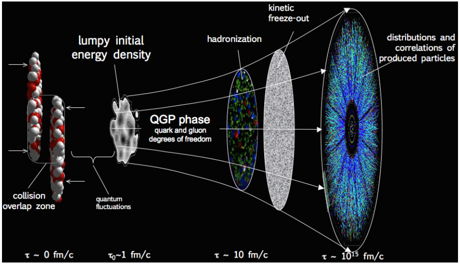
Big bang – little bang analogy





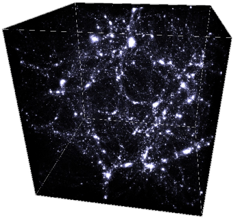
- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

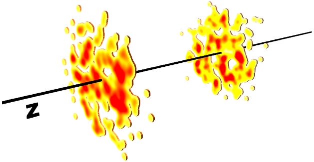
- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- dynamical description as a fluid

Symmetries in a statistical sense

- Concrete realization breaks symmetry
- Statistical properties are symmetric





Cosmology

- Cosmological principle: universe homogeneous and isotropic
- 3D translation and rotation

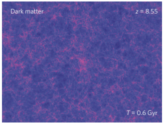
→ 3D Fourier expansion

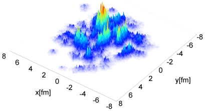
Heavy ion collisions

- 1D azimuthal rotation for central collisions
- 1D Bjorken boost (approximate)

→ Bessel-Fourier expansion [Floerchinger & Wiedemann (2013)]

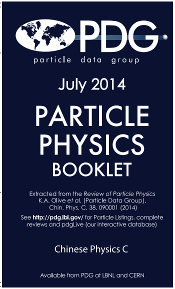
The problem of initial conditions





- Problem for cosmology and heavy ion physics: precise initial conditions for fluid dynamic description not known

	Planck+WP +highL	*Planck*+WP +highL+BAO	*WMAP*9+eCMB +BAO
$\Omega_b h^2$	0.02207 ± 0.00027	0.02214 ± 0.00024	0.02211 ± 0.00034
$\Omega_c h^2$	0.1198 ± 0.0026	0.1187 ± 0.0017	0.1162 ± 0.0020
$100\,\theta_{MC}$	1.0413 ± 0.0006	1.0415 ± 0.0006	–
n_s	0.958 ± 0.007	0.961 ± 0.005	0.958 ± 0.008
τ	$0.091^{+0.013}_{-0.014}$	0.092 ± 0.013	$0.079^{+0.011}_{-0.012}$
$\ln(10^{10}\Delta^2_{\mathcal{R}})$	3.090 ± 0.025	3.091 ± 0.025	3.212 ± 0.029
h	0.673 ± 0.012	0.678 ± 0.008	0.688 ± 0.008
σ_8	0.828 ± 0.012	0.826 ± 0.012	$0.822^{+0.013}_{-0.014}$
Ω_m	$0.315^{+0.016}_{-0.017}$	0.308 ± 0.010	0.293 ± 0.010
Ω_Λ	$0.685^{+0.017}_{-0.016}$	0.692 ± 0.010	0.707 ± 0.010



- Nevertheless, cosmology is now a precision science...
- How is that possible ?

Initial conditions in cosmology

- Perturbations are classified into scalars, vectors, tensors
- Vector modes are decaying, need not be specified
- Tensor modes are gravitational waves, can be neglected for most purposes
- Decaying scalar modes also not relevant
- Growing scalar modes are further classified by wavelength
- For relevant range of wavelength: close to Gaussian probability distribution
- Almost scale invariant initial spectrum

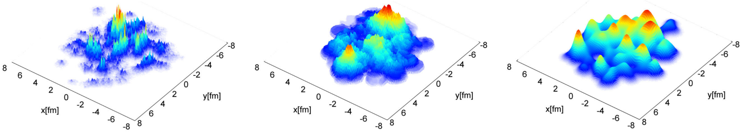
$$\langle \delta(\mathbf{k})\, \delta(\mathbf{k}') \rangle = P(k)\ \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

with

$$P(k) \sim k^{n_s - 1} \qquad\qquad n_s = 0.968 \pm 0.006$$ [Planck (2015)]

Initial conditions heavy ion collisions

- State of the art: Explicit realizations in terms of Monte-Carlo models



IP-Glasma MC-KLN MC-Glauber

[Schenke, Tribedy & Venugopalan, PRL 108, 252301 (2012)]

- Can heavy ion physics follow the successful approach used in cosmology?
  - Characterize statistical properties rather than explicit realizations
  - Focus on relevant wavelengths

- First attempts in this direction have been made
  [Teaney & Yan (2011), Coleman-Smith, Petersen & Wolpert (2012), Floerchinger & Wiedemann (2013), Yan & Ollitrault (2014), Bzdak & Skokov (2014), ...]

Mode expansion for fluid fields

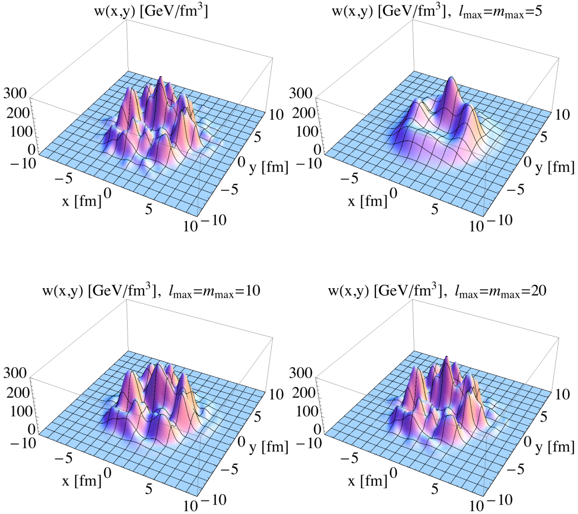
Bessel-Fourier expansion at fixed time τ

[Floerchinger & Wiedemann 2013, see also Coleman-Smith, Petersen & Wolpert 2012, Floerchinger & Wiedemann 2014]

$$w(r,\phi,\eta) = w_{\mathsf{BG}}(r) + w_{\mathsf{BG}}(r) \sum_{m,l} \int_k w_l^{(m)}(k)\, e^{im\phi + ik\eta}\, J_m\left(z_l^{(m)} \rho(r)\right)$$

- azimuthal wavenumber m, radial wavenumber l, rapidity wavenumber k
- $w_l^{(m)}$ dimensionless
- higher m and l correspond to finer spatial resolution
- coefficients $w_l^{(m)}$ can be related to eccentricienies
- works similar for vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



Statistics of initial density perturbations

Independent point-sources model (IPSM)

$$w(\vec{x}) = \left[\frac{1}{\tau_0}\frac{dW_{\mathsf{BG}}}{d\eta}\right]\frac{1}{N}\sum_{j=1}^{N}\delta^{(2)}(\vec{x}-\vec{x}_j)$$

- random positions $\vec{x}_j$, independent and identically distributed
- probability distribution $p(\vec{x}_j)$ reflects collision geometry
- possible to determine correlation functions analytically for *central* and *non-central* collisions [Floerchinger & Wiedemann (2014)]
- Long-wavelength modes (small m and l) that don't resolve differences between point-like and extended sources have *universal statistics*.

Cosmological perturbation theory

[Lifshitz, Peebles, Bardeen, Kosama, Sasaki, Ehler, Ellis, Hawking, Mukhanov, Weinberg, ...]

- Solves evolution equations for fluid + gravity
- Expands in perturbations around homogeneous background
- Detailed understanding how different modes evolve
- Diagramatic formalism for non-linear mode-mode interactions
- Very simple equations of state $p = w\,\epsilon$
- Viscosities usually neglected $\eta = \zeta = 0$
- Photons and neutrinos are free streaming

Fluid dynamic perturbation theory for heavy ions

[Floerchinger & Wiedemann, PLB 728, 407 (2014)]

- goal: understand dynamics of heavy ion collisions and determine QCD transport properties experimentally
- so far: numerical fluid simulations e.g. [Heinz & Snellings (2013)]
- new: solve fluid equations for smooth and symmetric background and order-by-order in perturbations
- good convergence properties [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]

Perturbative expansion

Write the hydrodynamic fields $h = (T, u^\mu, \pi^{\mu\nu}, \pi_{\text{Bulk}}, \dots)$

- at initial time τ_0 as

$$h = h_0 + \epsilon\, h_1$$

  with background h_0, fluctuation part $\epsilon\, h_1$

- at later time $\tau > \tau_0$ as

$$h = h_0 + \epsilon\, h_1 + \epsilon^2 h_2 + \epsilon^3 h_3 + \dots$$

Solve for time evolution in this scheme

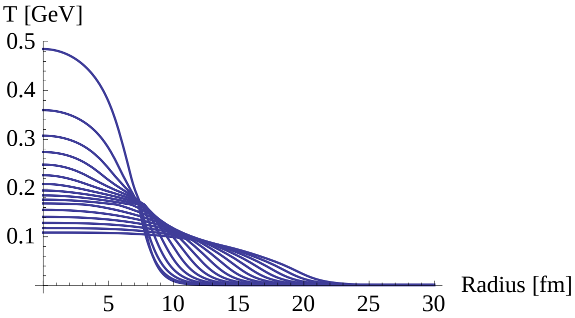
- h_0 is solution of full, non-linear hydro equations in symmetric situation: azimuthal rotation and Bjorken boost invariant
- h_1 is solution of linearized hydro equations around h_0, can be solved mode-by-mode
- h_2 can be obtained by from interactions between modes etc.

Background evolution

System of coupled $1 + 1$ dimensional non-linear partial differential equations for

- enthalpy density $w(\tau, r)$ (or temperature $T(\tau, r)$)
- fluid velocity $u^\tau(\tau, r), u^r(\tau, r)$
- two independent components of shear stress $\pi^{\mu\nu}(\tau, r)$

Can be easily solved numerically

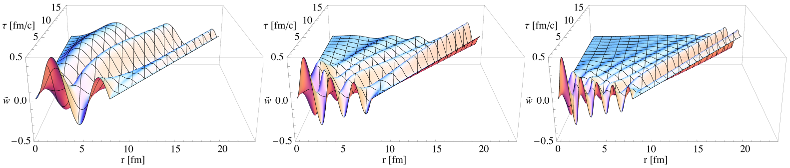


Evolving perturbation modes

- Linearized hydro equations: set of coupled $3+1$ dimensional, linear, partial differential equations.
- Use Fourier expansion

$$h_j(\tau, r, \phi, \eta) = \sum_m \int \frac{dk_\eta}{2\pi} \, h_j^{(m)}(\tau, r, k_\eta) \, e^{i(m\phi + k_\eta \eta)}.$$

- Reduces to $1+1$ dimensions.
- Can be solved numerically for each initial Bessel-Fourier mode.



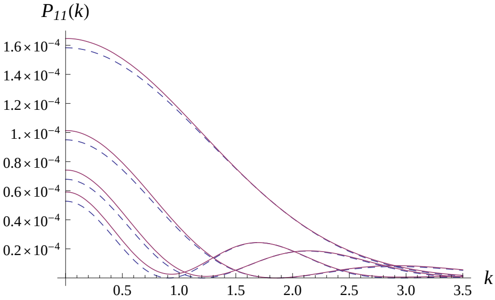
Mode interactions

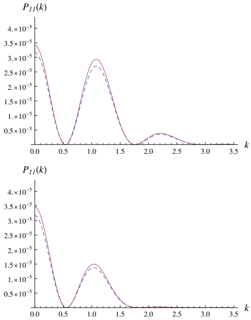
- Non-linear terms in the evolution equations lead to mode interactions.
- Quadratic and higher order in initial perturbations.
- Can be determined from iterative solution but has not been fully worked out yet.
- Convergence can be tested with numerical solution of full hydro equations.

Evolution of spectrum of density perturbations

Density-density spectrum

$$P_{11}(\vec{k}) = \int d^2x \, e^{-i\vec{k}(\vec{x}-\vec{y})} \, \langle \, d(\vec{x}_1) \, d(\vec{x}_2) \, \rangle_c$$





dashed: linear evolution, solid: including first non-linear correction

left: $\eta/s = 0.08$, $\tau = 1.5, 2.5, 3.5, 4.5$ fm/c, right: $\eta/s = 0.08$ and $\eta/s = 0.8$, $\tau = 7.5$ fm/c

[Brouzakis, Floerchinger, Tetradis & Wiedemann, PRD 91, 065007 (2015)]

Backreaction: General idea

- for $0+1$ dimensional, non-linear dynamics

$$\dot{\varphi} = f(\varphi) = f_0 + f_1\,\varphi + \tfrac{1}{2} f_2\,\varphi^2 + \ldots$$

- one has for expectation values $\bar{\varphi} = \langle \varphi \rangle$

$$\dot{\bar{\varphi}} = f_0 + f_1\,\bar{\varphi} + \tfrac{1}{2} f_2\,\bar{\varphi}^2 + \tfrac{1}{2} f_2\,\langle (\varphi - \bar{\varphi})^2 \rangle + \ldots$$

- evolution equation for expectation value $\bar{\varphi}$ depends on two-point correlation function or spectrum $P_2 = \langle (\varphi - \bar{\varphi})^2 \rangle$
- evolution equation for spectrum depends on bispectrum and so on
- more complicated for higher dimensional theories
- more complicated for gauge theories such as gravity

Backreaction in gravity

- Einstein's equations are non-linear.
- Important question [G. F. R. Ellis (1984)]: If Einstein's field equations describe small scales, including inhomogeneities, do they also hold on large scales?
- Is there a sizable backreaction from inhomogeneities to the cosmological expansion?
- Difficult question, has been studied by many people [Ellis & Stoeger (1987); Mukhanov, Abramo & Brandenberger (1997); Unruh (1998); Buchert (2000); Geshnzjani & Brandenberger (2002); Schwarz (2002); Wetterich (2003); Räsänen (2004); Kolb, Matarrese & Riotto (2006); Brown, Behrend, Malik (2009); Gasperini, Marozzi & Veneziano (2009); Clarkson & Umeh (2011); Green & Wald (2011); ...]
- Recent reviews: [Buchert & Räsänen, Ann. Rev. Nucl. Part. Sci. 62, 57 (2012); Green & Wald, Class. Quant. Grav. 31, 234003 (2014)]
- No general consensus but most people believe now that **gravitational backreaction is rather small**.
- In the following we look at a new **backreaction on the matter side** of Einstein's equations.

Fluid equation for energy density

First order viscous fluid dynamics

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu - \zeta\Theta^2 - 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} = 0$$

For $\vec{v}^2 \ll c^2$ and Newtonian potentials $\Phi, \Psi \ll 1$

$$\dot{\epsilon} + \vec{v}\cdot\vec{\nabla}\epsilon + (\epsilon+p)\left(3\frac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right)$$
$$= \frac{\zeta}{a}\left[3\frac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right]^2 + \frac{\eta}{a}\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}(\vec{\nabla}\cdot\vec{v})^2\right]$$

Fluid dynamic backreaction in Cosmology

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

Expectation value of energy density $\bar{\epsilon} = \langle \epsilon \rangle$

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p} - 3\bar{\zeta}H\right) = D$$

with dissipative backreaction term

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}\partial_i v_i \partial_j v_j\right] \rangle + \frac{1}{a^2}\langle \zeta [\vec{\nabla}\cdot\vec{v}]^2 \rangle + \frac{1}{a}\langle \vec{v}\cdot\vec{\nabla}\,(p - 6\zeta H)\rangle$$

- D vanishes for unperturbed homogeneous and isotropic universe
- D has contribution from shear & bulk viscous dissipation and thermodynamic work done by contraction against pressure gradients
- dissipative terms in D are positive semi-definite
- for spatially constant viscosities and scalar perturbations only

$$D = \frac{\bar{\zeta} + \frac{4}{3}\bar{\eta}}{a^2} \int d^3q \; P_{\theta\theta}(q)$$

Dissipation of perturbations

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- Dissipative backreaction does not need negative effective pressure

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p}_{\text{eff}}\right) = D$$

- D is an integral over perturbations, could become large at late times.
- Can it potentially accelerate the universe?
- Need additional equation for scale parameter a
- Use trace of Einstein's equations $R = 8\pi G_{\text{N}} T^{\mu}{}_{\mu}$

$$\frac{1}{a}\dot{H} + 2H^2 = \frac{4\pi G_{\text{N}}}{3}\left(\bar{\epsilon} - 3\bar{p}_{\text{eff}}\right)$$

  does not depend on unknown quantities like $\langle(\epsilon + p_{\text{eff}})u^{\mu}u^{\nu}\rangle$
- To close the equations one needs equation of state $\bar{p}_{\text{eff}} = \bar{p}_{\text{eff}}(\bar{\epsilon})$ and dissipation parameter D

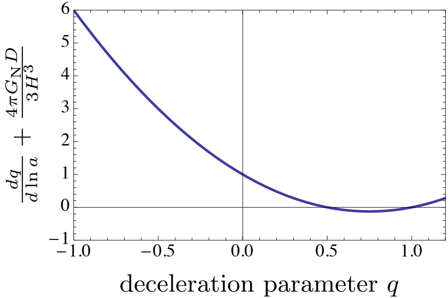
Deceleration parameter

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- assume now vanishing effective pressure $\bar{p}_{\text{eff}} = 0$
- obtain for deceleration parameter $q = -1 - \frac{\dot{H}}{aH^2}$

$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \frac{1}{2}\right) = \frac{4\pi G_{\text{N}} D}{3H^3}$$

- for $D = 0$ attractive fixed point at $q_* = \frac{1}{2}$ (deceleration)
- for $D > 0$ fixed point shifted towards $q_* < 0$ (acceleration)



Estimating viscous backreaction D

- For $\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx 4$ one could explain the current accelerated expansion ($q \approx -0.6$) by dissipative backreaction.
- Is this possible?
- In principle one can determine D for given equation of state and viscous properties from dynamics of structure formation.
- So far only rough estimates. If shear viscosity dominates:

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle \approx c_D \bar{\eta} H^2$$

  with $c_D = \mathcal{O}(1)$. Corresponds to $\Delta v \approx 100\,\mathsf{km/s}$ for $\Delta x \approx 1\,\mathsf{MPc}$
- Leads to

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D \bar{\eta} H}{2\rho_c}$$

  with $\rho_c = \frac{3H^2}{8\pi G_{\mathsf{N}}}$

Viscosities

- Relativistic particles / radiation contribute to shear viscosity

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

  - prefactor $c_\eta = \mathcal{O}(1)$
  - energy density of radiation ϵ_R
  - mean free time τ_R

- Bulk viscosity vanishes in situations with conformal symmetry but can be large when conformal symmetry is broken.
- For massive scalar particles with $\lambda\varphi^4$ interaction [Jeon & Yaffe (1996)]

$$\zeta \sim \frac{m^6}{\lambda^4 T^3} e^{2m/T}, \qquad \eta \sim \frac{m^{5/2} T^{1/2}}{\lambda^2} \qquad \text{for} \qquad \frac{T}{m} \ll 1$$

Estimating viscous backreaction D

Consider shear viscosity from radiation

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

Backreaction term

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D c_\eta}{2} \frac{\epsilon_R}{\rho_c} \tau_R H$$

- fluid approximation needs $\tau_R H < 1$
- for sizeable effect one would need $\epsilon_R / \rho_c = \mathcal{O}(1)$
- unlikely that D becomes large enough in this scenario

Needed refinements:

- full dynamics of perturbations
- second order fluid dynamics
- complete model(s)

Dissipation from the effective action

- Dissipative effects are usually discusses on the level of equations of motion.
- For some questions one would like to have a formulation in terms of an effective action
  - causality & stability analysis
  - fluctuations
  - renormalization
  - effective field theories
  - coupling to gravity
- One possibility: Schwinger-Keldysh double time path formalism
- Another possibility: Analytic continuation of the 1PI effective action [Floerchinger, 1603.07148]
  - Theories in approximate local equilibrium
  - General covariance and energy-momentum conservation
  - Local form of second law of thermodynamics
  - Effective action for fluid dynamics including viscosity terms

Local equilibrium & partition function

- Local equilibrium description with $T(x)$ and $u^\mu(x)$

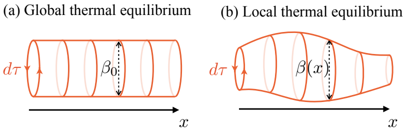
$$\beta^\mu(x) = \frac{u^\mu(x)}{T(x)}$$

- Use similarity between local density matrix and translation operator

$$e^{\beta^\mu(x)\mathscr{P}_\mu} \qquad \longleftrightarrow \qquad e^{i\Delta x^\mu \mathscr{P}_\mu}$$

to represent partition function as functional integral with periodicity in imaginary direction such that

$$\phi(x^\mu - i\beta^\mu(x)) = \pm\phi(x^\mu)$$



- Partition function $Z[J]$, Schwinger functional $W[J]$ in Euclidean domain

$$Z[J] = e^{W_E[J]} = \int D\phi\, e^{-S_E[\phi]+\int_x J\phi}$$

One particle irreducible effective action

- In Euclidean domain defined by standard Legendre transform

$$\Gamma_E[\Phi] = \int_x J_a(x)\Phi_a(x) - W_E[J]$$

with expectation values

$$\Phi_a(x) = \frac{1}{\sqrt{g}(x)} \frac{\delta}{\delta J_a(x)} W_E[J]$$

- Euclidean field equation

$$\frac{\delta}{\delta \Phi_a(x)} \Gamma_E[\Phi] = \sqrt{g}(x)\, J_a(x)$$

resembles classical equation of motion for $J = 0$.

- Need analytic continuation to obtain a viable equation of motion.

Analytic continuation 1

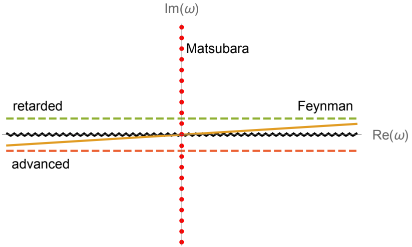
- Define for homogeneous background field and in global equilibrium

$$\frac{\delta^2}{\delta J_a(-p)\delta J_b(q)} W_E[J] = G_{ab}(p)\ (2\pi)^4 \delta^{(4)}(p-q)$$
$$\frac{\delta^2}{\delta \Phi_a(-p)\delta \Phi_b(q)} \Gamma_E[\Phi] = P_{ab}(p)\ (2\pi)^4 \delta^{(4)}(p-q)$$

- From definition of effective action

$$\sum_b G_{ab}(p) P_{bc}(p) = \delta_{ac}$$

- Correlation functions can be analytically continued in $\omega = -u^\mu p_\mu$.
- Branch cut on real frequency axis $\omega \in \mathbb{R}$.



Analytic continuation 2

- Decompose inverse two-point function

$$P_{ab}(p) = P_{1,ab}(p) - i s_{\mathsf{I}}(-u^\mu p_\mu)\, P_{2,ab}(p),$$

  with $s_{\mathsf{I}}(\omega) = \mathsf{sign}(\mathsf{Im}\ \omega)$.
- In position space, replace

$$\begin{aligned} & s_{\mathsf{I}}\left(-u^\mu p_\mu\right) = \mathsf{sign}\left(\mathsf{Im}(-u^\mu p_\mu)\right) \\ & \to \mathsf{sign}\left(\mathsf{Im}\left(i u^\mu \tfrac{\partial}{\partial x^\mu}\right)\right) = \mathsf{sign}\left(\mathsf{Re}\left(u^\mu \tfrac{\partial}{\partial x^\mu}\right)\right) = s_{\mathsf{R}}\left(u^\mu \tfrac{\partial}{\partial x^\mu}\right) \end{aligned}$$

- This symbol appears also in $\Gamma[\Phi]$
- *Real and causal* field equations follow from [Floerchinger, 1603.07148]

$$\frac{\delta\Gamma[\Phi]}{\delta\Phi_a(x)}\bigg|_{\mathsf{ret}} = 0$$

  with certain algebraic rules for $s_{\mathsf{R}}\left(u^\mu \frac{\partial}{\partial x^\mu}\right) \to \pm 1$.
- Energy momentum conservation, entropy production, fluid dynamics, ...

Gravitational growth of perturbations

- Small initial density perturbations

$$\delta = \frac{\Delta\epsilon}{\bar{\epsilon}} \ll 1$$

- At photon decoupling (CMB)

$$\delta \approx 10^{-5}$$

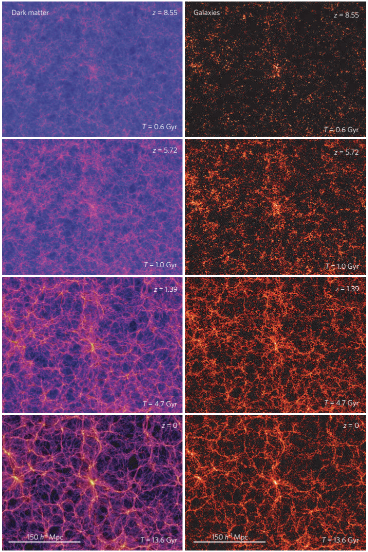
- Structure growth due to attractive gravitational interaction
- Perturbative treatment possible up to

$$\delta \approx 1$$

- For late times and small wavelengths

$$\delta \gg 1$$

Dark matter | Visible galaxies



[Springel, Frenk & White, Nature 440, 1137 (2006)]

The dark matter fluid

- Heavy ion collisions

$$\mathscr{L}_{\text{QCD}} \quad \rightarrow \quad \text{fluid properties}$$

- Late time cosmology

$$\text{fluid properties} \quad \rightarrow \quad \mathscr{L}_{\text{dark matter}}$$

- Until direct detection of dark matter, it can only be observed via

$$T^{\mu\nu}_{\text{dark matter}}$$

Formation of large scale structure

- Formation of large scale structure is interesting
  - tests physics of dark matter
  - tests physics of dark energy
  - gets tested by missions like Euclid, ...
- Cosmological perturbation theory breaks down when density contrast

$$\delta(\mathbf{k}) = \frac{\delta\rho(\mathbf{k})}{\bar{\rho}} \gg 1$$

  grows large at late times and for small scales.
- Numerical simulations (N-body) are expensive and time-consuming
- One would like to have better analytical understanding

Renormalization group apprach

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]
[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]

- Start from ideal fluid approximation
- Large scale structure formation can be formulated as classical field theory with stochastic initial conditions
- Leads to classical statistical field theory
- Initial state fluctuations can be treated by functional renormalization group, similar to thermal or quantum fluctuations in other contexts [Matarrese & Pietroni (2007)]
- Modify theory by cutting off the initial spectrum in the IR

$$P_k^0(\mathbf{q}) = P^0(\mathbf{q})\,\Theta(|\mathbf{q}| - k)$$

- Use flow equation for 1PI effective action [Wetterich (1993)]

$$\partial_k \Gamma_k[\phi, \chi] = \frac{1}{2}\,\mathrm{Tr}\left\{ \left(\Gamma_k^{(2)}[\phi, \chi] - i\left(P_k^0 - P^0\right)\right)^{-1} \partial_k P_k^0 \right\}$$

Renormalization of effective viscosity and pressure

- Effective theory at scale k has additional terms in equations of motion
- Order them by derivative expansion.
- Lowest order: ideal fluid
- Next-to-lowest order: effective sound velocity parameter

$$\gamma_s = \frac{c_s^2}{\mathcal{H}^2} = \frac{dp/d\rho}{\mathcal{H}^2}.$$

and effective viscosity parameter

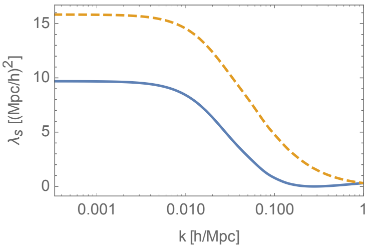
$$\gamma_\nu = \frac{4\eta/3 + \zeta}{(\rho + p)\mathcal{H}a}.$$

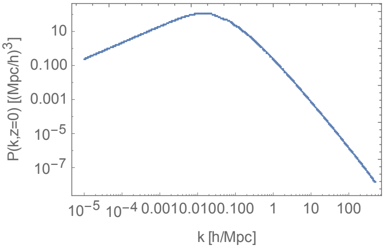
- Both depend on cosmological time or scale factor a

$$\gamma_s = \lambda_s \, a^\kappa, \qquad \gamma_\nu = \lambda_\nu \, a^\kappa$$

with exponent $\kappa \approx 2$.

RG flow of effective sound velocity parameter



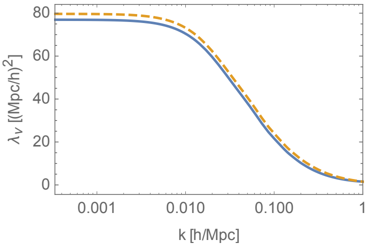


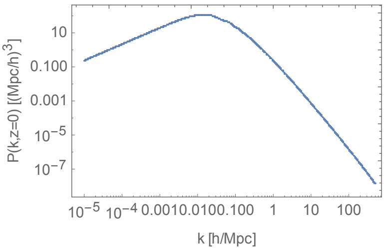
- left: RG flow of effective sound velocity today $\lambda_s = \frac{c_s^2}{H_0^2} = \frac{dp/d\rho}{H_0^2}$
  - dashed line: one-loop approximation

$$\partial_k \lambda_s = -\frac{4\pi}{3}\frac{31}{70}P^0(k)$$

  - solid line: functional RG
- right: linear density power spectrum

RG flow of effective viscosity parameter



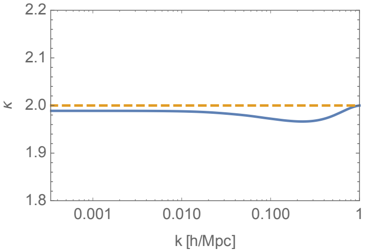


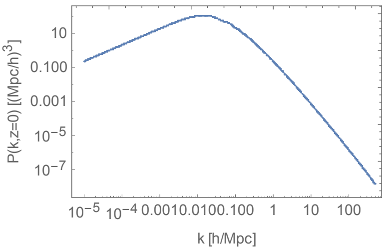
- left: RG flow of effective viscosity today $\lambda_\nu = \frac{4\eta/3+\zeta}{(\rho+p)_0 H_0}$
  - dashed line: one-loop approximation

$$\partial_k \lambda_\nu = -\frac{4\pi}{3}\frac{78}{35}P^0(k)$$

  - solid line: functional RG
- right: linear density power spectrum

RG flow of exponent κ



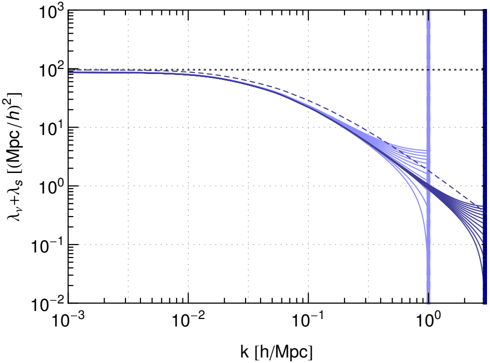


- left: RG flow of exponent κ
  - dashed line: one-loop approximation

$$\partial_k \kappa = \frac{4\pi}{3} P^0(k) \frac{78(\kappa - 2)}{35\tilde{\lambda}_\nu}$$

  - solid line: functional RG
- right: linear density power spectrum

Fixed point behavior



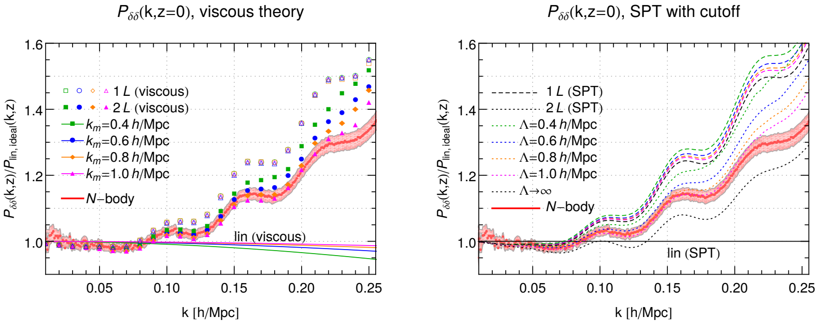
- growing mode is sensitive to $\lambda_s + \lambda_\nu$
- functional RG has IR fixed points

Functional RG + perturbation theory

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

[Floerchinger, Garny, Tetradis & Wiedemann, 1607.03453]

- RG evolution to determine effective viscosity and sound velocity at intermediate scale k_m
- Perturbation theory for power spectrum for scales $0 < |\mathbf{q}| < k_m$
- Theory with effective parameters



Conclusions

- Interesting parallels between cosmology and heavy ion collisions.
- Analog of cosmological perturbation theory can help to solve the fluid dynamics of heavy ion collisions.
- Dissipation of perturbations can have interesting effects in cosmology.
- Analytically continued one-particle irreducible effective action contains dissipative effects.
- Modified variational principle leads to real and causal equations of motion.
- Renormalization group and description as an effective fluid can help to understand large scale structure formation.

Backup slides

Ideal fluid versus collision-less gas

- Many codes used in cosmology describe dark matter as **ideal, cold and pressure-less fluid**

$$T^{\mu\nu} = \epsilon \, u^\mu u^\nu$$

- Equation of state $p = 0$
- No shear stress and bulk viscous pressure $\pi^{\mu\nu} = \pi_{\text{bulk}} = 0$
- Dark matter is also modeled as **collision-less gas** of massive particles, interacting via gravity only
- Two pictures are in general **not consistent**

Dissipative properties

Viscosities

- Diffusive transport of momentum [Maxwell (1860)]
- Depend strongly on interaction properties
- Example: non-relativistic gas of particles with mass m, mean peculiar velocity $\bar{v}$, elastic $2 \to 2$ cross-section σ_{el}

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{\text{el}}} \qquad\qquad \zeta = 0$$

- Interesting additional **information about dark matter**

How is structure formation modified?

Linear dynamics

- energy conservation ($\theta = \vec{\nabla} \cdot \vec{v}$)

$$\dot{\delta\epsilon} + 3\tfrac{\dot{a}}{a}\delta\epsilon + \bar{\epsilon}\,\theta = 0$$

- Navier-Stokes equation

$$\bar{\epsilon}\left[\dot{\theta} + \tfrac{\dot{a}}{a}\theta - k^2\psi\right] + \tfrac{1}{a}\left(\zeta + \tfrac{4}{3}\eta\right)k^2\theta = 0$$

- Poisson equation

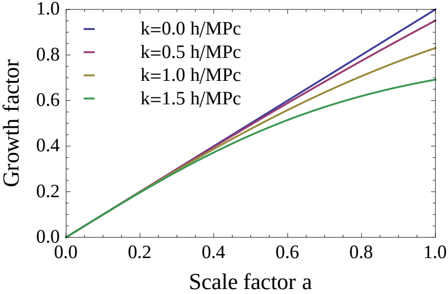
$$-k^2\psi = 4\pi G_{\mathsf{N}} a^2 \delta\epsilon$$

Scalar perturbations ($\delta = \frac{\delta\epsilon}{\bar{\epsilon}}$)

$$\ddot{\delta} + \left[\frac{\dot{a}}{a} + \frac{\zeta + \frac{4}{3}\eta}{a\bar{\epsilon}}k^2\right]\dot{\delta} - 4\pi G_{\mathsf{N}}\bar{\epsilon}\,\delta = 0$$

Viscosites slow down gravitational collapse but do not wash out structure

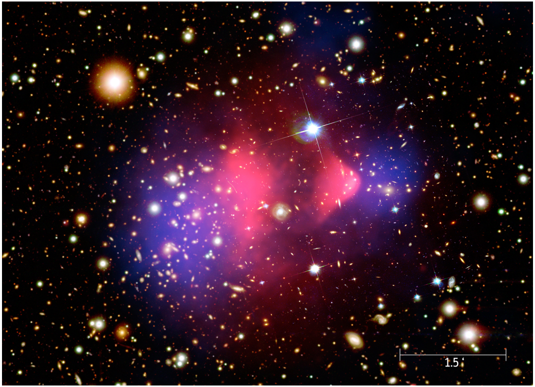
Structure formation with viscosities



[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP 1511, 049 (2015)]

- k-dependent growth factor for scalar modes
- Could be tested by observation of large scale structure
- Depends on $\zeta + \frac{4}{3}\eta$ as function of time (or density)

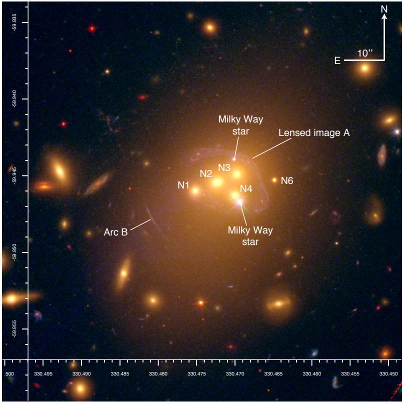
Material properties of dark matter

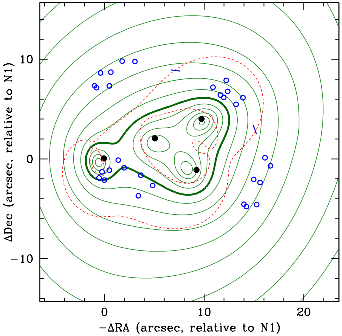


Gravitational lensing and x-ray image of “bullet cluster” 1E0657-56

- so far: dark matter is non-interacting $\rightarrow$ can collide without stopping
- Future decade: analysis of colliding galaxy clusters will refine this picture
- Dark energy self interacting
  - $\rightarrow$ modification of equation of state
  - $\rightarrow$ dissipation

Is dark matter self-interacting?





Galaxy cluster Abell 3827

[Massey *et al.*, MNRAS 449, 3393 (2015)]

- Offset between stars and dark matter falling into cluster
- Is this a first indication for a dark matter self interaction?
  [Kahlhoefer, Schmidt-Hoberg, Kummer & Sarkar, MNRAS 452, 1 (2015)]

$$\frac{\sigma}{m_{\text{DM}}} \approx 3\,\frac{\text{cm}^2}{\text{g}} \approx 0.5\,\frac{\text{b}}{\text{GeV}} \qquad \text{(under debate)}$$

Precision cosmology can measure shear stress

- Scalar excitations in gravity

$$ds^2 = a^2 \left[-(1+2\psi)d\eta^2 + (1-2\phi)dx_i dx_i\right]$$

  with two Newtonian potentials ψ and ϕ.

- Einsteins equations imply

$$\left(\partial_i\partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)(\phi - \psi) = 8\pi G_{\mathsf{N}} a^2 \; \pi_{ij}\big|_{\mathsf{scalar}}$$

  with scalar part of shear stress

$$\pi_{ij}\big|_{\mathsf{scalar}} = \left(\partial_i\partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)\tilde{\pi}$$

- Detailed data at small redshift e.g. from Euclid satellite (esa, 2020) [Amendola *et al.* (2012)]
  - ψ can be measured via acceleration of matter
  - $\psi + \phi$ can be meaured by weak lensing and Sachs-Wolfe effect
  - fluid velocity can be accessed by redshift space distortions
- New quantitative precise insights into fluid properties of dark matter!

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = (\epsilon + p + \pi_{\text{bulk}})u^\mu u^\nu + (p + \pi_{\text{bulk}})g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition w. r. t. fluid velocity u^μ
- pressure $p = p(\epsilon, n)$
- constitutive relations for viscous terms in derivative expansion
  - bulk viscous pressure $\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \ldots$
  - shear stress $\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right] + \ldots$
  - diffusion current $\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta \left(\frac{\mu}{T}\right) + \ldots$

Fluid dynamic equations from covariant **conservation laws**

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu N^\mu = 0.$$

Bulk viscosity

- Bulk viscous pressure is negative for expanding universe

$$\pi_{\mathsf{bulk}} = -\zeta\,\nabla_\mu u^\mu = -\zeta\,3H < 0$$

- Negative effective pressure

$$p_{\mathsf{eff}} = p + \pi_{\mathsf{bulk}} < 0$$

  would act similar to dark energy in Friedmann's equations
  [Murphy (1973), Padmanabhan & Chitre (1987), Fabris, Goncalves & de Sa Ribeiro (2006), Li & Barrow (2009), Velten & Schwarz (2011), Gagnon & Lesgourgues (2011), ...]

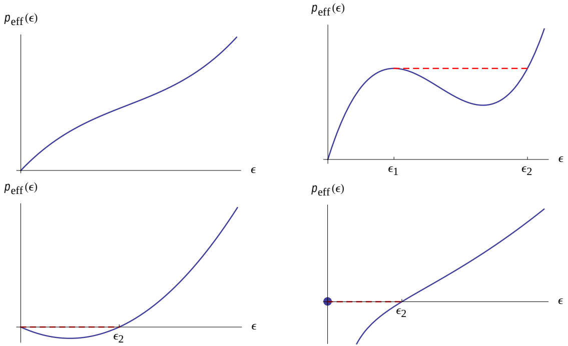
- **Is negative effective pressure physical?**
- In context of heavy ion physics: instability for $p_{\mathsf{eff}} < 0$ ("cavitation")
  [Torrieri & Mishustin (2008), Rajagopal & Tripuraneni (2010), Buchel, Camanho & Edelstein (2014), Habich & Romatschke (2015), Denicol, Gale & Jeon (2015)]
- What precisely happens at the instability?

Is negative effective pressure physical?

- Kinetic theory

$$p_{\text{eff}}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E_{\vec{p}}} f(x,\vec{p}) \geq 0$$

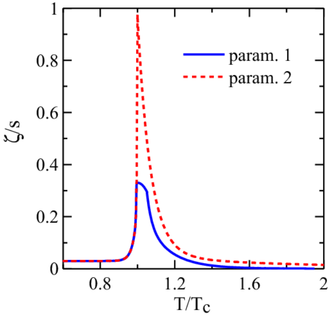
- Stability argument



If there is a vacuum with $\epsilon = p_{\text{eff}} = 0$, phases with $p_{\text{eff}} < 0$ cannot be mechanically stable. (But could be metastable.)

Bulk viscosity in heavy ion physics

- In heavy ion physics people start now to consider bulk viscosity.
- Becomes relevant close to chiral crossover



[Denicol, Gale & Jeon (2015)]

- Is there a first-order phase transition triggered by the expansion?
- What is the relation to chemical and kinetic freeze-out?
- More detailed understanding needed, both for heavy ion physics and cosmology

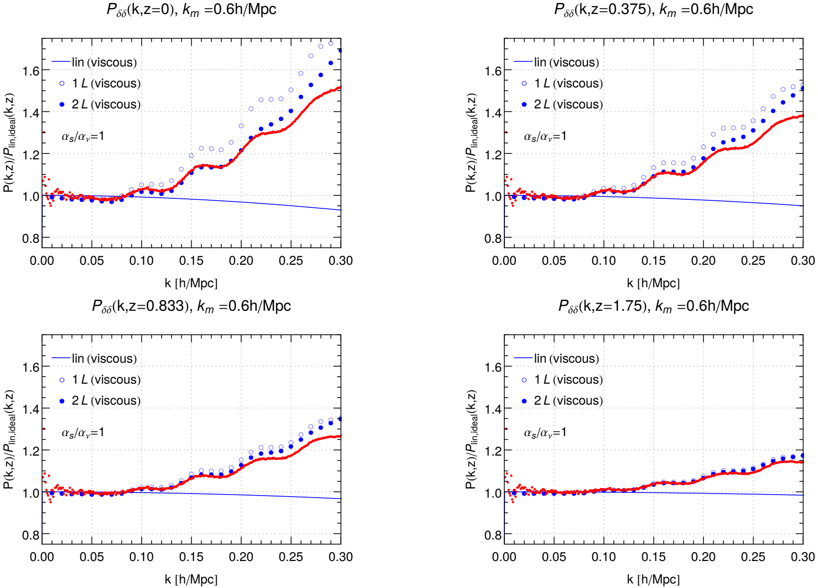
"Fundamental" and "effective" viscosity

Two types of viscosities for cosmological fluid

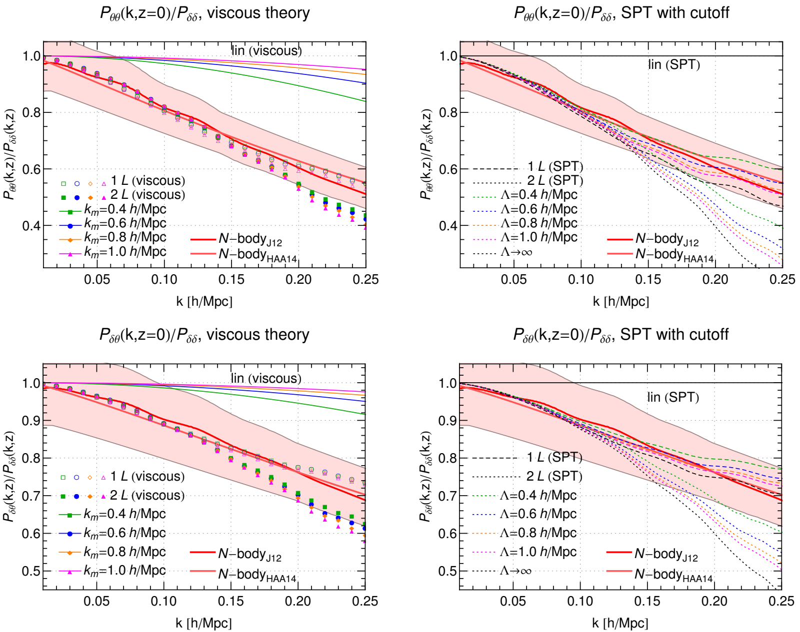
1. Momentum transport by particles or radiation
   - governed by interactions
   - from linear response theory [Green (1954), Kubo (1957)]
   - close to equilibrium

2. Momentum transport in the inhomogeneous, coarse-grained fluid
   - governed by non-linear fluid mode couplings
   - determined perturbatively [Blas, Floerchinger, Garny, Tetradis & Wiedemann]
   - non-equilibrium
   - heavy ions: anomalous plasma viscosity [Asakawa, Bass & Müller (2006)]
     eddy viscosity [Romatschke (2008)]

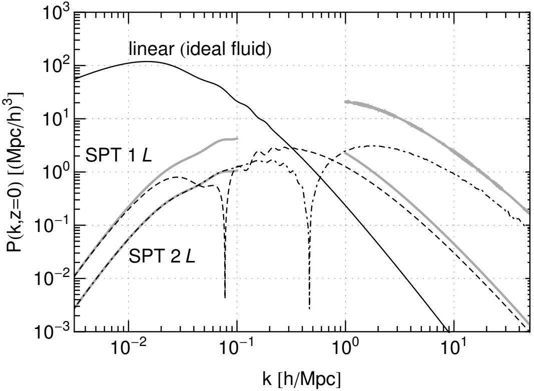
Power spectrum at different redshifts



Velocity spectra



Power spectrum, standard perturbation theory



[D. Blas, M. Garny and T. Konstandin, JCAP 1309 (2013) 024]

Could viscous backreaction lead to ΛCDM*-type expansion?*

[Floerchinger, Tetradis & Wiedemann, 1506.00407]

- Backreaction term $D(z)$ will be *some* function of redshift.
- For given dissipative properties $D(z)$ can be determined, but calculation is involved.
- One may ask simpler question: For what form of $D(z)$ would the expansion be as in the ΛCDM model?
- The *ad hoc* ansatz $D(z) = \text{const} \cdot H(z)$ leads to modified Friedmann equations

$$\bar{\epsilon} - \frac{D}{4H} = \frac{3}{8\pi G_\mathsf{N}} H^2, \qquad \bar{p}_\mathsf{eff} - \frac{D}{12H} = -\frac{1}{8\pi G_\mathsf{N}} \left(2\frac{1}{a}\dot{H} + 3H^2 \right)$$

- In terms of $\hat{\epsilon} = \bar{\epsilon} - \frac{D}{3H}$ one can write

$$\frac{1}{a}\dot{\hat{\epsilon}} + 3H(\hat{\epsilon} + \bar{p}_\mathsf{eff}) = 0, \qquad R + \frac{8\pi G_\mathsf{N} D}{3H} = -8\pi G_\mathsf{N}(\hat{\epsilon} - 3\bar{p}_\mathsf{eff})$$

- For $\bar{p}_\mathsf{eff} = 0$ these are standard equations for ΛCDM model with

$$\Lambda = \frac{2\pi G_\mathsf{N} D}{3H}$$

Modification of Friedmann's equations by backreaction 1

- For universe with fluid velocity inhomogeneities one cannot easily take direct average of Einstein's equations.
- However, fluid equation for energy density and trace of Einstein's equations can be used.
- By integration one finds modified Friedmann equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3} \left[\bar{\epsilon}(\tau) - \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)} \right)^4 a(\tau') D(\tau') \right]$$

- Additive deviation from Friedmann's law for $D(\tau') > 0$
- Part of the total energy density is due to dissipative production

$$\bar{\epsilon} = \bar{\epsilon}_{\mathsf{nd}} + \bar{\epsilon}_{\mathsf{d}}$$

- Assume for dissipatively produced part

$$\dot{\bar{\epsilon}}_{\mathsf{d}} + 3\frac{\dot{a}}{a}(1 + \hat{w}_{\mathsf{d}})\bar{\epsilon}_{\mathsf{d}} = aD$$

Modification of Friedmann's equations by backreaction 2

Leads to another variant of Friedmann's equation

$$H(\tau)^2 = \frac{8\pi G_\mathsf{N}}{3}\left[\bar{\epsilon}_\mathsf{nd}(\tau) + \int_{\tau_\mathsf{I}}^{\tau} d\tau' \left[\left(\frac{a(\tau')}{a(\tau)}\right)^{3+3\hat{w}_\mathsf{d}} - \left(\frac{a(\tau')}{a(\tau)}\right)^4\right] a(\tau')D(\tau')\right]$$

- If the dissipative backreaction D produces pure radiation, $\hat{w}_\mathsf{d} = 1/3$, it does not show up in effective Friedmann equation at all!
- For $\hat{w}_\mathsf{d} < 1/3$ there is a new component with positive contribution on the right hand side of the effective Friedmann equation.
- To understand expansion, parametrize for late times

$$D(\tau) = H(\tau)\left(\frac{a(\tau)}{a(\tau_0)}\right)^{-\kappa}\tilde{D}$$

  with constants $\tilde{D}$ and κ.
- Hubble parameter as function of $(a_0/a) = 1 + z$

$$H(a) = H_0\sqrt{\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_D\left(\frac{a_0}{a}\right)^\kappa}$$

- For $\kappa \approx 0$ the role of Ω_Λ and Ω_D would be similar.

Inhomogeneities in heavy ion collisions

Inhomogeneities are main source of information in cosmology.

Similarly, in heavy ion collisions:

- **Initial fluid perturbations**: Event-by-event fluctuations around averaged fluid fields at time τ_0 and their evolution:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n, electric charge density, electromagnetic fields, ...
- governed by universal evolution equations
- determine particle distributions after freeze-out, e.g. $v_n(p_T)$
- usefull to constrain **thermodynamic and transport properties of QCD**
- contain interesting information from early times

First steps towards fluid dynamic perturbation or response theory

- Linear perturbations around Bjorken flow [Floerchinger & Wiedemann (2011)]
- Linear perturbations around Gubser solution for conformal fluids
  [Gubser & Yarom (2010), Staig & Shuryak (2011), Springer & Stephanov (2013)]
- More detailed investigation of linear perturbations and first steps towards non-linear perturbations around Gubser solution
  [Hatta, Noronha, Torrieri, Xiao (2014)]
- Linear perturbations around general azimuthally symmetric initial state, realistic equation of state
  [Floerchinger & Wiedemann (2013)]
- Characterization of initial conditions by Bessel-Fourier expansion
  [Coleman-Smith, Petersen & Wolpert (2012), Floerchinger & Wiedemann (2013)]
- Comparison to full numerical solution shows good convergence properties of perturbative expansion
  [Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando (2013)]
- Related response formalism for expansion in eccentricities
  [Teaney & Yan (2012), Yan & Ollitrault (2015]

Gravity and thermalization

Consider ensemble of massive particles interacting via gravity only. Start with some velocity distribution. Is there **equilibration/thermalization**...

- ... in Newtonian gravity?
- ... in classical General relativity?
- ... in quantized gravity?

Analogy to other gauge theories suggests that **quantum properties are important for thermalization**

Dissipation by gravity

- Gravitational waves in viscous fluid have life time [Hawking (1966)]

$$\tau_G = \frac{1}{16\pi G_\mathsf{N}\eta}$$

- Diffusive momentum transport by graviton radiation induces viscosity

$$\eta \approx \epsilon_G\, \tau_G$$

  with energy density of gravitational field ϵ_G

- Can be solved for η and τ_G [Weinberg (1972)]

$$\eta = \sqrt{\frac{\epsilon_G}{16\pi G_\mathsf{N}}}, \qquad \tau_G = \sqrt{\frac{1}{16\pi G_\mathsf{N}\epsilon_G}}$$

- Can this really be independent of dark matter mass and density?
- Thermalization time $\sim m_\mathsf{P}/T^2$ is very large
- What determines dissipation on shorter time scales, when classical fields dominate?