Fluid dynamic propagation of initial baryon number perturbations

Stefan Flörchinger (Heidelberg U.)

Initial Stages 2016, Lisbon,

mainly based on

- S. Floerchinger & M. Martinez: *Fluid dynamic propagation of initial baryon number perturbations on a Bjorken flow background* [Phys. Rev. C 92 (2015), 064906]
- S. Floerchinger & U. A. Wiedemann: *Kinetic freeze-out, particle spectra and harmonic flow coefficients from mode-by-mode hydrodynamics* [Phys. Rev. C 89 (2014) 034914]

Baryon number & fluctuations

- Total baryon number $B - \bar{B}$ is conserved.
- For ^{208}Pb - ^{208}Pb collisions $B - \bar{B} = 416$
  Determines total integrated baryon number
- Small compared to total number of baryons $B + \bar{B}$ and other produced particles at at RHIC or LHC energies.
  $\Rightarrow$ Standard assumption:

$$\mu_B = 0$$

- What about local and event-by-event fluctuations?
- Dynamics should be governed by universal fluid dynamics.

Evolution of baryon number in fluid dynamics

- Small perturbation in static medium with $u^\mu = (1,0,0,0)$

$$\frac{\partial}{\partial t}\delta n(t,\vec{x}) = D\vec{\nabla}^2\delta n(t,\vec{x})$$

- Baryon number diffusion constant

$$D = \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

- Heat capacity κ appears here because

baryon diffusion in Landau frame	$\hat{=}$	heat conduction in Eckart frame

- Is D finite for $n \to 0$?

Heat conductivity

- Heat conductivity of QCD rather poorly understood theoretically so far.
- From perturbation theory [Danielewicz & Gyulassy, PRD 31, 53 (1985)]

$$\kappa \sim \frac{T^4}{\mu^2 \alpha_s^2 \ln \alpha_s} \qquad (\mu \ll T)$$

- From AdS/CFT [Son & Starinets, JHEP 0603 (2006)]

$$\kappa = 8\pi^2 \frac{T}{\mu^2}\eta = 2\pi \frac{sT}{\mu^2} \qquad (\mu \ll T)$$

- Baryon diffusion constant D finite for $\mu \to 0$!

Relativistic fluid dynamics

- Evolution of baryon number density from conservation law

$$u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

- Diffusion current ν^α determined by heat conductivity κ

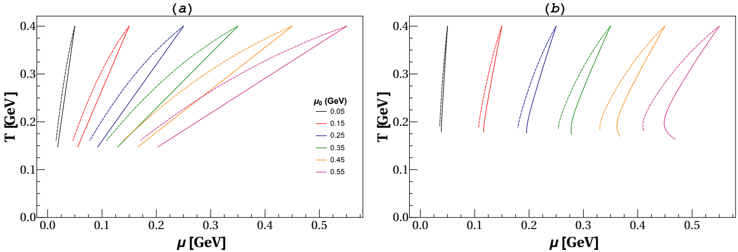
$$\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right)$$

Bjorken expansion

- Consider Bjorken type expansion

$$\partial_\tau \epsilon + (\epsilon + p)\frac{1}{\tau} - \left(\tfrac{4}{3}\eta + \zeta\right)\frac{1}{\tau^2} = 0$$
$$\partial_\tau n + n\frac{1}{\tau} = 0$$

- Heat conductivity κ does not enter by symmetry argument
- Compare ideal gas to lattice QCD equation of state
  [Borsanyi et al., JHEP 08 (2012) 053]



Perturbations around Bjorken expansion

- Consider situation with $\langle n(x) \rangle = \langle \mu(x) \rangle = 0$
- Local event-by-event fluctuation $\delta n \neq 0$
- Concentrate now on Bjorken flow profile for u^μ
- Consider perturbation δn

$$\partial_\tau \delta n + \frac{1}{\tau} \delta n - D(\tau) \left(\partial_x^2 + \partial_y^2 + \frac{1}{\tau^2} \partial_\eta^2 \right) \delta n = 0$$

- Structures in transverse and rapidity directions are “flattened out” by heat conductive dissipation

Solution by Bessel-Fourier expansion

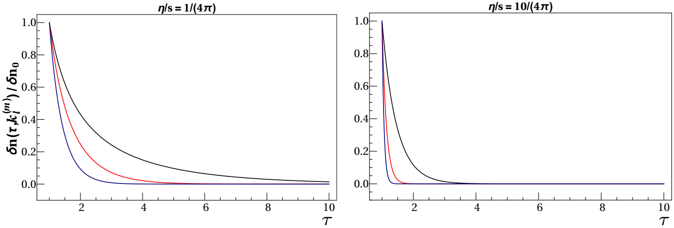
- Expand perturbations like

$$\delta n(\tau, r, \phi, \eta) = \int_0^\infty dk\, k \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \delta n(\tau, k, m, q)\, e^{i(m\phi + q\eta)} J_m(kr)$$

- Leads to ODE

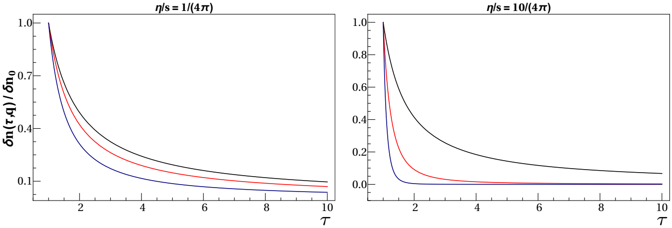
$$\partial_\tau \delta n + \frac{1}{\tau}\delta n + D(\tau)\left(k^2 + \frac{q^2}{\tau^2}\right)\delta n = 0.$$

- For $q = 0$ and different $k \approx 1/\text{fm}$, AdS/CFT value $\kappa = 8\pi^2 \frac{T}{\mu^2}\eta$

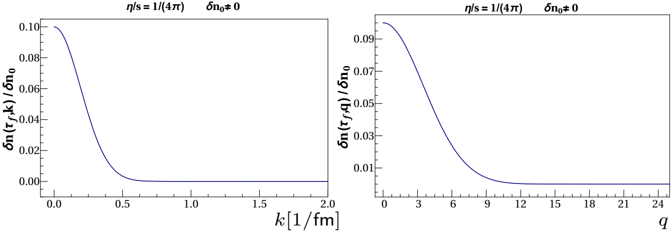


Evolution of perturbations

- For $k = 0$ and different $q = 1, 3, 5$, AdS/CFT value $\kappa = 8\pi^2 \frac{T}{\mu^2}\eta$



- At $\tau = 10\text{fm/c}$



- Only long-range fluctuations survive diffusive damping.

Initial baryon number fluctuation

- Initial baryon number density fluctuations must be known to learn about diffusive transport properties...
- What can be said from first principles?
- How are baryon number fluctuations generated by QCD processes?
- What is the dynamics at very early times?
- Maybe answers at this conference... ?

Glauber type model

- Fluctuations due to nucleon positions: used so far for energy density

$$\epsilon(\tau_0, \mathbf{x}, \eta) = \sum_{i=1}^{N_{\text{part}}} \hat{\epsilon}_w(\mathbf{x} - \mathbf{x}_i)$$

- Can be generalized to baryon number fluctuations

$$n(\tau_0, \mathbf{x}, \eta) = \sum_{i=1}^{N_{\text{part}}} \hat{n}_w(\mathbf{x} - \mathbf{x}_i)$$

- Would generate baryon number fluctuations on nucleon scale
- More general origin of fluctuations is initial state physics and early-time, non-equilibrium dynamics

Baryon number fluctuations at freeze-out

- On freeze-out surface

$$\frac{dN_i}{d^3p d^3x} = f_i(p^\mu; T, u^\mu, \mu, \pi^{\mu\nu}, \pi_{\mathsf{bulk}}, \nu^\mu)$$

- Close-to-equilibrium expansion

$$f_i = f_{i,\mathsf{eq}} + \delta f_i$$

- Equilibrium distribution functions

$$f_{i,\mathsf{eq}} = \frac{1}{e^{\frac{-p_\nu u^\nu - \mu_i}{T}} \pm 1}$$

Baryons and anti-baryons have opposite baryon chemical potential

- Non-equilibrium correction

$$\begin{aligned}\delta f_i =& p_\mu p_\nu \pi^{\mu\nu}\, \tilde{g}_i(p_\mu u^\mu, T, \mu_i) + p_\mu p_\nu \Delta^{\mu\nu}\, \pi_{\mathsf{bulk}}\, \tilde{h}_i(p_\mu u^\mu, T, \mu_i) \\ &+ p_\mu \nu^\mu\, \tilde{k}_i(p_\mu u^\mu, T, \mu_i)\end{aligned}$$

Fluctuations at freeze-out

- Background-perturbation splitting can also be used at freeze-out
- Interesting observable is net baryon number

$$n(\phi, \eta) = (B - \bar{B})(\phi, \eta)$$

- Correlation functions and distributions contain information about baryon number fluctuations
- Two-particle correlation function of baryons minus anti-baryons

$$C_{\text{Baryon}}(\phi_1 - \phi_2, \eta_1 - \eta_2) = \langle n(\phi_1, \eta_1)\, n(\phi_2, \eta_2) \rangle_c$$

Baryon number correlation function

- In Fourier representation

$$C_{\text{Baryon}}(\Delta\phi, \Delta\eta) = \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \tilde{C}_{\text{Baryon}}(m,q)\, e^{im\Delta\phi + iq\Delta\eta}$$

heat conductivity leads to exponential suppression

$$\tilde{C}_{\text{Baryon}}(m,q) = e^{-m^2 I_1 - q^2 I_2}\; \tilde{C}_{\text{Baryon}}(m,q)\big|_{\kappa=0}$$

- I_1 and I_2 can be approximated as

$$I_1 \approx \int_{\tau_0}^{\tau_f} d\tau\, \frac{2}{R^2}\, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

$$I_2 \approx \int_{\tau_0}^{\tau_f} d\tau\, \frac{2}{\tau^2}\, \kappa \left[\frac{nT}{\epsilon + p}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_\epsilon$$

- $I_2 \gg I_1$ would lead to long-range correlations in rapidity direction ("baryon number ridge")

Conclusions

- Baryon number diffusion constant $\sim$ heat conductivity is well defined transport property of the quark-gluon plasma for $\mu \to 0$
- Baryon number fluctuations could allow to constrain it
- Early time baryon diffusion should lead to long-range rapidity correlations in net baryon number
- More knowledge about initial state welcome
- Seems to be interesting topic for further experimental and theoretical studies