Composite chiral fermions from the renormalization group

Stefan Flörchinger (CERN)

ERG 2014, Lefkada, 23/09/2014

Remaining problems of the standard model

- Standard model of elementary particle physics works surprisingly well.
- Seems to describe all measurements at the LHC so far.
- Contains 18 free parameters (without neutrino masses)
  - 3 gauge couplings for U(1), SU(2) and SU(3)
  - 1 Higgs field vacuum expectation value
  - 1 Higgs field self coupling
  - 3 lepton masses
  - 6 quark masses
  - 3 CKM mixing angles + 1 phase

  13 out of 18 parameters are determined by the Yukawa couplings.
- Open questions are:
  - Why are there three generations?
  - What explains the Yukawa-coupling hierarchy between generations?
  - What gives mass to neutrinos?
  - What determines the Higgs VEV? (Hierarchy problem)

Are leptons and quarks composite?

- It seems plausible that there is some structure underlying the standard model that explains the Yukawa couplings.
- Quarks and leptons before electroweak symmetry breaking are chiral: left-handed and right-handed fields in different gauge representations
- Chiral symmetry forbids a mass term.
- Can chiral fermions be composite?
- In principle yes, there is at least no good argument against it.
- Some constrains come from anomaly matching ['t Hooft (1979)].
- However, a formalism to describe this and to determine whether chiral bound states form in a given theory, is lacking.
- For example it is clear that Schrödingers equation cannot be used.

Constituents have not been found so far...

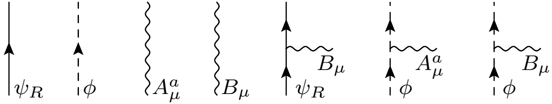
- If leptons and quark consist of more elementary constituents the question arises why these have never been found.
- In principle a confining theory with strong interactions at a very high energy scale could do the job.
- Can only work if this theory has unbroken chiral symmetry in contrast to QCD.

There is no obvious candidate for a theory underlying the standard model so let us sharpen knifes by asking some questions on the standard model itself.

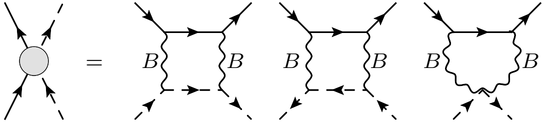
Right-handed fermions and scalar bosons

Start from

- right-handed lepton ψ_R: SU(2) singlet, U(1)$_Y$ charge g'
- mass-less scalar boson ϕ: SU(2) doublet, U(1)$_Y$ charge $-\frac{1}{2}g'$
- gauge fields B_μ for U(1)$_Y$ and A^a_μ for SU(2)



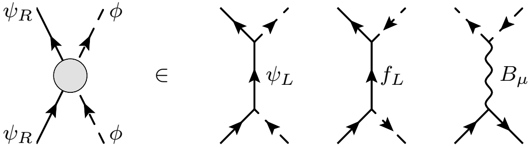
Quantum fluctuations induce fermion-boson vertex $\lambda_{\phi R}$



- all particles in the loop are mass-less
- perturbative one-loop contributions linearly infrared divergent

Composite fields

- What can be composite particles of ψ_R and ϕ?
- Or: What substructures can fermion-boson vertex $\lambda_{\phi R}$ have?



  - left handed lepton ψ_L: SU(2) doublet, U(1)$_Y$ charge $\frac{1}{2}g'$
  - left-handed fermion f_L: SU(2) doublet, U(1)$_Y$ charge $\frac{3}{2}g'$
  - vector boson of B_μ type
- ψ_R and ϕ have opposite U(1)$_Y$ charge or attractive interaction, in favor of bound state ψ_L

Fermionic Hubbard-Stratonovich transformation

- perform Hubbard-Stratonovich transformation with respect to the attractive channel
- field for ψ_L is introduced as auxiliary field with quadratic "Lagrangian"

$$\mathscr{L}_{\mathsf{HS}} = i(\bar{\psi}_L - \bar{\xi}_L)\,\bar{\sigma}^\mu D_\mu\, q_L\, (-D_\nu D^\nu)\ (\psi_L - \xi_L)$$

  - D_μ is covariant derivative appropriate for ψ_L
  - ξ_L is quadratic in right-handed fermion and scalar fields, $\xi_L \sim \phi\psi_R$
  - the function

    $$q_L(p^2) = 1 + \nu_L^2/p^2$$

    contains a non-local mass ν_L
  - for large ν_L the fermion ψ_L is heavy and plays no role

Effective theory after HS transformation

- Right-handed fermions as before, standard kinetic term.
- Left-handed fermions with kinetic term and non-local mass term ν_L

$$\begin{aligned}\mathscr{L}_{\psi_L} =& i\,(\bar{\psi}_L)_{\dot{a}}\,(\bar{\sigma}^\mu)^{\dot{a}b}\left(\partial_\mu - iA^a_\mu t^a_L - iB_\mu y_L\right)(\psi_L)_b \\ &+ i\,\nu_L^2\,(\bar{\psi}_L)_{\dot{a}}\left(\left[\sigma^\mu D_\mu\right]^{-1}\right)^{\dot{a}b}(\psi_L)_b\end{aligned}$$

- Yukawa interactions

$$\mathscr{L}_{\mathsf{Yukawa}} = -h\,\left[(\bar{\psi}_L)_{\dot{a}}\,\phi\,(\psi_R)^{\dot{a}} + (\bar{\psi}_R)^a\,\phi^\dagger\,(\psi_L)_a\right].$$

- Boson-Fermion interaction vertex

$$\mathscr{L}_{\phi R} = i\,(\bar{\psi}_R)^a\,\phi^\dagger\,\lambda_{\phi R}\,(-D^\nu D_\nu)\,(\sigma^\mu)_{a\dot{b}}\,D_\mu\,\phi\,(\psi_R)^{\dot{b}}$$

- Kinetic terms for scalars and gauge fields as before.

Adapting parameters

- Boson-fermion vertex has two contributions

$$\lambda_{\phi R} = (\lambda_{\phi R})_{\text{loops}} - \frac{h^2}{p^2 + \nu_L^2}$$

  - first term generated by radiative corrections / loops
  - second term from HS transformation
- Idea is now to adapt h and ν_L such that $\lambda_{\phi R} = 0$.
- One-loop calculation with IR cutoff Λ gives

$$(\lambda_{\phi R})_{\text{loops}} = \frac{g'^4}{16\pi^2}\left[\frac{1}{4\Lambda^2} - p^2\frac{7}{12\Lambda^4} + \mathcal{O}(p^4)\right].$$

which cancels to the given order in p^2 for

$$h_\Lambda^2 = \frac{3g'^4}{448\pi^2}, \qquad \nu_{L,\Lambda}^2 = \frac{3}{7}\Lambda^2.$$

  - for $g'^2 = \alpha\frac{4\pi}{\cos^2\theta_W}$ with the fine structure constant $\alpha(M_Z) = 1/128$ and $\sin^2\theta_W(M_Z) = 0.23126$ one finds $h_\Lambda = 0.0033$
  - surprisingly close to Yukawa coupling of τ-lepton $h_\tau = 0.0072$
  - non-local mass ν_L vanishes for $\Lambda \to 0$

Exact flow equation with HS transformation

For functional RG study one needs flow equation that implements k-dependent HS transformation [Floerchinger & Wetterich, PLB 680, 371 (2009), see also Gies & Wetterich (2002), Pawlowski (2007)]

$$\begin{aligned}\partial_k \Gamma_k &= \frac{1}{2}\mathsf{STr}\left\{(\Gamma_k^{(2)} + R_k)^{-1}\left(\partial_k R_k - R_k(\partial_k Q^{-1})R_k\right)\right\} \\ &\quad -\frac{1}{2}\overleftarrow{\Gamma}_k^{(1)}\left(\partial_k Q^{-1}\right)\overrightarrow{\Gamma}_k^{(1)}\end{aligned}$$

- exact flow equation that generalizes Wetterich equation
- $\Gamma_k^{(1)}$ is functional derivative with respect to the composite field
- $\partial_k Q^{-1}$ can be chosen arbitrary
- works also for fermionic composite fields

Regulator functions

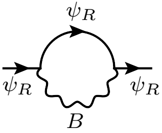
- all relevant diagrams are UV finite
- simple IR regulators are sufficient

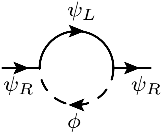
$$
\begin{aligned}
\Delta\mathscr{L}_k = & - i\, k^2\, (\bar{\psi}_L)_{\dot{a}} \left([\sigma^\mu \partial_\mu]^{-1} \right)^{\dot{a}b} (\psi_L)_b \\
& - i\, k^2\, (\bar{\psi}_R)^a \left([\bar{\sigma}^\mu \partial_\mu]^{-1} \right)_{a\dot{b}} (\psi_R)^{\dot{b}} \\
& + k^2 \phi^\dagger \phi \\
& - k^2 \frac{1}{2} \left(A^{a\mu} A^a{}_\mu + B^\mu B_\mu \right) + k^2 \bar{c}^a c^a
\end{aligned}
$$

- regulator functions break gauge invariance
- results presented in the following are for fixed gauge: Feynman gauge

Flow equations for anomalous dimensions

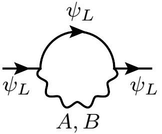
- anomalous dimension right-handed fermions

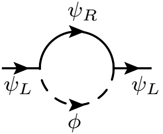




$$(\eta_R)_{\text{loops}} = \frac{1}{16\pi^2}\left[4g'^2 + 2h^2\frac{k^2}{\nu_L^2}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right)\right]$$

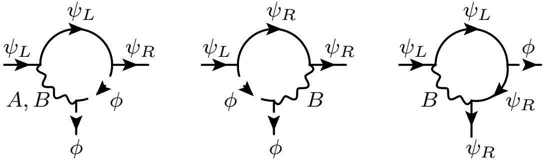
- anomalous dimension left-handed fermions





$$(\eta_L)_{\text{loops}} = \frac{1}{16\pi^2}\left[\left(3g^2 + g'^2\right)\frac{k^2}{\nu_L^2}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right) + 2h^2\right]$$

Flow equations Yukawa coupling



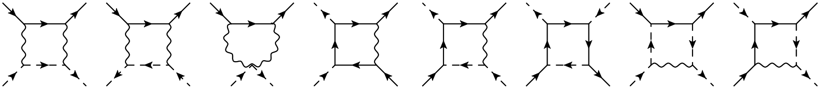
Yukawa coupling at vanishing momentum

$$(\partial_t h)_{\mathsf{loops}} = \frac{1}{16\pi^2}\Bigg[- h\left(3g^2 - g'^2\right)\frac{k^2}{\nu_L^2}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right) - 2\,h\,g'^2 - 8\,h\,g'^2\frac{k^2}{\nu_L^2}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right)\Bigg]$$

First derivative with respect to fermion momentum p^2

$$(\partial_t h')_{\mathsf{loops}} = \frac{1}{16\pi^2}\Bigg[h\left(\frac{3}{4}g^2 - \frac{1}{4}g'^2\right)\Bigg[-12\frac{k^2}{\nu_L^4} + 6\frac{2k^4+k^2\nu_L^2}{\nu_L^6} \times \ln\left(\frac{k^2+\nu_L^2}{k^2}\right)\Bigg] + \frac{1}{2}\,h\,g'^2\frac{1}{k^2}\Bigg]$$

Flow equation boson-fermion vertex



at vanishing momentum

$$(\partial_t \lambda_{\phi R})_{\text{loops}} = \frac{1}{16\pi^2}\Bigg[-\tfrac{1}{2} g'^4 \frac{1}{k^2} + 8h^2 g'^2 \frac{1}{k^2+\nu_L^2} - 3h^4 \frac{k^2}{\nu_L^4} \ln\left(\frac{(2\nu_L^2+k^2)k^2}{(\nu_L^2+k^2)}\right) \\ - h^2\left(\tfrac{3}{2}g^2 + \tfrac{1}{2}g'^2\right)\left[\frac{3k^2+2\nu_L^2}{\nu_L^2(\nu_L^2+k^2)} - \frac{3k^2}{\nu_L^4}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right)\right]\Bigg]$$

first derivative with respect to fermion momentum p^2

$$(\partial_t \lambda'_{\phi R})_{\text{loops}} = \frac{1}{16\pi^2}\Bigg[\tfrac{7}{3} g'^4 \frac{1}{k^4} + 2h^2 g'^2 \frac{k^2+2\nu_L^2}{(k^2+\nu_L^2)^2 k^2} - h^2\left(\frac{3}{2}g^2 + \frac{1}{2}g'^2\right) \\ \times\left[-\frac{24k^2}{\nu_L^6} - \frac{2}{k^2\nu_L^2} + \frac{2}{k^2(k^2+\nu_L^2)} + \frac{12k^2(2k^2+\nu_L^2)}{\nu_L^8}\ln\left(\frac{k^2+\nu_L^2}{k^2}\right)\right]\Bigg]$$

Scale-dependent HS transformation

- choose parameters of k-dependent HS transformation such that

$$\partial_k \lambda_{\phi R}(p^2)\big|_{p^2=0} = 0, \qquad \partial_k \lambda'_{\phi R}(p^2)\big|_{p^2=0} = 0.$$

- choose also p-dependent wave-function renormalization for composite field $\psi_L(p)$ such that

$$\partial_k h(p^2)\big|_{p^2=0} = 0.$$

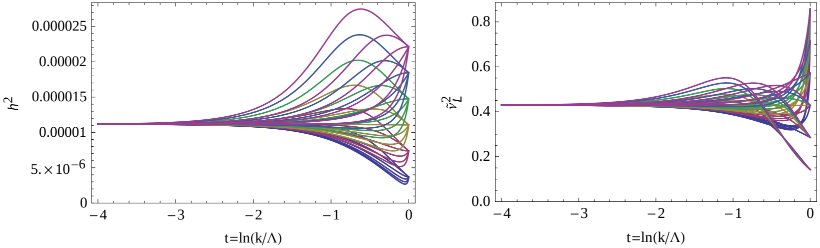
- that gives final flow equations for non-local mass

$$\begin{aligned}\partial_t \nu_L^2 =& (\eta_L)_{\text{loops}}\, \nu_L^2 + \frac{\nu_L^4}{h^2}(\partial_t \lambda_{\phi_R})_{\text{loops}} + \frac{\nu_L^6}{h^2}(\partial_t \lambda'_{\phi R})_{\text{loops}} \\ &+ \frac{2\nu_L^4}{h}(\partial_t h')_{\text{loops}}\end{aligned}$$

and the Yukawa coupling

$$\begin{aligned}\partial_t h^2 =& 2h\,(\partial_t h)_{\text{loops}} + h^2\left[(\eta_R)_{\text{loops}} + (\eta_L)_{\text{loops}}\right] \\ &+ 2\nu_L^2\,(\partial_t \lambda_{\phi R})_{\text{loops}} + \nu_L^4 (\partial_t \lambda'_{\phi R})_{\text{loops}} + \nu_L^2\, 2h\,(\partial_t h')_{\text{loops}}\end{aligned}$$

Solution of flow equations



- for fixed gauge couplings $g(M_Z) = 0.651$ and $g'(M_Z) = 0.807$
- fixed point approximately at

$$h^{*2} = \frac{3g'^4}{448\pi^2} \approx 0.000011, \qquad \tilde{\nu}_L^{*2} = \frac{\nu_L^2}{k^2} = \frac{3}{7} \approx 0.43$$

- non-local mass parameter ν_L vanishes with k
- Yukawa coupling related to $\mathrm{U}(1)_Y$ gauge coupling
- numerical value $h^* = 0.0033$ close to $h_{\tau\text{-lepton}} = 0.0072$

Flow of gauge couplings

- One loop perturbative flow equations

$$\partial_t g = -\frac{\frac{22}{3} - \frac{1}{3}(n_{l_L} + 3n_{q_L}) - \frac{1}{6}}{16\pi^2} g^3,$$

$$\partial_t g' = \frac{\frac{2}{3}\left(\frac{1}{2}n_{l_L} + n_{l_R^e} + \frac{1}{6}n_{q_L} + \frac{4}{3}n_{q_R^u} + \frac{1}{3}n_{q_R^d}\right) + \frac{1}{6}}{16\pi^2} g'^3,$$

  where the fermion content is

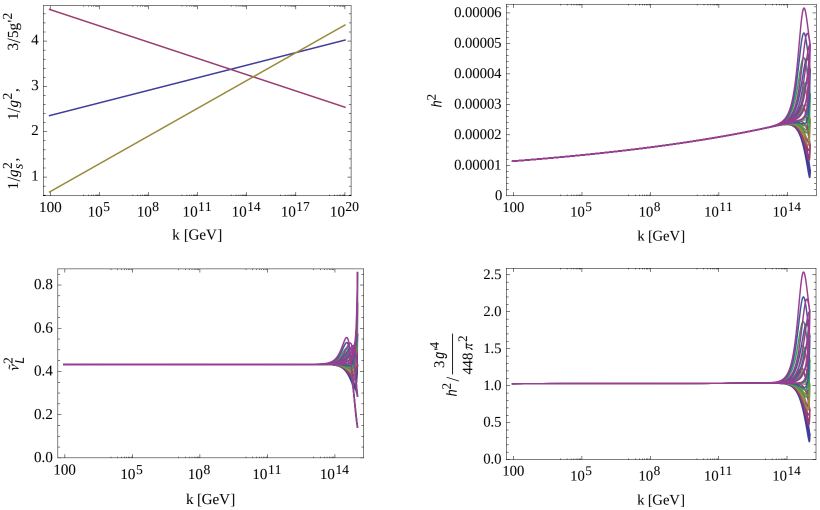
  - n_{l_L} left-handed leptons,
  - $n_{l_R^e}$ right-handed leptons of electron type,
  - n_{q_L} left-handed quarks,
  - $n_{q_R^u}$ right-handed quarks of up-type,
  - $n_{q_R^d}$ right-handed quarks of down-type

- For the standard model with complete fermion content

$$g^2(k) = \frac{1}{\frac{1}{g^2(k_0)} + \frac{19}{96\pi^2}\ln(k/k_0)},$$

$$g'^2(k) = \frac{1}{\frac{1}{g'^2(k_0)} - \frac{41}{96\pi^2}\ln(k/k_0)}.$$

Flow with flowing gauge couplings

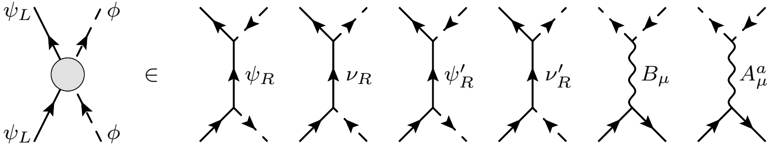


Remarks on anomalies

- it is known that theories with only right-handed fermions (or only left-handed fermions) lead to gauge anomalies
- on first sight this seems to make an initial theory with only right-handed fermions inconsistent
- on the other side, the auxiliary fields that are added by the Hubbard-Stratonovich transformation can also contribute to the anomaly and might even cancel it
- quite generally, theories with composite chiral fermions must fulfill anomaly matching conditions ['t Hooft (1979)]
- these issues need more study

Composite right-handed fermions

- also right-handed fermions might be composite



- combinations of left-handed fermions ψ_L and scalars ϕ
  - right-handed fermion ψ_R: SU(2) singlet, U(1)$_Y$ charge g'
  - right-handed fermion ν_L: SU(2) singlet, U(1)$_Y$ charge 0
  - right-handed fermion ψ'_R: SU(2) triplet, U(1)$_Y$ charge g'
  - right-handed fermion ν'_L: SU(2) triplet, U(1)$_Y$ charge 0
  - vector boson of B_μ type
  - vector boson of A^a_μ type
- ψ_L and ϕ can be bound by U(1)$_Y$ or SU(2) interactions
- attractive U(1)$_Y$ interaction favors right-handed neutrino type ν_R

Conclusions

- Left-handed τ-lepton could be composite of scalar doublet and right-handed τ-lepton!
- Yukawa coupling can be predicted and agrees up to factor ~ 2 with experimental value but good agreement could be partly accidental.
- Theoretical uncertainties still high:
  - Fierz ambiguities in Hubbard-Stratonovich transformation
  - Effect of scalar field self interaction and vacuum expectation value
- Flow equation with scale-dependent Hubbard-Stratonovich transformation can be used to investigate this interesting physics.
- More detailed analysis needed to investigate possibilities for other bound states (right-handed neutrinos ?).
- Question of anomalies needs further studies.