Backreaction effects on the matter side of Einsteins field equations

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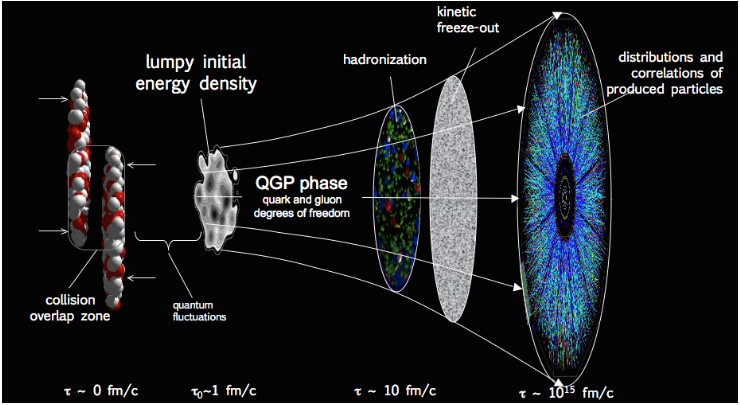
Seminar at Geneva University, May 22, 2015.

based on:

- S. Floerchinger, N. Tetradis and U. A. Wiedemann, *Accelerating Cosmological Expansion from Shear and Bulk Viscosity*, [Phys. Rev. Lett. 114, 091301 (2015)]
- work in progress with Diego Blas, Mathias Garny, Nikolaos Tetradis and Urs A. Wiedemann

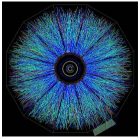
From heavy ion collisions to cosmology

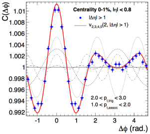


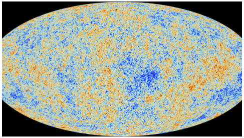
- goal of heavy ion program is to understand QFT in- and out-of-equilibrium
- needed e. g. for Condensed matter physics, Cosmology
- ongoing experiments at RHIC and at the LHC
- standard model of heavy ion collision is now in terms of **viscous relativistic fluid dynamics**

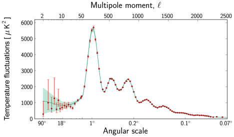
Fluid dynamic perturbation theory for heavy ions

proposed in: [Floerchinger & Wiedemann, PLB 728, 407 (2014)]









- goal: determine transport properties experimentally
- so far: numerical fluid simulations e.g. [Heinz & Snellings (2013)]
- new: solve fluid equations for smooth and symmetric background and order-by-order in perturbations
- less numerical effort – more systematic studies
- good convergence properties [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]
- similar technique used in cosmology since many years
- some insights from heavy ion physics might be useful for cosmology

Backreaction: General idea

- for $0+1$ dimensional, non-linear dynamics

$$\dot{\varphi} = f(\varphi) = f_0 + f_1\,\varphi + \tfrac{1}{2} f_2\,\varphi^2 + \ldots$$

- one has for expectation values $\bar{\varphi} = \langle \varphi \rangle$

$$\dot{\bar{\varphi}} = f_0 + f_1\,\bar{\varphi} + \tfrac{1}{2} f_2\,\bar{\varphi}^2 + \tfrac{1}{2} f_2\,\langle (\varphi - \bar{\varphi})^2 \rangle + \ldots$$

- evolution equation for expectation value $\bar{\varphi}$ depends on two-point correlation function or spectrum $P_2 = \langle (\varphi - \bar{\varphi})^2 \rangle$
- evolution equation for spectrum depends on bispectrum and so on
- more complicated for higher dimensional theories
- more complicated for gauge theories such as gravity

Backreaction in gravity

- Einstein's equations are non-linear.
- Important question [Ellis (1984)]: If Einstein's field equations describe small scales, including inhomogeneities, do they also hold on large scales?
- Is there a sizable backreaction from inhomogeneities to evolution of background fields, i.e. to the cosmological expansion?
- Difficult question, has been studied by many people [Ellis & Stoeger (1987); Mukhanov, Abramo & Brandenberger (1997); Unruh (1998); Buchert (2000); Geshnzjani & Brandenberger (2002); Schwarz (2002); Wetterich (2003); Räsänen (2004); Kolb, Matarrese & Riotto (2006); Brown, Behrend, Malik (2009); Gasperini, Marozzi & Veneziano (2009); Clarkson & Umeh (2011); Green & Wald (2011); ...]
- Recent reviews: [Buchert & Räsänen, Ann. Rev. Nucl. Part. Sci. 62, 57 (2012); Green & Wald, Class. Quant. Grav. 31, 234003 (2014)]
- No general consensus but most people believe now that **gravitational backreaction is rather small**.
- Here we look at a **backreaction on the matter side** of Einstein's equations.

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = (\epsilon + p + \pi_{\mathsf{bulk}})u^\mu u^\nu + (p + \pi_{\mathsf{bulk}})g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- energy density ϵ, pressure $p = p(\epsilon)$, number density n
- fluid velocity u^μ
- constitutive relations for viscous terms in derivative expansion
  - bulk viscous pressure $\pi_{\mathsf{bulk}} = -\zeta\, \nabla_\mu u^\mu + \dots$
  - shear stress $\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right] + \dots$
  - diffusion current $\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta \left(\frac{\mu}{T}\right) + \dots$

Fluid dynamic equations from covariant **conservation laws**

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu N^\mu = 0.$$

Fluid equation for energy density

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu - \zeta\Theta^2 - 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} = 0$$

with

- bulk viscosity ζ, shear viscosity η
- expansion scalar

$$\Theta = \nabla_\mu u^\mu = \frac{1}{a}\left[3\frac{\dot a}{a} + \vec\nabla\cdot\vec v - \Psi\vec\nabla\cdot\vec v - 3\frac{\dot a}{a}\Psi - 3\dot\Phi - 3\vec v\cdot\vec\nabla\Phi + \dots\right]$$

- shear stress $(\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu)$

$$\sigma^{\mu\nu} = \left[\tfrac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta} + \tfrac{1}{2}\Delta^{\mu\beta}\Delta^{\nu\alpha} - \tfrac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\nabla_\alpha u_\beta$$

For small fluid velocity $\vec v^2 \ll c^2$ and Newtonian potentials $\Phi, \Psi \ll 1$

$$\dot\epsilon + \vec v\cdot\vec\nabla\epsilon + (\epsilon + p)\left(3\frac{\dot a}{a} + \vec\nabla\cdot\vec v\right)$$
$$= \frac{\zeta}{a}\left[3\frac{\dot a}{a} + \vec\nabla\cdot\vec v\right]^2 + \frac{\eta}{a}\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}(\vec\nabla\cdot\vec v)^2\right]$$

Fluid dynamic backreaction in Cosmology

Expectation value of energy density $\bar{\epsilon} = \langle \epsilon \rangle$

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p} - 3\bar{\zeta}H\right) = D$$

with dissipative backreaction term

$$D = \frac{1}{a^2}\langle \eta\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle$$
$$+ \frac{1}{a^2}\langle \zeta [\vec{\nabla}\cdot\vec{v}]^2\rangle + \frac{1}{a}\langle \vec{v}\cdot\vec{\nabla}\left(p - 6\zeta H\right)\rangle$$

- D vanishes for unperturbed homogeneous and isotropic universe
- D has contribution from shear viscosity, bulk viscosity and thermodynamic work done by contraction against pressure gradients
- viscous terms in D are positive semi-definite
- bulk viscous pressure $\pi_{\text{bulk}} = -3\bar{\zeta}H$ already present in background
- for $\frac{1}{a}\vec{\nabla}\vec{v} \sim H$ backreaction term at same order
- for spatially constant viscosities and scalar perturbations only

$$D = \frac{\bar{\zeta} + \frac{4}{3}\bar{\eta}}{a^2}\int d^3q \; P_{\theta\theta}(q)$$

Why is dissipation interesting for (late time) cosmology?

- Ideal cosmological fluid can be characterized by the equation of state. Allows to distinguish and constrain different components
  - radiation
  - cold and warm dark matter
  - dark energy
- Dissipative or transport properties can provide finer details
  - example: Silk damping
- Constraints of viscous properties in the dark sector such as
  - shear and bulk viscosity
  - heat conductivity
  - relaxation times ...

  could lead to nice constraints of dark matter models
- For curiosity one may also ask whether qualitatively new effects can arise from dissipation

Dissipative effects in cosmological expansion

So far only homogenous expansion + linear perturbations investigated

- for homogeneous and isotropic expansion only effect from bulk viscosity
- bulk viscous pressure is negative

$$p_{\text{eff}} = p + \pi_{\text{bulk}} = p - 3\zeta H$$

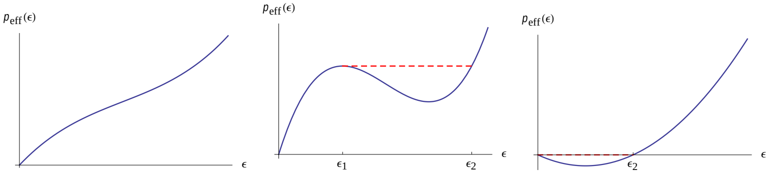
- negative viscous pressure $\pi_{\text{bulk}} < 0$ acts similar to dark energy and can be used to accelerate the universe
  [Murphy (1973); Padmanabhan & Chitre (1987); Fabris, Goncalves & de Sa Ribeiro (2006); Li & Barrow (2009); Velten & Schwarz (2011); Gagnon & Lesgourgues (2011); ...]
- Model based on scalar particles with small self interaction leads to phenomenologically plausible cosmology
  [Gagnon & Lesgourgues (2011)]

Is negative effective pressure physical?

- Accelerated expansion from bulk viscous pressure for homogeneous universe needs negative effective pressure $p_{\text{eff}} = p - 3\zeta H < 0$
- Is this physically plausible?
- Kinetic theory gives

$$p_{\text{eff}}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E_{\vec{p}}} f(x, \vec{p}) \geq 0$$

- There is also a fluid dynamic stability argument against $p_{\text{eff}} < 0$



If there is a vacuum with $\epsilon = p_{\text{eff}} = 0$, negative pressure phases cannot be stable. (But could be metastable.)

Dissipation of perturbations

- The dissipative backreaction does not need negative effective pressure

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p}_{\mathsf{eff}}\right) = D$$

- D is an integral over perturbations, could become large at late times.
- Can it potentially accelerate the universe?
- Need equation for scale parameter.
- Use trace of Einstein's equations $R = 8\pi G_{\mathsf{N}} T^{\mu}{}_{\mu}$

$$\frac{1}{a}\dot{H} + 2H^2 = \frac{4\pi G_{\mathsf{N}}}{3}\left(\bar{\epsilon} - 3\bar{p}_{\mathsf{eff}}\right)$$

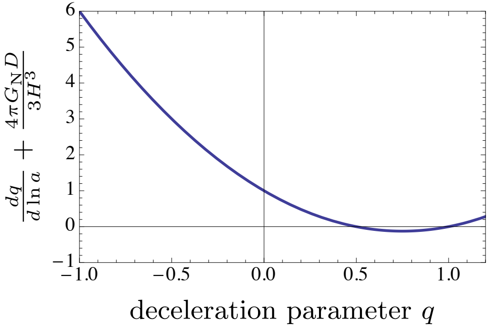
  does not depend on unknown quantities like $\langle(\epsilon + p_{\mathsf{eff}})u^{\mu}u^{\nu}\rangle$
- To close the equations one needs equation of state $\bar{p}_{\mathsf{eff}} = \bar{p}_{\mathsf{eff}}(\bar{\epsilon})$ and dissipation parameter D

Deceleration parameter

- assume now simple equation of state $p_{\text{eff}} = \hat{w}\ \epsilon$
- obtain for deceleration parameter $q = -1 - \frac{\dot{H}}{aH^2}$

$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \tfrac{1}{2}(1+3\hat{w})\right) = \frac{4\pi G_{\text{N}} D(1-3\hat{w})}{3H^3}$$

- for $D = 0$ attractive fixed point at $q_* = \frac{1+3\hat{w}}{2}$ and in particular $q_* = \frac{1}{2}$ for matter domination (deceleration)
- for $D > 0$ the fixed point is shifted to larger values



Could viscous backreaction lead to ΛCDM *expansion?*

- Backreaction term $D(z)$ will be *some* function of redshift.
- For given dissipative properties $D(z)$ can be determined, but calculation is involved.
- One may ask simpler question: For what form of $D(z)$ would the expansion be as in the ΛCDM model?
- The *ad hoc* ansatz $D(z) = \text{const} \cdot H(z)$ leads to modified Friedmann equations

$$\bar{\epsilon} - \frac{D}{4H} = \frac{3}{8\pi G_{\mathsf{N}}} H^2, \qquad \bar{p}_{\text{eff}} - \frac{D}{12H} = -\frac{1}{8\pi G_{\mathsf{N}}} \left(2\frac{1}{a}\dot{H} + 3H^2 \right)$$

- In terms of $\hat{\epsilon} = \bar{\epsilon} - \frac{D}{3H}$ one can write

$$\frac{1}{a}\dot{\hat{\epsilon}} + 3H(\hat{\epsilon} + \bar{p}_{\text{eff}}) = 0, \qquad R + \frac{8\pi G_{\mathsf{N}} D}{3H} = -8\pi G_{\mathsf{N}}(\hat{\epsilon} - 3\bar{p}_{\text{eff}})$$

- For $\bar{p}_{\text{eff}} = 0$ these are standard equations for ΛCDM model with

$$\Lambda = \frac{2\pi G_{\mathsf{N}} D}{3H}$$

Estimating viscous backreaction D

- For $\frac{4\pi G_\mathsf{N} D}{3H^3} \approx 4$ one could explain the current accelerated expansion ($q \approx -0.6$) by dissipative backreaction.
- Is this possible?
- In principle one can determine D for given equation of state and viscous properties from dynamics of structure formation.
- So far only rough estimates. If shear viscosity dominates:

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle \approx \sigma \bar{\eta} H^2$$

  with $\sigma = \mathcal{O}(1)$. Corresponds to $\Delta v \approx 100\,\mathsf{km/s}$ for $\Delta x \approx 1\,\mathsf{MPc}$
- Leads to

$$\frac{4\pi G_\mathsf{N} D}{3H^3} \approx \frac{\sigma \bar{\eta} H}{2\rho_c}$$

  with $\rho_c = \frac{3H^2}{8\pi G_\mathsf{N}}$
- Need also estimate for viscosity η

Viscosities

- Shear and bulk viscosity arise from transport of momentum.
- Relativistic particles / radiation contribute to shear viscosity

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

  - prefactor $c_\eta = \mathcal{O}(1)$
  - energy density of radiation ϵ_R
  - mean free time τ_R, large for small interaction cross section
  - fluid approximation requires $\tau_R H < 1$
- Bulk viscosity vanishes in situations with conformal symmetry but can be large when conformal symmetry is broken.
- For massive scalar particles with $\lambda\varphi^4$ interaction [Jeon & Yaffe (1996)]

$$\zeta \sim \frac{m^6}{\lambda^4 T^3} e^{2m/T}, \qquad \eta \sim \frac{m^{5/2} T^{1/2}}{\lambda^2} \qquad \text{for} \qquad \frac{T}{m} \ll 1$$

- Bulk viscosity can become large when inelastic collisions become rare

Estimating viscous backreaction D

Concentrate on shear viscosity induced by radiation

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{\sigma\bar{\eta}H}{2\rho_c} = \frac{\sigma c_\eta}{2}\frac{\epsilon_R}{\rho_c}\,\tau_R H$$

- could only become of order one for $\epsilon_R/\rho_c = \mathcal{O}(1)$, $\tau_R H = \mathcal{O}(1)$
- photons or rel. neutrinos have too long mean free times, $\tau_R H \gg 1$
- curiously, gravitons have mean free time [Hawking (1966)]

$$\tau_G = \frac{1}{16\pi G_{\mathsf{N}}\eta}$$

- allows to solve for η and τ_G [Weinberg (1972)] and leads to

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \sigma\sqrt{\frac{c_\eta \epsilon_G}{24\rho_c}}$$

- presumably too small to have an interesting effect

Considerations illustrate how D is determined by the properties of different components of the universe or could be used to constrain them.

How is structure formation modified?

- Viscosities modify dynamics of structure formation.
- On linear level this can be easily taken into account.
  - from energy conservation ($\theta = \vec{\nabla} \cdot \vec{v}$)

$$\dot{\delta\epsilon} + 3\frac{\dot{a}}{a}\delta\epsilon + \bar{\epsilon}\,\theta = 0$$

  - from Navier-Stokes equation

$$\bar{\epsilon}\left[\dot{\theta} + \frac{\dot{a}}{a}\theta - q^2\psi\right] + \frac{1}{a}\left(\zeta + \frac{4}{3}\eta\right)q^2\theta = 0$$

  - Poisson equation

$$-q^2\psi = 4\pi G_{\mathsf{N}} a^2 \delta\epsilon$$

- In terms of $\delta = \frac{\delta\epsilon}{\bar{\epsilon}}$

$$\ddot{\delta} + \left[\frac{\dot{a}}{a} + \frac{\zeta + \frac{4}{3}\eta}{a\bar{\epsilon}}q^2\right]\dot{\delta} - 4\pi G_{\mathsf{N}}\bar{\epsilon}\,\delta = 0$$

  The viscosites slow down gravitational collapse but do not wash out structure.
- More detailed investigations in progress.

Conclusions

- Backreaction term D describes how energy from perturbation is dissipated into background energy density.
- It is clear that there are dissipative effects of some size also in the dark sector, but unclear of what size.
- Fluid velocity gradients grow due to gravitational collapse but could be dissipated by viscosity.
- If dissipative effects are large they could potentially explain the observed accelerated expansion.
- Dissipative backreaction is triggered by structure formation so it would be natural that it sets in at late times.
- If dissipative effects are smaller they are still interesting in order to learn more about the dark sector.

Backup

Modification of Friedmann's equations 1

- For universe with fluid velocity inhomogeneities one cannot easily take direct average of Einstein's equations.
- However, fluid equation for energy density and trace of Einstein's equations can be used.
- By integration one finds modified Friedmann equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}(\tau) - \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)}\right)^4 a(\tau')D(\tau')\right]$$

- Additive deviation from Friedmann's law for $D(\tau') > 0$
- Part of the total energy density is due to dissipative production

$$\bar{\epsilon} = \bar{\epsilon}_{\mathsf{nd}} + \bar{\epsilon}_{\mathsf{d}}$$

- Assume for dissipatively produced part

$$\dot{\bar{\epsilon}}_{\mathsf{d}} + 3\frac{\dot{a}}{a}(1+\hat{w}_{\mathsf{d}})\bar{\epsilon}_{\mathsf{d}} = aD$$

Modification of Friedmann's equations 2

Leads to another variant of Friedmann's equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}_{\mathsf{nd}}(\tau) + \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left[\left(\frac{a(\tau')}{a(\tau)}\right)^{3+3\hat{w}_{\mathsf{d}}} - \left(\frac{a(\tau')}{a(\tau)}\right)^{4}\right] a(\tau') D(\tau')\right]$$

- If the dissipative backreaction D produces pure radiation, $\hat{w}_{\mathsf{d}} = 1/3$, it does not show up in effective Friedmann equation at all!
- For $\hat{w}_{\mathsf{d}} < 1/3$ there is a new component with positive contribution on the right hand side of the effective Friedmann equation.
- To understand expansion, parametrize for late times

$$D(\tau) = H(\tau)\left(\frac{a(\tau)}{a(\tau_0)}\right)^{-\kappa}\tilde{D}$$

with constants $\tilde{D}$ and κ.

- Hubble parameter as function of $(a_0/a) = 1 + z$

$$H(a) = H_0\sqrt{\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_D\left(\frac{a_0}{a}\right)^{\kappa}}$$

- For $\kappa \approx 0$ the role of Ω_Λ and Ω_D would be similar.