Isotropization from Color Field Condensate in heavy ion collisions

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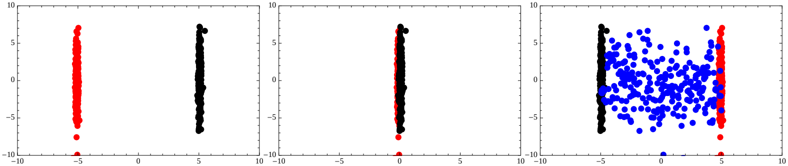
RBRC Workshop on "The Approach to Equilibrium in Strongly Interacting Matter", BNL, April 2, 2014.

based on:
S. Floerchinger and C. Wetterich, *Isotropization from Color Field Condensate in heavy ion collisions*, [JHEP 03 (2014) 121].

Heavy Ion Collisions



- ions are strongly Lorentz-contracted
- *some* medium is produced after collision
- medium expands in longitudinal direction and gets diluted

Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...
- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down
- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more
- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more

The puzzle of thermalization / isotropization

- Hydrodynamic description works well when started at $\tau_0 \approx 0.5\,\mathrm{fm/c}$.
- Perturbative time-scale for thermalization is much longer [Baier, Mueller, Schiff, Son (2001)].
- Effective hydrodynamic description for some quantities may also be possible without local equilibrium and detailed balance.
- Some quantities e.g. pressure may thermalize faster than others: "Prethermalization" [Berges, Borsanyi, Wetterich (2004)].
- In praxis hydro description *does* assume early local equilibrium and it works rather well with that.
- There must be some nontrivial mechanism of thermalization / isotropization to be understood.
- Another puzzle is: How does entropy and particle production work?

Could macroscopic / classical fields be the solution?

- Field expectation value or "classical field" has influence on quasi-particle excitations and leads to
  - modified vertices
  - modified dispersion relations / self energies
- That could lead to higher scattering rates and faster thermalization.
- Dynamical evolution of classical fields itself might also contribute to isotropization.
- Classical fields can also induce instabilities / particle production.

Large occupation numbers versus condensate

- In thermodynamic limit (stationary, infinite volume) a classical field corresponds to large occupation number of zero-mode: a *condensate*.
- For realistic heavy-ion collision one may have
  - non-equilibrium situation
  - finite size
  - finite number of gluons.
- Distinction between condensate and large occupation numbers for a few modes is not so clear.
- Nevertheless, condensate picture may be easiest way to capture important features of situation with large occupation numbers.
- Gluon condensate were also discussed in kinetic theory framework. [Blaizot, Gelis, Liao, McLerran, Venugopalan, Epelbaum, Berges, Schlichting, Sexty, Kurkela, Moore,...]

Is a homogeneous and isotropic color field possible?

- Expectation value for vector field $\langle \mathbf{A}_\mu \rangle$ breaks rotation invariance except for $\mu = 0$ component.
- But $\mathbf{A_0}$ is gauge degree of freedom.
- One can choose Weyl or temporal gauge, $\mathbf{A_0 = 0}$.
- Seems to suggest that homogeneous and isotropic color field is *not* possible.

Modified rotation symmetry

- One can combine rotations with gauge transformations into a modified rotation transformation [Reuter, Wetterich (1994)].
- Group theoretic: embed $SU(2) \in SU(3)$.
- Gauge singlets rotate in the normal way.
- There are two inequivalent embeddings of this type. For one of them Lie algebra of $SU(2)$ spanned by Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$.
- Contains a singlet

$$(A_j)_{mn} = \sigma \, \epsilon_{jmn}$$

- More general, temporal part $\mathbf{A}_0$ transforms like

$$\mathbf{8} = \mathbf{5} + \mathbf{3},$$

and spatial part $\mathbf{A}_j$ like

$$\mathbf{24} = \mathbf{7} + 2 \times \mathbf{5} + 2 \times \mathbf{3} + \mathbf{1}.$$

Field configurations with cylindrical symmetry

- There is only one candidate for isotropic condensate σ, i.e. a singlet under three-dimensional rotations.
- For cylindrical symmetry, i.e. reduced symmetry under
  - rotations in the transverse plane of x_1, x_2,
  - rotations of 180° around x_1 or x_2 axis,

  one has two more condensate candidates, $\tilde{\gamma}^A$ and $\tilde{\gamma}^B$.
- For space parity transformations $\mathsf{P}(\mathbf{A_0}, \mathbf{A_j}) = (\mathbf{A_0}, -\mathbf{A_j})$ one has

$$\mathsf{P}\,\sigma = -\sigma, \qquad \mathsf{P}\,\tilde{\gamma}^{\mathsf{A}} = -\tilde{\gamma}^{\mathsf{A}}, \qquad \mathsf{P}\,\tilde{\gamma}^{\mathsf{B}} = -\tilde{\gamma}^{\mathsf{B}}.$$

- For color charge conjugation $\mathsf{C}\,\mathbf{A}_\mu = -\mathbf{A}_\mu^*$ one has

$$\mathsf{C}\,\sigma = \sigma, \qquad \mathsf{C}\,\tilde{\gamma}^{\mathsf{A}} = -\tilde{\gamma}^{\mathsf{A}}, \qquad \mathsf{C}\,\tilde{\gamma}^{\mathsf{B}} = \tilde{\gamma}^{\mathsf{B}}.$$

  and accordingly for CP

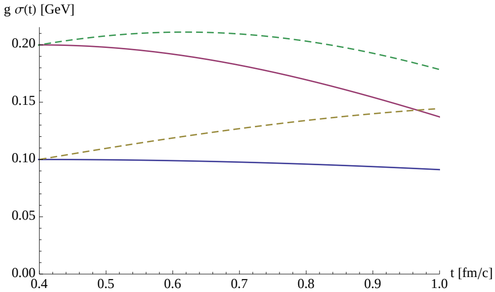
$$\mathsf{CP}\,\sigma = -\sigma, \qquad \mathsf{CP}\,\tilde{\gamma}^{\mathsf{A}} = \tilde{\gamma}^{\mathsf{A}}, \qquad \mathsf{CP}\,\tilde{\gamma}^{\mathsf{B}} = -\tilde{\gamma}^{\mathsf{B}}.$$

Time evolution of condensates 1

- Time evolution of condensate in general quite complicated due to quantum effects.
- Qualitative guiding from classical Yang-Mills equations.
- For isotropic and homogeneous condensate σ

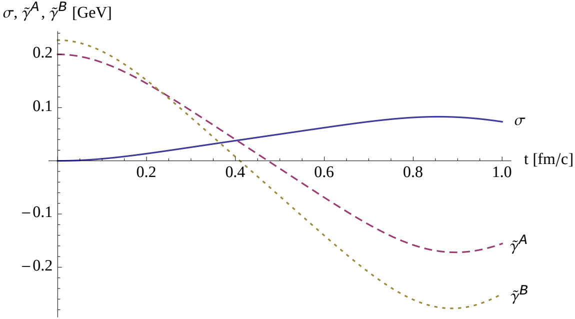
$$\partial_t^2 \sigma = -2g^2\sigma^3.$$

Anharmonic oscillator, solution in terms of Jacobi elliptic functions.



Time evolution of condensate 2

- Isotropic and cylindric condensates have coupled evolution equations.
- Can be easily solved numerically.
- Isotropic condensate σ can be generated from $\tilde{\gamma}^A$, $\tilde{\gamma}^B$. For example:



Energy-momentum tensor due to condensates

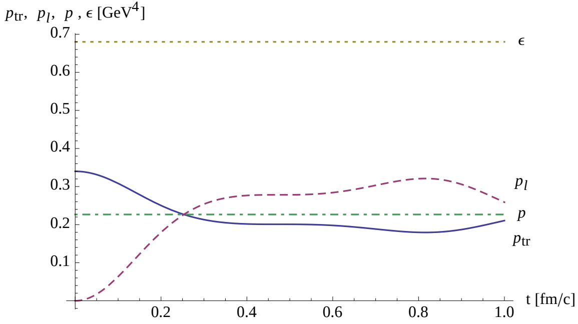
- Energy-momentum tensor due to condensates

$$T^{\mu\nu} = 2\text{tr}\, \mathbf{F}^{\rho\mu}\mathbf{F}_{\rho}{}^{\nu} - \frac{1}{2}g^{\mu\nu}\ \text{tr}\ \mathbf{F}^{\alpha\beta}\mathbf{F}_{\alpha\beta}.$$

- Assume that energy and momentum are dominated by this.

$$T^{\mu\nu} = \text{diag}(\epsilon, p_{tr}, p_{tr}, p_l).$$

For same example as above:



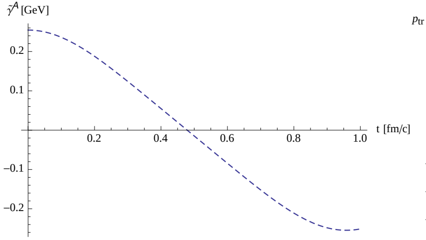
- Condensates can contribute to quick isotropization!

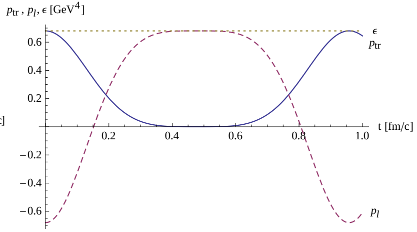
CP-*even cylindrical condensate*

- Initial condition with only $\tilde{\gamma}^A$ is CP symmetric.
- CP-breaking isotropic condensate σ not generated.
- Initial energy momentum tensor of the form

$$T^{\mu\nu} = \mathrm{diag}(\epsilon, p_{tr}, p_{tr}, p_l) = \mathrm{diag}(\epsilon, \epsilon, \epsilon, -\epsilon).$$

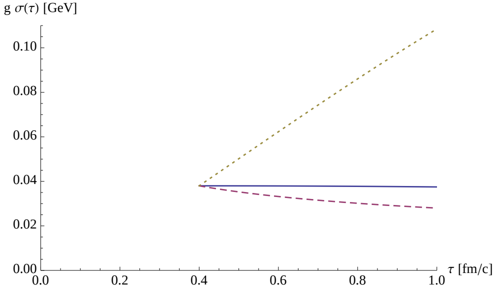
- Leads to oscillations between p_{tr} and p_l





Longitudinal expansion

- In realistic heavy ion collision the time evolution is modified by several effects, in particular by longitudinal expansion.
- Condensates will be diluted.
- That will probably hinder oscillations.
- Compare here only different scenarios for time evolution to $1/\tau^{1/3}$ dilution.



Excitations

- Consider now excitations of other field modes in the presence of isotropic condensate σ.
- Classify them according to the transformation behavior under modified rotations.
- Investigate in particular dispersion relations for excitations in the presence of isotropic condensate σ

Decomposition of gauge field 1

Write spatial and temporal parts of gauge field

$$
\begin{aligned}
(A_j)_{mn} =& \kappa_{jmn} + \gamma^A_{mk}\epsilon_{kjn} + \gamma^A_{nk}\epsilon_{kjm} + i\,\gamma^B_{jk}\epsilon_{kmn} \\
& + (\beta^A_m + i\,\beta^B_m)\,\delta_{jn} + (\beta^A_n - i\,\beta^B_n)\,\delta_{jm} - \frac{2}{3}\beta^A_j\delta_{mn} + i\,\sigma\,\epsilon_{jmn} \\
(A_0)_{mn} =& \gamma^C_{mn} + i\,\beta^C_l\,\epsilon_{lmn}
\end{aligned}
$$

with

- κ_{jmn} is real, completely symmetric, three-dimensional tensor of rank three, traceless with respect to all contractions,
- γ^A_{jk}, γ^B_{jk} and γ^C_{jk} are real, symmetric and traceless three-dimensional tensors,
- β^A_m, β^B_m and β^C_m are real, three-dimensional vectors.

In summary

$$
\begin{aligned}
\mathbf{24} =& \mathbf{7} + 2\times\mathbf{5} + 2\times\mathbf{3} + \mathbf{1}, \\
\mathbf{8} =& \mathbf{5} + \mathbf{3}.
\end{aligned}
$$

Decomposition of gauge field 2

To analyze dispersion relations it is useful to decompose further

- vectors

$$\beta_m = \partial_m \beta + \hat{\beta}_m$$

  - β is a real scalar,
  - $\hat{\beta}_m$ is a real, divergence-less vector.

- tensors of rank two

$$\gamma_{mn} = \hat{\gamma}_{mn} + \partial_m \hat{\gamma}_n + \partial_n \hat{\gamma}_m + (\partial_m \partial_n - \frac{1}{3}\delta_{mn}\partial_j^2)\gamma$$

  - $\hat{\gamma}_{mn}$ is real, traceless and divergence-less tensor
  - $\hat{\gamma}_m$ is real, divergence-less vector
  - γ is a real scalar.

- and tensors of rank three

$$\kappa_{jmn} = \hat{\kappa}_{jmn} + \partial_j \hat{\kappa}_{mn} + \partial_m \hat{\kappa}_{jn} + \partial_n \hat{\kappa}_{jm} + \dots$$

Decomposition of gauge field 3

- Discrete symmetries C and P classify fields further.
- Fields in different representations du not mix on linear level.
- Gauge fixing to Weyl gauge implies $\mathbf{A}_0 = 0$ or $\gamma^C_{mn} = \beta^C_m = 0$.
- At this point we are left with
  - C-even scalars $\sigma, \beta^B, \gamma^B$
  - C-odd scalars $\beta^A, \gamma^A, \kappa$
  - C-even vectors $\hat{\beta}^B_m, \hat{\gamma}^B_m$
  - C-odd vectors $\hat{\beta}^A_m, \hat{\gamma}^A_m, \hat{\kappa}_m$
  - C-even rank-two tensors $\hat{\gamma}^B_{mn}$
  - C-odd rank-two tensors $\hat{\gamma}^A_{mn}, \hat{\kappa}_{mn}$
  - C-odd rank-three tensor $\hat{\gamma}_{jmn}$

  which makes 24 real degrees of freedom.
- To reduce to 16 d.o.f. one needs the Gauss constraint.

Constraint equations

Variation of action with respect to $\mathbf{A}_0$ yields the Gauss constraint

$$\partial_j (E_j)_{mn} - ig(A_j)_{mk}(E_j)_{kn} + ig(E_j)_{mk}(A_j)_{kn} = D_j(E_j)_{mn} = 0.$$

Linearize this around constant background σ and decomposed further

- tensor constraint

$$\partial_0 \Big[\partial_j\, \kappa_{jmn} + \epsilon_{kjn}(\partial_j \gamma^A_{mk}) + \epsilon_{kjm}(\partial_j \gamma^A_{nk}) + \partial_m \beta^A_n + \partial_n \beta^A_m - \frac{2}{3}\partial_j \beta^A_j\, \delta_{mn} - 6\, g\sigma\, \gamma^A_{mn} \Big] = 0,$$

- vector constraint

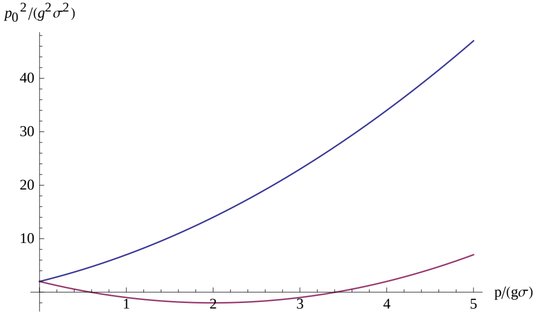
$$\partial_0 \Big[\partial_k \gamma^B_{jk} + \epsilon_{jmn}\partial_n \beta^B_m + \partial_j \delta\sigma - 2\, g\sigma\, \beta^B_j \Big] = 0.$$

Instabilities and particle production

One can now determine the dispersion relations for independent excitation modes. For example, for symmetric tensor of rank three $\hat{\kappa}_{jmn}$

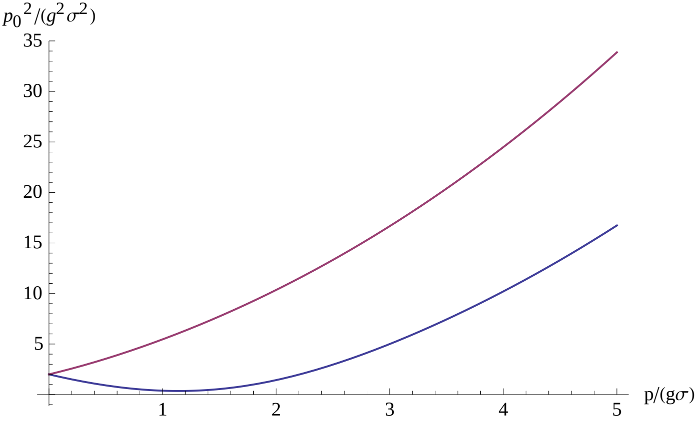
$$p_0^2 = \vec{p}^2 + 2g^2\sigma^2 \pm 4pg\sigma,$$

- One mode gapped with $\Delta = \sqrt{2g^2\sigma^2}$.
- Other mode has Nielsen-Olesen instability for intermediate momenta.
- Particles will be produced in that momentum regime.
- Time scale for particle production $\tau_{\text{pp}} \approx 1/\sqrt{g^2\sigma^2} \approx 1\,\text{fm/c}$.



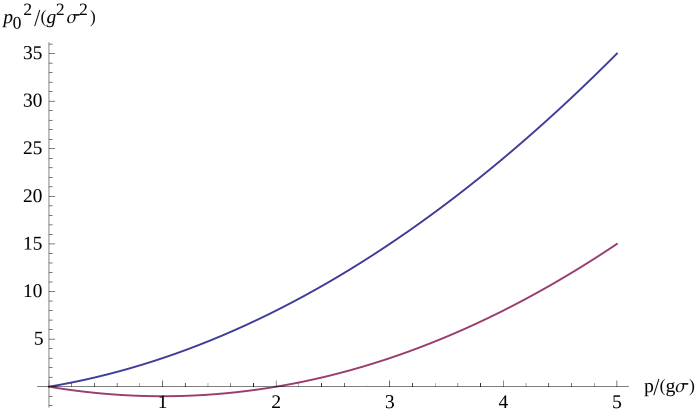
Dispersion relations 1

C-odd tensors of rank two



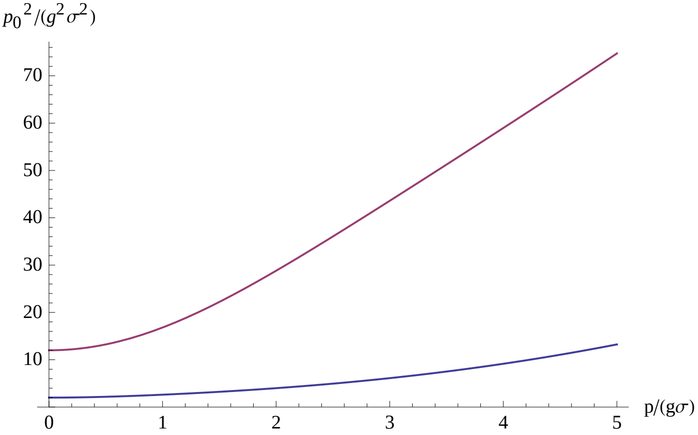
Dispersion relations 2

C-even tensors of rank two



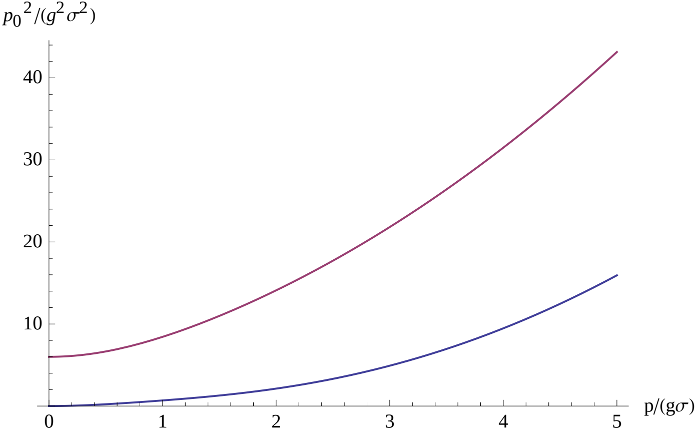
Dispersion relations 3

C-odd scalars



Dispersion relations 4

C-even scalars



Conclusions

- Color field condensate may be simple qualitative description for state with high gluon occupation numbers.
- Modified rotation symmetry (involving a gauge transformation) provides powerful ordering principle.
- Collective dynamics provides efficient mechanism for approximate isotropization.
- Nielsen-Olesen type instabilities can trigger decay of color field condensate into quasi-particle excitations.
- Particle production from decay of isotropic condensate can be approximately isotropic, as well.

BACKUP

Alternative embedding of $SU(2) \in SU(3)$

- Lie algebra of $SU(2)$ spanned by Gell-Mann matrices $\lambda_1, \lambda_2, \lambda_3$.
- Contains singlets
  - in spatial part
$$(A_j)_{mn} = \sigma(\lambda_j)_{mn}.$$
  - in temporal part
$$(A_0)_{mn} = \sigma'(\lambda_8)_{mn}$$
- More general decomposition of gauge field according to
  - temporal part $\mathbf{8} = \mathbf{3} + 2 \times \mathbf{2} + \mathbf{1}$,
  - spatial part $\mathbf{24} = \mathbf{5} + 2 \times \mathbf{4} + 2 \times \mathbf{3} + 2 \times \mathbf{2} + \mathbf{1}$.

Parametric resonances

- Here we considered excitations around constant background σ.
- For oscillating condensate one has additional parametric resonance phenomenon leading to an additional instability band [Berges et al, PRD 85, 034507 (2012)].
- Parametric resonance instability subleading compared to Nielsen-Olesen instability.