Mode-by-mode hydrodynamics

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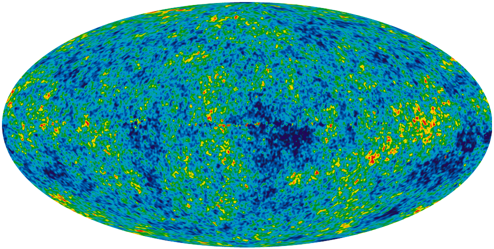
based on work with Urs Achim Wiedemann

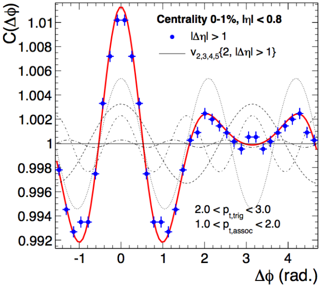
- Mode-by-mode fluid dynamics for relativistic heavy ion collisions [arXiv:1307.3453]
- Characterization of initial fluctuations for the hydrodynamical description of heavy ion collisions, [arXiv:1307.7611]
- Fluctuations around Bjorken Flow and the onset of turbulent phenomena, [JHEP 11, 100 (2011)]

What fluctuations are interesting and why?

- **Initial hydro fluctuations**: Event-by-event perturbations around the average of hydrodynamical fields at time τ_0:
  - energy density ϵ
  - fluid velocity u^μ
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n_B, electric charge density, electromagnetic fields, ...
- measure for deviations from equilibrium
- contain interesting information from early times
- governed by universal evolution equations
- can be used to constrain thermodynamic and transport properties

Similarities to cosmic microwave background





- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- can lead to detailed understanding of evolution and properties
- could trigger precision era in heavy ion physics

A complete story about fluctuations

1. initial fluctuations at initialization time of hydro should be characterized and quantified completely
2. fluctuations have to be propagated through the hydrodynamical regime
3. contribution of different fluctuations to the particle spectra must be understood and quantified
4. fluctuations generated from non-hydro sources (such as jets) have to be taken into account

Background-fluctuation splitting

- Background or average over many events is described by smooth fields

$$w_{\mathsf{BG}} = \langle w \rangle$$
$$u^\mu_{\mathsf{BG}} = \langle u^\mu \rangle$$

- Fluctuations are added on top

$$w = w_{\mathsf{BG}} + \delta w$$
$$u^\mu = u^\mu_{\mathsf{BG}} + \delta u^\mu$$

- For background one may assume Bjorken boost and azimuthal rotation invariance

$$w_{\mathsf{BG}} = w_{\mathsf{BG}}(\tau, r)$$
$$u^\mu_{\mathsf{BG}} = (u^\tau_{\mathsf{BG}}, u^r_{\mathsf{BG}}, 0, 0)$$

Characterization of transverse density 1

Fluctuations in initial transverse enthalpy density $w(r,\phi)$ can be characterized in terms of eccentricities $\epsilon_{n,m}$ and angles $\psi_{n,m}$ [Ollitrault, Teaney, Luzum, and others]

$$\epsilon_{n,m}\, e^{im\,\psi_{n,m}} = \frac{\int dr\ \int_0^{2\pi} d\varphi\, r^{n+1}\, e^{im\varphi}\, w(r,\varphi)}{\int dr\ \int_0^{2\pi} d\varphi\, r^{n+1}\, w(r,\varphi)}$$

- $w(r,\phi)$ completely determined by set of all $\epsilon_{n,m}$ and $\psi_{n,m}$
- closely related method is based on cumulants [Teaney, Yan]
- no positive transverse density can be associated to small set of cumulants (beyond Gaussian order) such that higher order cumulants vanish
- generalization to velocity and shear fluctuations not known

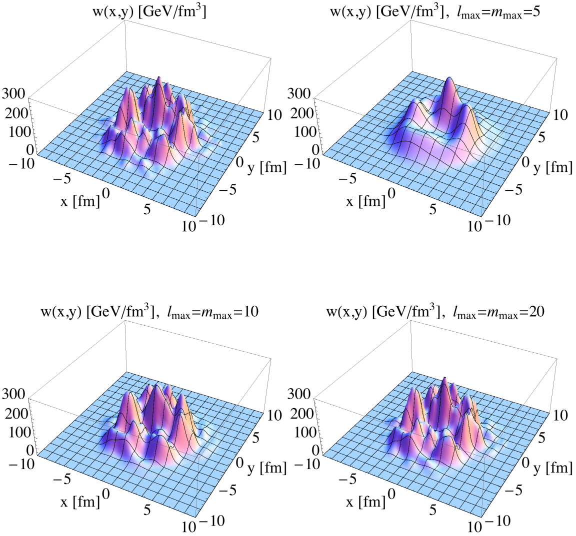
Characterization of transverse density 2

Characterizations based on orthonormal functions exist
[Gubser & Yarom, Shuryak & Staig, Floerchinger & Wiedemann, Coleman-Smith, Petersen & Wolpert]
Based on orthonormal set of functions and background density:
[Floerchinger & Wiedemann, 2013]

$$w(r,\varphi) = w_{\mathsf{BG}}(r) + w_{\mathsf{BG}}(r) \sum_{m=-m_{\mathsf{max}}}^{m_{\mathsf{max}}} \sum_{l=1}^{l_{\mathsf{max}}} \tilde{w}_l^{(m)}\, e^{im\varphi}\, J_m(k_l^{(m)} r)$$

- $w(r,\phi)$ completely determined by set of all $\tilde{w}_l^{(m)}$
- higher l correspond to smaller spatial resolution
- single or few coefficients $\tilde{w}_l^{(m)}$ lead to positive density
- single modes can be propagated in hydro
- works similar for vectors (velocity) and tensors (shear stress)

Transverse density from Glauber model



Velocity fluctuation

- initial velocity fluctuations at $\tau_0 \approx 0.5\,\text{fm/c}$ are conceivable
- characterization similar as for density fluctuations. Two polarizations

$$u^r = u^r_{\mathsf{BG}} + \frac{1}{\sqrt{2}}(\tilde{u}^- + \tilde{u}^+)$$
$$u^\phi = \frac{i}{\sqrt{2}\,r}(\tilde{u}^- - \tilde{u}^+)$$

with

$$\tilde{u}^-(r,\phi) = \sum_{m,l} \tilde{u}_l^{-(m)}\, e^{im\phi}\, J_{m-1}\left(k_l^{(m)} r\right)$$
$$\tilde{u}^+(r,\phi) = \sum_{m,l} \tilde{u}_l^{+(m)}\, e^{im\phi}\, J_{m+1}\left(k_l^{(m)} r\right)$$

- would be interesting to search for them in experimental data

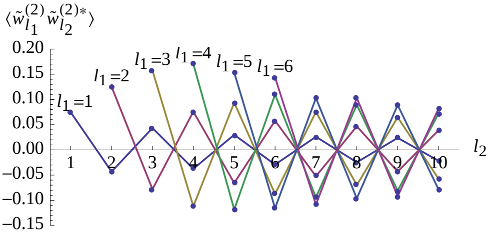
Event ensembles

- Event ensembles can be characterized in terms of functional probability distribution $p_{\tau_0}[w, u^\mu, \pi^{\mu\nu}, \ldots]$.
- Simplest case is Gaussian form

$$p_{\tau_0} \sim \exp\left[-\frac{1}{2} \sum_{m=-m_{\max}}^{m_{\max}} \sum_{l1,l2=1}^{l_{\max}} T^{(m)}_{l_1 l_2} \; \tilde{w}^{(m)*}_{l_1} \tilde{w}^{(m)}_{l_2} \right]$$

- Fully determined by correlator

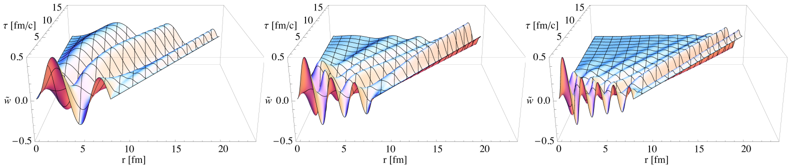
$$(T^{(m)})^{-1}_{l_1 l_2} = \langle \tilde{w}^{(m)}_{l_1} \tilde{w}^{(m)*}_{l_2} \rangle$$



Evolving fluctuations

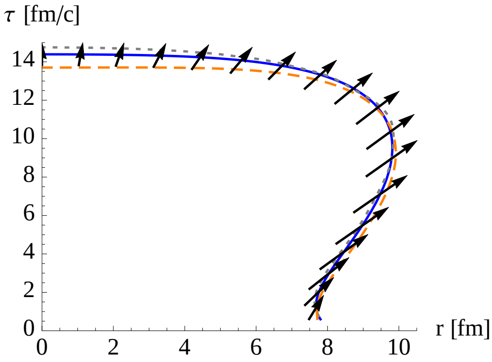
Bessel expansion can also be used to solve evolution equations

- expand enthalpy density, fluid velocity and shear in modes
- leads to set of coupled ordinary differential equations for expansion coefficients
- truncated set can be solved numerically
- do this here for linearized equations



Freeze-out surface

Background and fluctuations are propagated until $T_{\mathrm{fo}} = 120\,\mathrm{MeV}$ is reached.



(solid: $\eta/s = 0.08$, dotted: $\eta/s = 0$, dashed: $\eta/s = 0.3$)

Distribution functions are determined and free streaming is assumed for later times [Cooper & Frye]

Contribution of modes to "single event spectrum"

Particle spectrum (or its logarithm) can be expanded in contribution from different modes

$$\ln\left(\frac{dN^{\text{single event}}}{p_T dp_T d\phi dy}\right) = \underbrace{\ln S_0(p_T)}_{\text{from background}} + \underbrace{\sum_{m,l} \tilde{w}_l^{(m)} e^{im\phi} \theta_l^{(m)}(p_T)}_{\text{from fluctuations}}$$

- each mode has it's own angle $\tilde{w}_l^{(m)} = |\tilde{w}_l^{(m)}| \, e^{im\psi_l^{(m)}}$
- p_T dependence of different modes described by $\theta_l^{(m)}(p_T)$

Harmonic flow coefficients

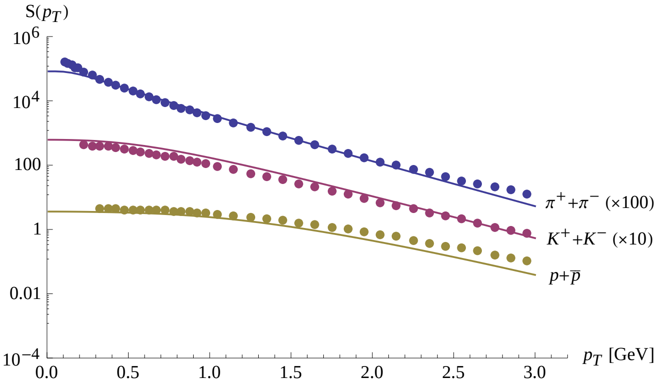
Double differential harmonic flow coefficient to lowest order

$$v_m^2\{2\}(p_T^a, p_T^b) = \sum_{l_1,l_2=1}^{l_{\text{max}}} \theta_{l_1}^{(m)}(p_T^a)\, \theta_{l_2}^{(m)}(p_T^b)\, \langle \tilde{w}_{l_1}^{(m)} \tilde{w}_{l_2}^{(m)*} \rangle$$

- intuite matrix expression
- in general no factorization
- higher order corrections important for non-central collisions

One-particle spectrum

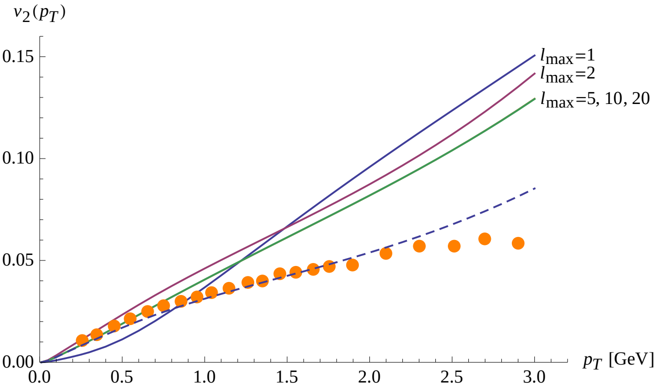
$$S(p_T) = dN/(2\pi p_T dp_T d\eta d\phi)$$



Points: 5% most central collisions, ALICE [PRL 109, 252301 (2012)]
Curves: Our calculation, no hadron rescattering and decays after freeze-out.

Harmonic flow coefficients for central collisions

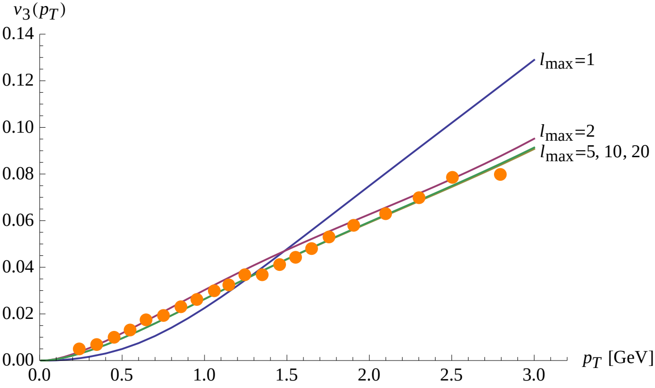
Elliptic flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Solid curves: Different maximal resolution l_{max}
Dashed curve: Mode $(m = 2, l = 1)$ suppressed by factor 0.7

Harmonic flow coefficients for central collisions

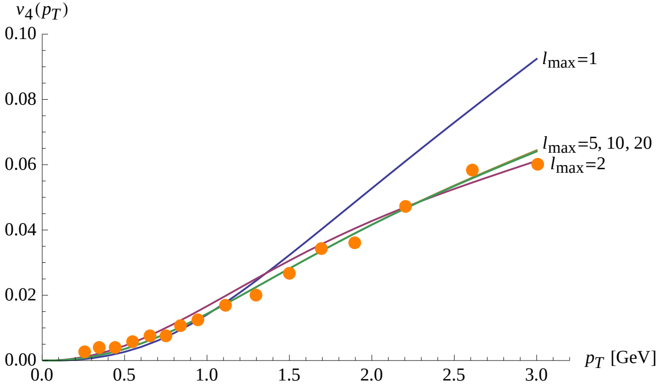
Triangular flow for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients for central collisions

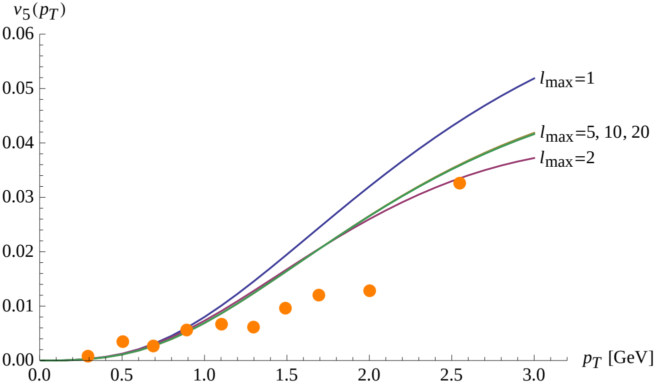
Flow coefficient v_4 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

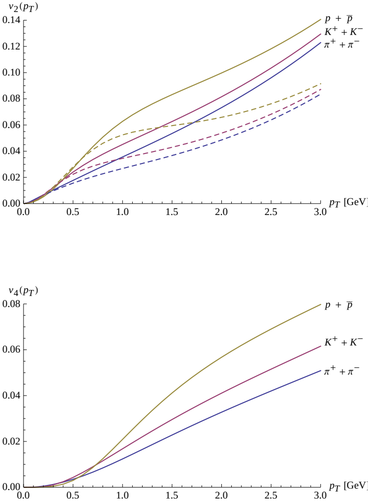
Harmonic flow coefficients for central collisions

Flow coefficient v_5 for charged particles



Points: 2% most central collisions, ALICE [PRL 107, 032301 (2011)]
Curves: Different maximal resolution l_{max}

Harmonic flow coefficients, central, particle identified



Conclusions

- Method to characterize and propagate initial fluctuations in hydrodynamical fields has been developed
- First study for enthalpy density fluctuations in Glauber model
  - yields good description of $v_m(p_T)$ for central collisions
  - shows that fluctuations up to $l_{\text{max}} \approx 5$ can be resolved
- Fluctuations to be studied:

	transverse plane	rapidity direction
enthalpy density / entropy	✓	-
fluid velocity	-	-
shear stress	-	-
baryon number density	-	-
electromagnetic fields	-	-
electric charge density	-	-
chiral order parameter	-	-

Backup

Linear vs. non-linear

- Non-linearities can arise from
  - hydrodynamic evolution
  - freeze-out
  - hadron decay and rescattering phase
- Formalism can be generalized in two ways
  - background with elliptic flow, small (linear) fluctuations
  - background with radial flow only, non-linear evolution of fluctuations
  - leads to coupling between modes with different m in a way constrained by azimuthal symmetry