Ultracold quantum gases

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I. Introduction

Why are ultracold quantum gases interesting?

- As a phenomenon of nature
  - How does matter behave at very low temperatures?
  - However: Only metastable state. True ground state is solid. (Exception ^{3}He, ^{4}He.)
- As a quantum physics laboratory (due to good experimental control)
  - quantum information, Bell's inequalities, quantum computation
  - simulation of condensed matter physics (optical lattices)
  - simulation of fundamental physics like QCD matter

  Of course, only some features can be simulated. Nevertheless helpful to test some ideas, concepts, methods.

Typical numbers

- density
  - particle number $N = 10^6$
  - cloud volume $V = 10^{-9}$ cm^3
  - interparticle distance $d = 0.1$ μm
- temperature
  - temperature $T = 10^{-6} K$
  - thermal de-Broglie length $\lambda_T = 1$ μm
- interaction
  - interaction range $\lambda_{\text{vdW}} = 10^{-4}$ μm
  - scattering length $a = (0...\infty)$ μm

Universality

- effective range is small

$$d \gg \lambda_{\mathsf{vdW}}, \quad \lambda_T \gg \lambda_{\mathsf{vdW}}$$

- interaction strength can be large, as well
- many properties are independent of detailed form of interaction potential
- universal physics described in terms of a few parameters
  - dimensionless scattering length

$$c = a\, n^{1/3}$$

  - dimensionless temperature

$$\tilde{T} = \frac{2Mk_BT}{n^{2/3}}$$

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- We look for a unified description.
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QUANTUM FIELD THEORY.

Classical field theory

- Describes electro-magnetic fields, waves, ... ($\hbar \to 0$).
- Crucial object: classical action

$$S[\phi] = \int dt \int d^d x \; \mathcal{L}(\phi, \partial_t \phi, \vec{\nabla}\phi, \dots)$$

- Classical field equations from $\frac{\delta S}{\delta \phi} = 0$.
- Symmetries of S lead to conserved currents.
- All physical observables are easily obtained from S.

Quantum field theory

- Describes electrons, atoms, quarks, gluons, protons,...
  ...and cold quantum gases
- Crucial object: quantum effective action

$$\Gamma[\phi] = \int dt \int d^d x \; U(\phi) + ...$$

- Quantum field equations from $\frac{\delta\Gamma}{\delta\phi} = 0$
- Symmetries of Γ lead to conserved currents
- All physical observables are easily obtained from Γ
- Γ is generating functional of 1-PI Feynman diagrams and depends on external parameters like T, μ, or $\vec{B}$
- for interacting theories Γ is hard to calculate

How does non-relativistic QFT look like?

Lagrange density for Bose gas with pointlike interaction

$$-\mathcal{L} = \varphi^* \left(-i \frac{\partial}{\partial t} - \frac{\vec{\nabla}^2}{2M} \right) \varphi + \frac{1}{2} \lambda (\varphi^* \varphi)^2$$

- $\varphi = \varphi(t, \vec{x})$ is a complex scalar field
- dispersion relation is non-relativistic
- local contact interaction $\sim \lambda$

This describes a classical field theory that can be quantized, e.g. by canonical quantization or by the functional integral formalism.

II. Basic concepts of thermal quantum field theory

Blackboard part

- Grand canonical potential
- Functional integral representation
- Imaginary time and Matsubara formalism
- Schwinger functional, Effective action, Flowing action

II. Bose-Einstein Condensation

Blackboard part

- Effective potential
- Spontaneous symmetry breaking
- Superfluidity
- Phase transitions

IV. Quantum fluctuations and the renormalization group

Blackboard part

- Renormalization group equation
- Vacuum limit and few-body observables
- Feynman diagrams
- Triviality problem

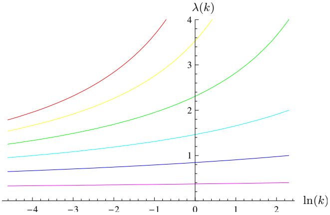
V. The superfluid Bose gas in two dimensions

Finite size as an infrared cutoff

- Renormalization group can be used for quantum systems at non-zero density and temperature
- Sometimes physics is better described by flowing action $\Gamma_k[\phi]$ with $k = 1/l$ instead of quantum effective action $\Gamma[\phi] = \Gamma_{k=0}[\phi]$
- For finite volume $V \approx l^3$ there are no quantum fluctuations with $k < 1/l$ to include

RG evolution of different quantities

Interaction strength

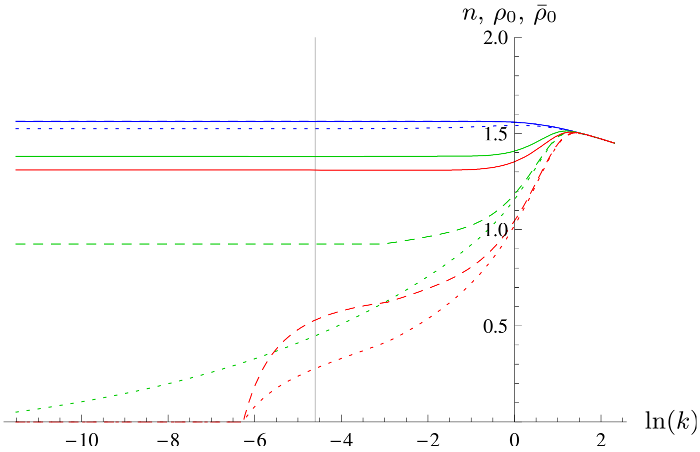


$$\lambda(k) = \frac{1}{\frac{1}{\lambda(\Lambda)} + c\ln(\Lambda/k)}$$

- goes to zero for $k \to 0$
- shows that scale of experiments is important
- for experiments effectively

$$\lambda = \lambda(1/l)$$

density n (solid)
superfluid density ρ_0 (dashed)
condensate density $\bar{\rho}_0$ (dotted)



$T = 0$ (blue) $T < T_c$ (green) $T > T_c$ (red)

Kosterlitz-Thouless phase transition

- for increasing system size $k = 1/l \to 0$
  - $T = 0$ — ρ_0 and $\bar{\rho}_0$ remain non-zero
  - $0 < T < T_c$ — $\bar{\rho}_0 \to 0$ and ρ_0 remains non-zero
  - $T > T_c$ — $\bar{\rho}_0 = 0$ and $\rho_0 = 0$ for scales $k < k_c$
- superfluid density non-zero for $T < T_c$
- condensate density goes to zero for $T > 0$
- needed to fulfill Mermin-Wagner theorem:
  No long range order in $d = 2$ for $T > 0$
- experiments at finite $k = 1/l$ can find non-zero condensate density $\bar{\rho}_0 > 0$

VI. Fermions

Functional integral for fermions

- some alkali gases such as ^{6}Li are fermions
- functional integral can be extended to fermions
- integrals over (anti-commuting) Grassmann numbers is needed

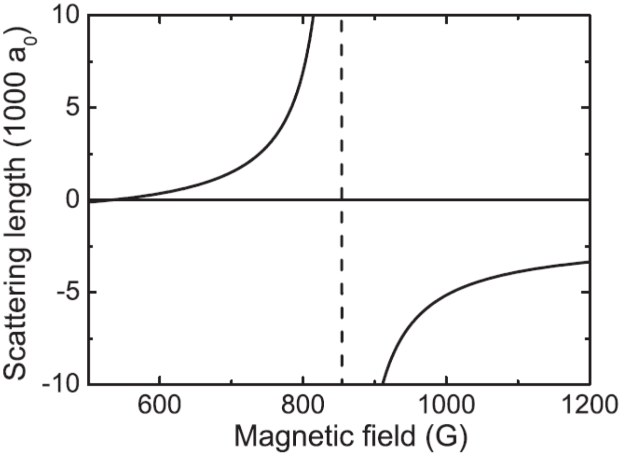
Fermi gases with different physics

- 1 component Fermi gas - no s-wave interaction (due to Pauli blocking)
- 2 component Fermi gas - BCS-BEC crossover well studied, will be discussed below
- 3 component Fermi gas - BCS-Trion-BEC transition current research, will also be discussed

VII. BCS-BEC Crossover

Feshbach resonances

allow to tune scattering length in a wide range

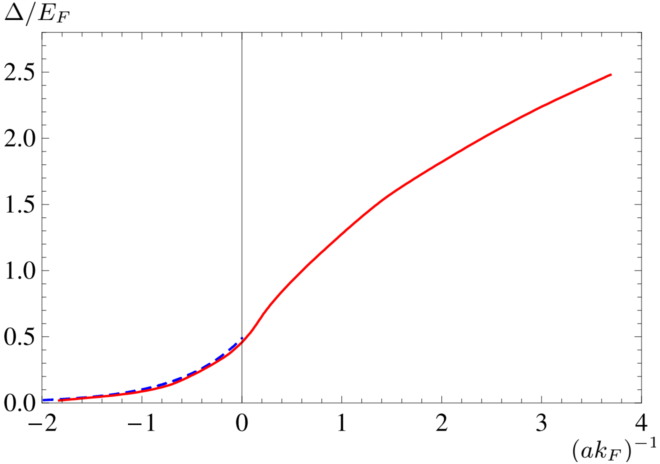


Blackboard part

- Microscopic model and Hubbard-Stratonovich transformation
- Scattering physics and bound states
- BCS limit
- BEC limit

Crossover

Gap at temperature $T = 0$ from RG study



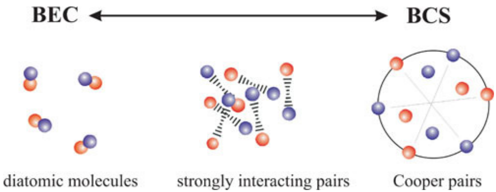
(Floerchinger, Scherer and Wetterich, PRA **81**, 063619 (2010).)

Unitarity point and universality

the point with $(ak_F)^{-1} = 0$ (divergent scattering length a) is particular interesting

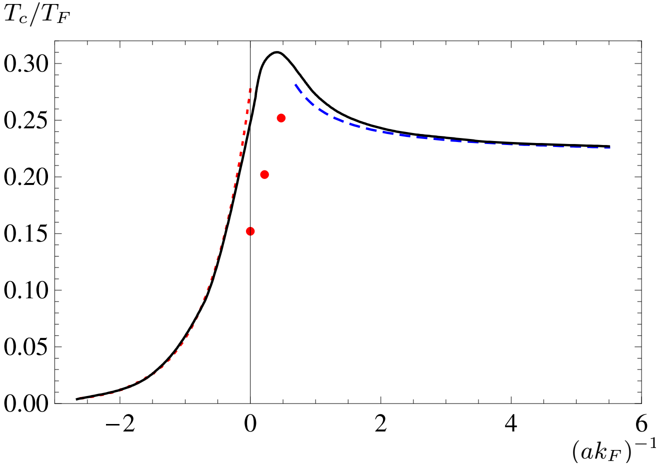
- scattering physics is governed by unitarity of S-matrix
- no scale except temperature T and density n
- example for non-relativistic conformal field theory

Summary of BCS-BEC Crossover



- Small negative scattering length $a \to 0_-$
  - Formation of Cooper pairs in momentum space
  - BCS-theory valid
  - superfluid at small temperatures
  - order parameter $\varphi \sim \psi_1\psi_2$
- Small positive scattering length $a \to 0_+$
  - Formation of dimers or molecules in position space
  - Bosonic mean field theory valid
  - superfluid at small temperatures
  - order parameter $\varphi \sim \psi_1\psi_2$
- Between both limits: Continuous *BCS-BEC Crossover*
  - scattering length becomes large: strong interaction
  - superfluid, order parameter $\varphi \sim \psi_1\psi_2$ at small T

Phase diagram



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VIII. Three component fermions

Three component Fermi gas

- For equal masses, densities etc. global SU(3) symmetry

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \to u \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \quad u \in \mathrm{SU(3)}.$$

  Similar to flavor symmetry in the Standard model!
- For small scattering length $|a| \to 0$
  - BCS ($a < 0$) or BEC ($a > 0$) superfluidity at small T.
  - order parameter is conjugate triplet $\bar{\mathbf{3}}$ under SU(3)

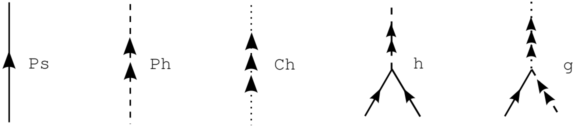
$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \begin{pmatrix} \psi_2\psi_3 \\ \psi_3\psi_1 \\ \psi_1\psi_2 \end{pmatrix}.$$

  - SU(3) symmetry is broken spontaneously for $\varphi \neq 0$.
- What happens for large $|a|$?

Simple truncation for fermions with three components

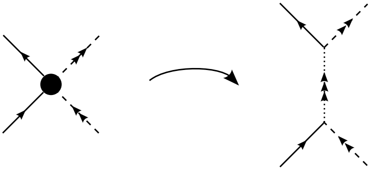
$$\Gamma_k = \int_x \psi^\dagger(\partial_\tau - \vec{\nabla}^2 - \mu)\psi + \varphi^\dagger(\partial_\tau - \frac{1}{2}\vec{\nabla}^2 + m_\varphi^2)\varphi$$
$$+\chi^*(\partial_\tau - \frac{1}{3}\vec{\nabla}^2 + m_\chi^2)\chi$$
$$+h\ \epsilon_{ijk}(\varphi_i^*\psi_j\psi_k + h.c.) \quad + g(\varphi_i\psi_i^*\chi + h.c.).$$

- Units are such that $\hbar = k_B = 2M = 1$
- Wavefunction renormalization for ψ, φ and χ is implicit.
- Γ_k contains terms for
  - fermion field $\psi = (\psi_1, \psi_2, \psi_3)$
  - bosonic field $\varphi = (\varphi_1, \varphi_2, \varphi_3) \sim (\psi_2\psi_3, \psi_3\psi_1, \psi_1\psi_2)$
  - trion field $\chi \sim \psi_1\psi_2\psi_3$

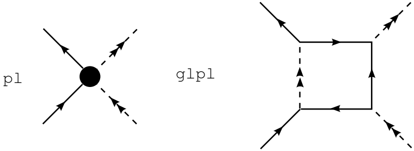


"Refermionization"

- Trion field is introduced via a generalized Hubbard-Stratonovich transformation



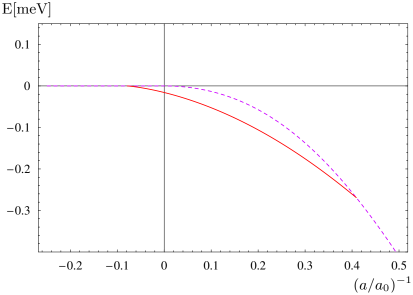
- Fermion-boson coupling is regenerated by the flow

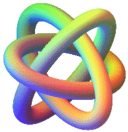


- Express this again by trion exchange
  (Gies and Wetterich, PRD **65**, 065001 (2002),
  Floerchinger and Wetterich, PLB **680**, 371 (2009).)

Binding energies

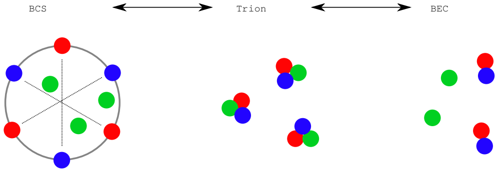
- Vacuum limit $T \to 0, \quad n \to 0.$





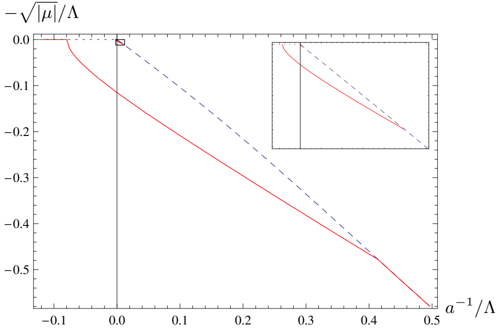
- Binding energy per atom for
  - molecule or dimer φ (dashed line)
  - trion or trimer χ (solid line)
- For large scattering length a trion is energetically favorable!
- Three-body bound state even for $a < 0$.

Quantum phase diagram



- BCS-Trion-BEC transition
  (Floerchinger, Schmidt, Moroz and Wetterich, PRA **79**, 013603 (2009)).
  - $a \to 0_-$: Cooper pairs, $SU(3) \times U(1) \to SU(2) \times U(1)$.
  - $a \to 0_+$: BEC of molecules, $SU(3) \times U(1) \to SU(2) \times U(1)$.
  - $a \to \pm\infty$: Trion phase, $SU(3)$ unbroken.
- Quantum phase transitions
  - from BCS to Trion phase
  - from Trion to BEC phase.

Efimov effect



- Self-similarity in energy spectrum.
- Efimov trimers become more and more shallow. At $a = \infty$

$$E_{n+1} = e^{-2\pi/s_0} E_n.$$

- Simple truncation: $s_0 \approx 0.82$.
- Advanced truncation: $s_0 \approx 1.006$ (exact result)

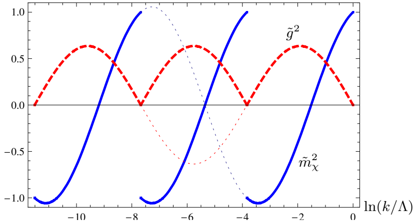
(Moroz, Floerchinger, Schmidt and Wetterich, PRA **79**, 042705 (2009).)

Renormalization group limit cycle

- For $\mu = 0$ and $a^{-1} = 0$ flow equations for rescaled couplings

$$k\frac{\partial}{\partial k}\begin{pmatrix}\tilde{g}^2\\ \tilde{m}_\chi^2\end{pmatrix} = \begin{pmatrix}7/25 & -13/25\\ 36/25 & 7/25\end{pmatrix}\begin{pmatrix}\tilde{g}^2\\ \tilde{m}_\chi^2\end{pmatrix}.$$

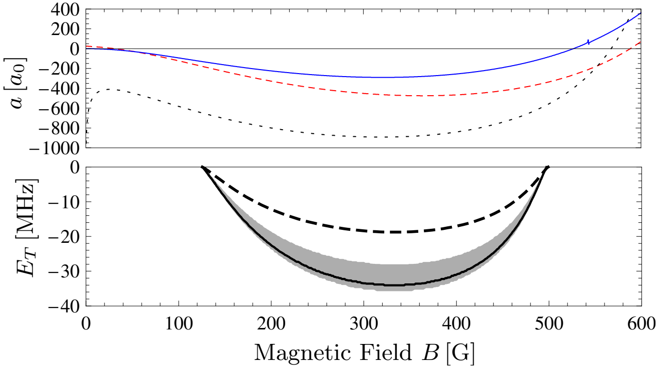
- Solution is log-periodic in scale.



- Every zero-crossing of $\tilde{m}_\chi^2$ corresponds to a new bound state.
- For $\mu \neq 0$ or $a^{-1} \neq 0$ limit cycle scaling stops at some scale k. Only finite number of Efimov trimers.

Contact to experiments

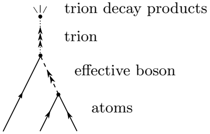
- Model can be generalized to case without SU(3) symmetry (Floerchinger, Schmidt and Wetterich, PRA A **79**, 053633 (2009)).
- Hyperfine states of ^{6}Li have large scattering lengths.

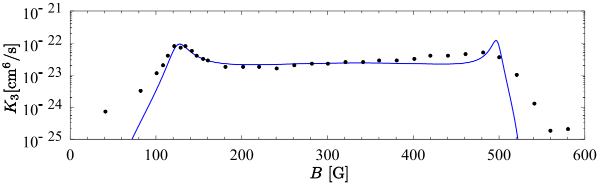


- Binding energies might be measured using RF-spectroscopy.
- Lifetime is quite short ~ 10ns.

Three-body loss rate

- Three-body loss rate measured experimentally (Ottenstein et al., PRL **101**, 203202 (2008); Huckans et al., PRL **102**, 165302 (2009))





- Trion may decay into deeper bound molecule states
- Calculate B-field dependence of loss process above.
- Left resonance (position and width) fixes model parameters.
- Form of curve for large B is prediction.
- Similar results obtained by other methods (Braaten, Hammer, Kang and Platter, PRL **103**, 073202 (2009); Naidon and Ueda, PRL **103**, 073203 (2009).)

Thank you very much for your attention!