Flowing bosonization and bound states

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Motivation

- Formation of bound states was one of the first problems discussed in quantum mechanics

1926. № 6.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 79.

1. ***Quantisierung als Eigenwertproblem;***
von E. Schrödinger.

- Bound state formation is much more difficult to treat in Quantum field theory.
- Bethe-Salpeter equation can be used to sum Ladder diagrams but it is difficult to go beyond.
- Look for an alternative approach!

Flow equations and Bound states

- Wetterich's flow equation was used by Ellwanger to study bound states in the Wick-Cutkosky model.
  (U. Ellwanger, Z. Phys. C **62**, 503 (1994).)
- Wegner's flow equation for Hamiltonians was used to investigate bound states in two dimensions
  (S. D. Glazek and K. G. Wilson, PRD **57**, 3558 (1998).)
- Partial bosonization and k-dependent, non-linear field transformations were used for the NJL-model
  (H. Gies and C. Wetterich, PRD **65**, 065001 (2002).)

Quantum field theory

- Describes also electrons, atoms, quarks, gluons, protons,...
- Crucial object: quantum effective action

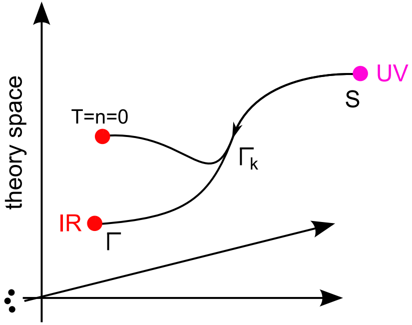
$$\Gamma[\phi] = S[\phi] + \frac{1}{2}\mathsf{Tr}\ln S^{(2)} + \ldots$$

- Quantum field equations from $\frac{\delta\Gamma}{\delta\phi} = 0$.
- All physical observables are easily obtained from Γ.
    - Few body physics
        - scattering amplitudes, renormalized masses, charges, ...
        - binding energies
    - Many-body properties
        - Phase diagram
        - Thermodynamic observables: pressure, density,...
        - Response functions
- Γ is generating functional of 1-PI Feynman diagrams.

How do we obtain the quantum effective action $\Gamma[\phi]$?

Idea of functional renormalization: $\Gamma[\phi] \rightarrow \Gamma_k[\phi]$

- k is additional infrared cutoff parameter.
- $\Gamma_k[\phi] \rightarrow \Gamma[\phi]$ for $k \rightarrow 0$.
- $\Gamma_k[\phi] \rightarrow S[\phi]$ for $k \rightarrow \infty$.
- Dependence on T, μ or $\vec{B}$ trivial for $k \rightarrow \infty$.



How the flowing action flows

Simple and exact flow equation (Wetterich 1993)

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathsf{STr} \left(\Gamma_k^{(2)}[\phi] + R_k \right)^{-1} \partial_k R_k .$$

- Differential equation for a functional.
- For most cases not solvable exactly.
- Approximate solutions can be found from Truncations.
  - Ansatz for Γ_k with a finite number of parameters.
  - Derive ordinary differential equations for this parameters or couplings from the flow equation for Γ_k.
  - Solve these equations numerically.

Truncations

- Derivative expansion

$$\Gamma_k = \int_x \varphi^*(-Z_k \partial_\mu \partial^\mu)\varphi + U_k(\varphi^*\varphi) + \dots$$

- Vertex expansion

$$\Gamma_k = \int_q \varphi^*(q) P_k(q) \varphi(q) + \int_{q_1..q_4} A_k(q_1,..,q_4)\, \varphi^*(q_1)\, \varphi(q_2)\, \varphi^*(q_3)\, \varphi(q_4) + \dots$$

- Momentum dependence of vertices is crucial!
- Key problem for the whole method!

Problems with momentum dependence

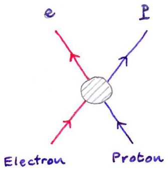
Numerical schemes to resolve the momentum dependence face various problems

- Symmetries / Ward identities
- Numerical effort
- Singularities
- Spontaneous symmetry breaking
- Analytic continuation to real frequencies
- Unitarity and Causality
- Physical interpretation

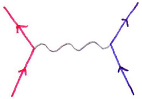
Idea followed here: Learn from Nature!

Four point function in QED

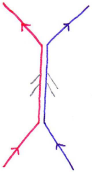
- Exact four point function in QED



- Two very different contributions
  - Photon exchange



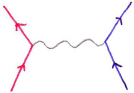
  - Bound state formation



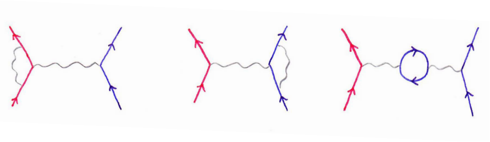
- Different physics with different description but both included in exact four-point function.

Perturbative QED point of view

- basic process

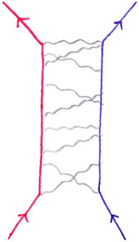


- gets renormalized by



- leads for example to

$$\frac{g-2}{2} = \frac{\alpha}{2\pi} \approx 0.0011614$$



- Bound state formation is non-perturbative
- Bethe-Salpeter equation allows to resum parts of this

Quantum mechanics point of view 1

- Integrate photon out, take non-relativistic limit

$$\frac{-e^2}{(\vec{p}-\vec{p}')^2} \sim \frac{-e^2}{4\pi|\vec{x}_1-\vec{x}_2|}$$

- Schrödinger equation

$$H\psi = E\psi$$

- Hamiltonian

$$H = \frac{1}{2(m_e+m_P)}(\vec{p}_e+\vec{p}_P)^2 + \frac{1}{2\mu}\vec{p}_r^{\,2} + V$$

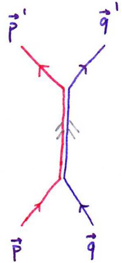
- Solution gives series of bound states

$$H\psi_{nlm} = E_n\psi_{nlm}$$

$$\psi_{nlm} = R_{nl}(r)Y_{lm}(\Omega_{\vec{r}})$$

Quantum mechanics point of view 2

- Four point function



$$\sum_{nlm} \frac{g_{nlm}(m_P\vec{p}\,' - m_e\vec{q}\,')\; g^*_{nlm}(m_P\vec{p} - m_e\vec{q})}{q_0 + p_0 - \frac{1}{2(m_e+m_P)}(\vec{p}+\vec{q})^2 - E_n}.$$

- Limits are
  - Only instantaneous interactions
  - No radiation corrections
  - Not Lorentz invariant

Unified treatment

Should describe both

- Perturbative QED (High energies / momenta)
- Bound states (Small energy / momenta)

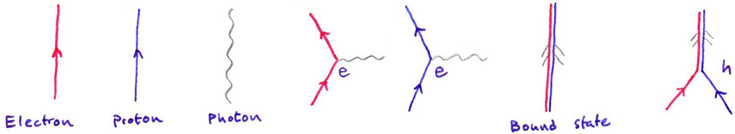
Basic ideas

- Introduce auxiliary fields for the orbitals
  - simple description of bound states
  - efficient treatment of singular momentum structure
- Keep photon exchange picture for interaction
  - retardation effects
  - radiation corrections
  - simple scattering theory for large energies
- On large scale only photon exchange
  - introduce orbitals gradually during flow

Can be done with flowing bosonization.

Flowing bosonization

- Start with QED + auxiliary fields for bound states



- Auxiliary fields decouple at the microscopic scale $h_\Lambda = 0$.
- Need one auxiliary field for every orbital $j = (n, l, m)$.
- For instantaneous photon ($c \to \infty$):
  - Yukawa vertex depends on relative velocity of electron and proton
    $$h_j = h_j\left(\vec{p}/m_e - \vec{q}/m_P\right)$$
    .
  - Propagator matrix depends on center of mass momentum
    $$G_{jj'} = G_{jj'}(p+q).$$

Flowing bosonization with exact flow equation 1

- Exact flow equation

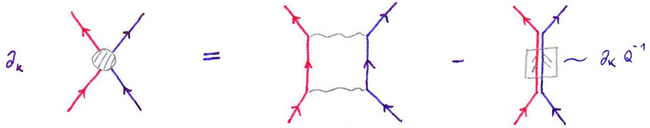
$$
\begin{aligned}
\partial_k \Gamma_k =& \frac{1}{2}\mathsf{STr}(\Gamma_k^{(2)} + R_k)^{-1}(\partial_k R_k - R_k(\partial_k Q^{-1})R_k) \\
&- \frac{1}{2}\Gamma_k^{(1)}(\partial_k Q^{-1})\Gamma_k^{(1)}.
\end{aligned}
$$

(S. Floerchinger and C. Wetterich, PLB **680**, 371 (2009).)

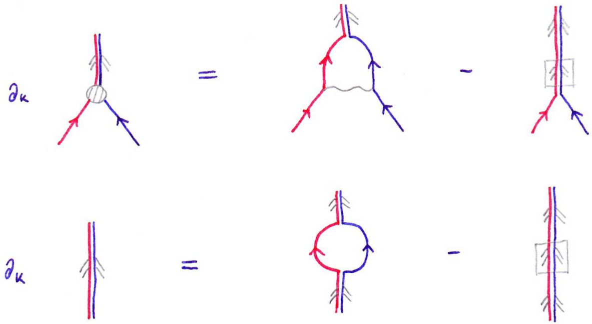
- Derived from k-dependent Hubbard-Stratonovich transformation.
- $\Gamma_k^{(1)}$ is functional derivative with respect to the composite field.
- $\partial_k Q^{-1}$ can be chosen arbitrary.

Flowing bosonization with exact flow equation 2

- Flow of four point function can be absorbed by convenient choice of $\partial_k Q^{-1}$.

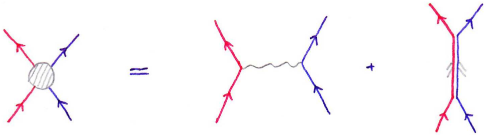


- This modifies flow of coupling h and bound state propagator



Flowing bosonization with exact flow equation 3

- For non-relativistic particles with instantaneous interaction one can solve the flow equations. Equivalence to Schrödinger equation can be shown (S. Floerchinger, arXiv:1001.4497.)
- For $k = 0$ the effective four-point function has two main contributions



- Fundamental fields and composite fields are treated equal.
- This allows to treat
  - Interactions between composite fields
  - Spontaneous symmetry breaking
  - Bound states of composite fields

Conclusions

- Flow equation approach for bound states was developed.
- Allows unified treatment of fundamental and composite fields.
- Useful and efficient formalism to treat momentum dependences is emerging.
- Equivalence of formalism to two-body Schrödinger equation for $c \to \infty$ can be proven.