

News from Bosonization

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Work in collaboration with C. Wetterich

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Introduction

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- At low temperature we expect some kind of order.
- Order parameter is a *bosonic* field φ .

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- Partial bosonization combines advantages of both approaches.

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- α differentiates also between particles ψ and holes ψ^* (or antiparticles $\bar{\psi}$).

Microscopic model

$$S = \frac{1}{2} \psi_\alpha P_{\alpha\beta} \psi_\beta + \frac{1}{4!} \lambda_{\alpha\beta\gamma\delta} \psi_\alpha \psi_\beta \psi_\gamma \psi_\delta$$

- Inverse propagator P

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- Fermion-fermion interaction λ
- Examples: Hubbard model, BCS-model, NJL-model,...

Hubbard-Stratonovich transformation

- Add Gaussian functional integral over boson field φ

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$$\frac{1}{4!} \lambda_{\alpha\beta\gamma\delta} + \frac{1}{2} (HQ^{-1}H)_{\alpha\beta\gamma\delta} = 0.$$

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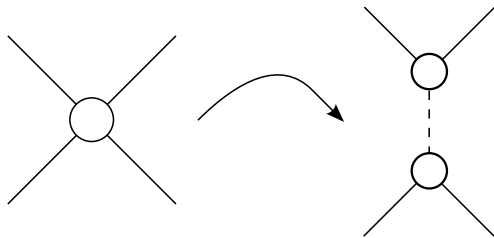
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- Arrive at theory for fermions and bosons

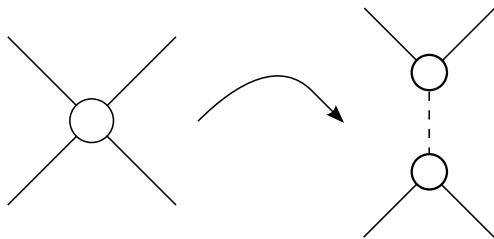
$$S = \frac{1}{2} \psi P \psi + \frac{1}{2} \varphi Q \varphi - \varphi H \psi \psi.$$

Hubbard-Stratonovich II



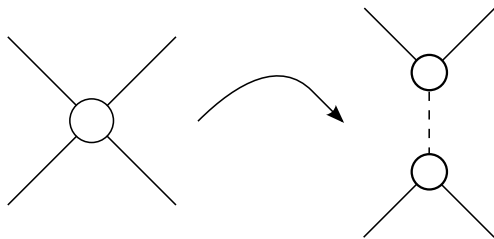
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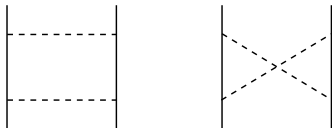
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- Fermion interaction is expressed as boson exchange.
- Need boson field for every channel!
- Divergence in λ corresponds to order parameter $\langle \varphi \rangle \neq 0$.

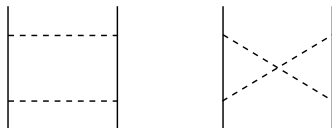
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- Fermion coupling λ is regenerated by renormalization flow.



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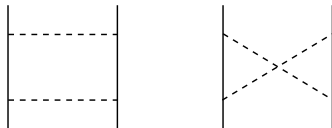
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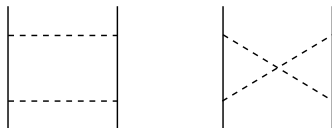
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- Related work: Pawłowski (2007).

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- Gies & Wetterich (2001): Use scale dependent boson fields $\varphi_k[\psi]$ to have $\lambda = 0$ on all scales.
- Related work: Pawłowski (2007).
- We will present an exact and simple one-loop flow equation for this task.
(S. Floerchinger and C. Wetterich, arXiv:0905.0915)

Nonlinear field transformation

Consider flow equation (Wetterich 1993)

$$\partial_k \Gamma_k[\psi, \varphi] \Big|_{\varphi} = \frac{1}{2} \text{STr} \left\{ (\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right\}$$

with k -dependent change of variables $\varphi = \varphi[\bar{\varphi}]$

$$\partial_k \Gamma_k \Big|_{\bar{\varphi}} = \partial_k \Gamma_k \Big|_{\varphi} - \int_q \frac{\delta \Gamma_k}{\delta \bar{\varphi}} \partial_k \bar{\varphi} \Big|_{\varphi}.$$

- $\partial_k \bar{\varphi} \sim \psi\psi$ allows for “rebosonization”.

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- Connection terms $\sim \frac{\delta^2 \varphi}{\delta \bar{\varphi} \delta \bar{\varphi}}$ destroy one-loop structure.
- $\Gamma_k[\bar{\varphi}] := \Gamma_k[\varphi[\bar{\varphi}]$ has different properties than $\Gamma_k[\varphi]$.

Scale-dependent bosonization

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- Scale-dependent Schwinger functional

$$e^{W_k[\eta, j]} = \int D\tilde{\psi} D\tilde{\varphi} e^{-S_k[\tilde{\psi}, \tilde{\varphi}] + \eta\tilde{\psi} + j\tilde{\varphi}}$$

with

$$\begin{aligned} S_k[\tilde{\psi}, \tilde{\varphi}] &= S_\psi[\tilde{\psi}] + \frac{1}{2}\tilde{\psi}(R_k^\psi)\tilde{\psi} + \frac{1}{2}\tilde{\varphi}(Q + R_k^\varphi)\tilde{\varphi} \\ &\quad + \frac{1}{2}(\tilde{\psi}\tilde{\psi}H)Q^{-1}(H\tilde{\psi}\tilde{\psi}) - \tilde{\varphi}(H\tilde{\psi}\tilde{\psi}). \end{aligned}$$

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- k -dependent are: R_k , H and Q .

Exact flow equation

$$\partial_k \Gamma_k = \frac{1}{2} \text{STr} \left\{ (\Gamma_k^{(2)} + R_k)^{-1} (\partial_k R_k - R_k (\partial_k Q^{-1}) R_k) \right\} - \frac{1}{2} \Gamma_k^{(1)} (\partial_k Q^{-1}) \Gamma_k^{(1)} + \gamma_k$$

- One-loop term is supplemented by “tree” term.

$$\partial_k \Gamma_k = \text{Diagram 1} + \text{Diagram 2}$$

The diagrammatic equation shows the derivative of the effective action as the sum of two terms. The first term is a circle with a double line, representing a one-loop diagram, with a square containing an 'X' on top. The second term is a tree-level diagram consisting of two solid black dots connected by a dashed line, with a square containing an 'X' in the middle of the dashed line.

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The diagram shows the decomposition of the derivative of the effective action. The first term is a one-loop diagram consisting of a circle with a cross inside. The second term is a tree-level diagram consisting of two black dots connected by a dashed line, with a square containing a cross in the middle of the dashed line.

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- One-loop term is supplemented by “tree” term.
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- Field-independent term γ_k can often be dropped.

$$\partial_k \Gamma_k = \text{Diagram 1} + \text{Diagram 2}$$

The diagram shows the decomposition of the derivative of the effective action. The first term is a one-loop diagram consisting of two concentric circles with a cross symbol (⊗) on the top arc. The second term is a tree-level diagram consisting of two solid black circles connected by a dashed line, with a square containing a cross symbol (⊗) in the middle of the dashed line.

Properties of exact flow equation

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- Independently chosen $\partial_k Q$ and $\partial_k H = (\partial_k F)H$ are equivalent to linear change in φ .
- Generalization to other composite operators are possible.
- For $R_k^\varphi = 0$ formalism is close to purely fermionic flow.

Flowing bosonization

$$\partial_k \mathcal{P}_\varphi = \text{diagram} + \text{diagram}$$

The first equation shows the derivative of the partition function \mathcal{P}_φ with respect to k . The left-hand side is equal to the sum of two diagrams. The first diagram is a circle with two external dashed lines and a cross inside. The second diagram is a square with two external dashed lines and a cross inside.

$$\partial_k h = \text{diagram} + \text{diagram}$$

The second equation shows the derivative of h with respect to k . The left-hand side is equal to the sum of two diagrams. The first diagram is a circle with two external dashed lines and two external solid lines, and a cross inside. The second diagram is a square with two external dashed lines and two external solid lines, and a cross inside.

$$\partial_k \lambda = \text{diagram} + \text{diagram}$$

The third equation shows the derivative of λ with respect to k . The left-hand side is equal to the sum of two diagrams. The first diagram is a circle with four external solid lines and a cross inside. The second diagram is a square with four external solid lines and a cross inside.

Flowing bosonization

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- $\partial_k Q^{-1}$ can be chosen such that $\partial_k \lambda = 0$.
- This gives corrections to $\partial_k h$ and $\partial_k \mathcal{P}_\varphi$.

Proof of flow equation

Schwinger functional

$$e^{W_k[\eta, j]} = \int D\tilde{\psi} D\tilde{\varphi} e^{-S_k[\tilde{\psi}, \tilde{\varphi}] + \eta\tilde{\psi} + j\tilde{\varphi}} \quad (1)$$

with

$$\begin{aligned} S_k[\tilde{\psi}, \tilde{\varphi}] &= S_\psi[\tilde{\psi}] + \frac{1}{2}\tilde{\psi}R_k^\psi\tilde{\psi} + \frac{1}{2}\tilde{\varphi}(Q + R_k^\varphi)\tilde{\varphi} \\ &\quad + \frac{1}{2}\chi Q^{-1}\chi - \tilde{\varphi}\chi. \end{aligned} \quad (2)$$

Shift in field $\tilde{\varphi}$

$$\begin{aligned} e^{W_k[\eta, j]} &= \int D\tilde{\psi} e^{-S_\psi[\tilde{\psi}] - \frac{1}{2}\tilde{\psi}R_k^\psi\tilde{\psi} + \eta\tilde{\psi}} \\ &\quad \times e^{\frac{1}{2}(j+\chi)(Q+R_k^\varphi)^{-1}(j+\chi) - \frac{1}{2}\chi Q^{-1}\chi} \\ &\quad \times \int D\tilde{\varphi} e^{-\frac{1}{2}\tilde{\varphi}(Q+R_k^\varphi)\tilde{\varphi}}. \end{aligned} \quad (3)$$

Equivalence of both equations yields

$$\langle \chi_\epsilon \rangle = Q_{\epsilon\rho} \varphi_\rho - l_\epsilon \quad (4)$$

with the modified source $l_\epsilon = j_\epsilon - (R_k^\varphi)_{\epsilon\sigma} \varphi_\sigma$. Similarly

$$\begin{aligned} \langle \chi_\epsilon \chi_\sigma \rangle &= [(Q + R_k^\varphi)(\delta_j \delta_j W_k)(Q + R_k^\varphi)]_{\epsilon\sigma} \\ &\quad + (Q\varphi - l)_\epsilon (Q\varphi - l)_\sigma - (Q + R_k^\varphi)_{\epsilon\sigma}, \end{aligned} \quad (5)$$

and

$$\begin{aligned} \langle \tilde{\varphi}_\epsilon \chi_\sigma \rangle &= \langle \tilde{\varphi}_\epsilon \tilde{\varphi}_\tau \rangle (Q + R_k^\varphi)_{\tau\sigma} - \varphi_\epsilon j_\sigma - \delta_{\epsilon\sigma} \\ &= \varphi_\epsilon (Q\varphi)_\sigma + [(\delta_j \delta_j W_k)(Q + R_k^\varphi)]_{\epsilon\sigma} - \varphi_\epsilon l_\sigma - \delta_{\epsilon\sigma}. \end{aligned} \quad (6)$$

We now turn to the scale-dependence of $W_k[\eta, j]$

$$\begin{aligned}
 \partial_k W_k &= -\frac{1}{2} \langle \tilde{\psi} (\partial_k R_k^\psi) \tilde{\psi} \rangle - \frac{1}{2} \langle \tilde{\varphi} (\partial_k R_k^\varphi + \partial_k Q) \tilde{\varphi} \rangle \\
 &\quad + \frac{1}{2} \langle \chi (\partial_k Q^{-1}) \chi \rangle \\
 &\quad - \langle \tilde{\varphi} Q (\partial_k Q^{-1}) \chi \rangle.
 \end{aligned} \tag{7}$$

Use now (2), (3), (4)

$$\begin{aligned}
 \partial_k W_k &= -\frac{1}{2} \psi (\partial_k R_k^\psi) \psi - \frac{1}{2} \varphi (\partial_k R_k^\varphi) \varphi \\
 &\quad - \frac{1}{2} \text{STr} \{ (\partial_k R_k^\psi) (\delta_\eta \delta_\eta W_k) \} \\
 &\quad - \frac{1}{2} \text{Tr} \{ [\partial_k R_k^\varphi - R_k^\varphi (\partial_k Q^{-1}) R_k^\varphi] (\delta_j \delta_j W_k) \} \\
 &\quad + \frac{1}{2} l (\partial_k Q^{-1}) l + \frac{1}{2} \text{Tr} \{ \partial_k Q^{-1} (Q - R_k^\varphi) \}.
 \end{aligned} \tag{8}$$

The average action is defined as the modified Legendre transform

$$\Gamma_k[\psi, \varphi] = \eta\psi + j\varphi - W_k[\eta, j] - \frac{1}{2}\psi R_k^\psi \psi - \frac{1}{2}\varphi R_k^\varphi \varphi. \quad (9)$$

As usual, one has

$$\frac{\delta}{\delta\psi_\alpha} \Gamma_k = \pm\eta_\alpha - (R_k^\psi)_{\alpha\beta} \psi_\beta, \quad (10)$$

and

$$\frac{\delta}{\delta\varphi_\epsilon} \Gamma_k = j_\epsilon - (R_k^\varphi)_{\epsilon\sigma} \varphi_\sigma = l_\epsilon. \quad (11)$$

This yields our central result

$$\begin{aligned} \partial_k \Gamma_k &= \frac{1}{2} \text{STr} \left\{ (\Gamma_k^{(2)} + R_k)^{-1} (\partial_k R_k - R_k (\partial_k Q^{-1}) R_k) \right\} \\ &\quad - \frac{1}{2} \Gamma_k^{(1)} (\partial_k Q^{-1}) \Gamma_k^{(1)} + \gamma_k \end{aligned} \quad (12)$$

with

$$\gamma_k = -\frac{1}{2} \text{Tr} \left\{ (\partial_k Q^{-1}) (Q - R_k) \right\}. \quad (13)$$

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- Thank you for your attention!