Strong Interactions in Extreme Conditions

Axel Maas

25th of March 2011
Darmstadt
Germany
Overview

• Strong interactions - quarks, gluons, and QCD

• Supported by the DFG
Overview

• Strong interactions - quarks, gluons, and QCD

• Particles in the sky
  • The early universe and neutron stars
  • The phase diagram of QCD

• Supported by the DFG
Overview

• Strong interactions - quarks, gluons, and QCD

• Particles in the sky
  • The early universe and neutron stars
  • The phase diagram of QCD

• Describing QCD from the bottom up
  • A correlation-function-based framework
  • Finite temperature and finite density

• Summary

• Supported by the DFG
Strong interactions
Strong interactions

- The constituents of matter
Strong interactions

• The constituents of matter
• Quarks and gluons
Strong interactions

- The constituents of matter
- Quarks and gluons
- Color
The structure of matter

- Common materials consist of atoms
The structure of matter

- Common materials consist of atoms
  - Which form molecules, crystals, cells,...
The structure of matter

(Atomic) Hydrogen gas

- Common materials consist of atoms
  - Which form molecules, crystals, cells,...
The structure of matter

Hydrogen atom
The structure of matter

- Each atom divides up in its electron
The structure of matter

Hydrogen atom

• Each atom divides up in its electron and a nucleus
The structure of matter

- Each atom divides up in its electron and a nucleus
  - Deduced from scattering experiments
  - Nucleus is 100000 times smaller than an atom
The structure of matter

• Each atom divides up in its electron and a nucleus
  • Deduced from scattering experiments
  • Nucleus is 100000 times smaller than an atom
• Scattering experiments on the nuclei show a similar behavior as for atoms

Hydrogen atom
The structure of matter

- Each atom divides up in its electron and a nucleus
  - Deduced from scattering experiments
  - Nucleus is 100000 times smaller than an atom
- Scattering experiments on the nuclei show a similar behavior as for atoms
  - Nuclei must also have a sub-structure
The structure of matter

Hydrogen atom
The structure of matter
The structure of matter

Resolution

Proton

Hydrogen atom
The structure of matter

- Proton consists of three particles
The structure of matter

- Proton consists of three particles
The structure of matter

- Proton consists of three particles
  - Quarks are fermions with spin 1/2
  - Two different types: up and down quarks
The structure of matter

- Proton consists of three particles
  - Quarks are fermions with spin 1/2
  - Two different types: up and down quarks

Masses:
- Up: 2-3 MeV
- Down: 4-6 MeV
The structure of matter

- Proton consists of three particles
  - Quarks are fermions with spin 1/2
  - Two different types: up and down quarks
  - Four more types of quarks are known

Proton

Resolution

Masses:
Up: 2-3 MeV
Down: 4-6 MeV
The structure of matter

Proton consists of three particles

- Quarks are fermions with spin 1/2
- Two different types: up and down quarks
- Four more types of quarks are known

Masses:
- Up: 2-3 MeV
- Down: 4-6 MeV
- Strange: 80-130 MeV
- Charm: 1270(10) MeV
- Bottom: 4190(200) MeV
- Top: 172000(1500) MeV
The structure of matter

Proton

Resolution
The structure of matter

Proton

Resolution
The structure of matter
The structure of matter
The structure of matter

- Quarks exchange gluons
The structure of matter

- Quarks exchange gluons
  - Massless bosons with spin 1, like photons

Resolution
The structure of matter

• Quarks exchange gluons
  • Massless bosons with spin 1, like photons
  • Carry a new force, like photons carry electromagnetism
  • Force is much stronger: Strong interaction
The structure of matter

- Quarks exchange gluons
  - Massless bosons with spin 1, like photons
  - Carry a new force, like photons carry electromagnetism
  - Force is much stronger: Strong interaction
  - Gluons interact
The structure of matter

- Quarks exchange gluons
  - Massless bosons with spin 1, like photons
  - Carry a new force, like photons carry electromagnetism
  - Force is much stronger: Strong interaction
  - Gluons interact: Yang-Mills theory
Color

- Photons mediate a force between charges
Color

• Photons mediate a force between charges – so do gluons
Color

- Photons mediate a force between charges – so do gluons
- Charge of the strong interactions is called color
Color

- Photons mediate a force between charges – so do gluons

- **Charge of the strong interactions is called color**
  - Three charges: Red, green, and blue (like the electron charge)
  - Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
Color

• Photons mediate a force between charges – so do gluons

• **Charge of the strong interactions is called color**
  • Three charges: Red, green, and blue (like the electron charge)
  • Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
  • One of each (anti-)color is neutral
Color

- Photons mediate a force between charges – so do gluons
- **Charge of the strong interactions is called color**
  - Three charges: Red, green, and blue (like the electron charge)
  - Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
  - One of each (anti-)color is neutral
  - Color and anti-color is neutral
Color

- Photons mediate a force between charges – so do gluons

**Charge of the strong interactions is called color**

- Three charges: Red, green, and blue (like the electron charge)
- Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
- One of each (anti-)color is neutral
- Color and anti-color is neutral
Color

- Photons mediate a force between charges – so do gluons
- **Charge of the strong interactions is called color**
  - Three charges: Red, green, and blue (like the electron charge)
  - Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
  - One of each (anti-)color is neutral
  - Color and anti-color is neutral
Color

- Photons mediate a force between charges – so do gluons

- **Charge of the strong interactions is called color**
  - Three charges: Red, green, and blue (like the electron charge)
  - Three anti-charges: Antired, antigreen, and antiblue (like the positron charge)
  - One of each (anti-)color is neutral
  - Color and anti-color is neutral
  - Gluons interact: Eight more charges
    - Neutral gluon combinations possible

**Hadrons**
- Baryons: Protons, Neutrons, Delta...
- Mesons: Pions, Kaons, Rhos...
Are there 'ionized' hadrons?

• There are electrically charged particles: Ions, electrons...
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1:\sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At 1:~10^{40}
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
Are there 'ionized' hadrons?

- There are electrically charged particles: Ions, electrons...
- Are there color-ionized particles? Free quarks?
- Experiment: No. At $1: \sim 10^{40}$
  - Theoretically not fully understood
- Particle creation happens before ionization
  - 100% efficient: “Confinement”
Another phenomenon: mass

• Energy required to split a hadron is roughly the total mass of the resulting hadrons
Another phenomenon: mass

- Energy required to split a hadron is roughly the total mass of the resulting hadrons
- Not the mass of the quarks
  - up 2-3 MeV, down 4-6 MeV
  - Proton: 938 MeV
  - Rho: 770 MeV
Another phenomenon: mass

- Energy required to split a hadron is roughly the total mass of the resulting hadrons
- Not the mass of the quarks
  - up 2-3 MeV, down 4-6 MeV
  - Proton: 938 MeV
  - Rho: 770 MeV
- Spontaneous generation of mass
Another phenomenon: mass

- Energy required to split a hadron is roughly the total mass of the resulting hadrons
- Not the mass of the quarks
  - up 2-3 MeV, down 4-6 MeV
  - Proton: 938 MeV
  - Rho: 770 MeV
- Spontaneous generation of mass
  - Strong interactions makes quarks condense
  - Every quark is roughly 350 MeV heavier
Another phenomenon: mass

- Energy required to split a hadron is roughly the total mass of the resulting hadrons
- Not the mass of the quarks
  - up 2-3 MeV, down 4-6 MeV
  - Proton: 938 MeV
  - Rho: 770 MeV
- Spontaneous generation of mass
  - Strong interactions makes quarks condense
  - Every quark is roughly 350 MeV heavier
Another phenomenon: mass

- Energy required to split a hadron is roughly the total mass of the resulting hadrons
- Not the mass of the quarks
  - up 2-3 MeV, down 4-6 MeV
  - Proton: 938 MeV
  - Rho: 770 MeV
- Spontaneous generation of mass
  - Strong interactions makes quarks condense
  - Every quark is roughly 350 MeV heavier
  - Associated with the spontaneous breaking of the so-called chiral symmetry
The theory

- Quantized theory of these interactions is QCD
  - Quantumchromodynamics
The theory

- Quantized theory of these interactions is QCD
  - Quantumchromodynamics
- Gauge theory
  - Like (classical) electrodynamics
The theory

- Quantized theory of these interactions is QCD
  - Quantumchromodynamics
- Gauge theory
  - Like (classical) electrodynamics
- Description in terms of a quantum field theory
The theory

- Quantized theory of these interactions is QCD
  - Quantumchromodynamics
- Gauge theory
  - Like (classical) electrodynamics
- Description in terms of a quantum field theory
- Embedded in the standard model of particle physics as the strong sector
The theory

• Quantized theory of these interactions is QCD
  • Quantumchromodynamics
• Gauge theory
  • Like (classical) electrodynamics
• Description in terms of a quantum field theory
• Embedded in the standard model of particle physics as the strong sector
• Ultimately describes nuclear physics
  • Residual strong interactions between (color-neutral) hadrons bind nuclei together
Particles in the sky
Particles in the sky

• Hot: The early universe
Particles in the sky

• Hot: The early universe
• Dense: Neutron stars
Particles in the sky

- Hot: The early universe
- Dense: Neutron stars
- The phase diagram of QCD
Hot: The early universe
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
- Almost as many particles as anti-particles
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
  - Temperatures are so high that hadrons have so much energy that they cannot be distinguished
    - Deconfinement
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
  - Temperatures are so high that hadrons have so much energy that they cannot be distinguished
    - Deconfinement
  - Mass generation ceases
    - Almost massless quarks
Hot: The early universe

- Fractions of a second after the big-bang the temperature of the universe was higher than the mass of the pion
  - Almost as many particles as anti-particles
- Strong interactions behave differently
  - Temperatures are so high that hadrons have so much energy that they cannot be distinguished
    - Deconfinement
  - Mass generation ceases
    - Almost massless quarks
- What happened during the cooldown of the universe?
Dense: Neutron stars

• Matter can be compressed
Dense: Neutron stars

• Matter can be compressed

• Highest gravitational stable densities occur in neutron stars
Dense: Neutron stars

- Matter can be compressed
- Highest gravitational stable densities occur in neutron stars
Dense: Neutron stars

- Matter can be compressed
- Highest gravitational stable densities occur in neutron stars
- Hadrons are very densely packed
- Changes the strong interaction
Dense: Neutron stars

• Matter can be compressed
• Highest gravitational stable densities occur in neutron stars
• Hadrons are very densely packed
• Changes the strong interaction
Dense: Neutron stars

• Matter can be compressed
• Highest gravitational stable densities occur in neutron stars
• Hadrons are very densely packed
• Changes the strong interaction
Dense: Neutron stars

• Matter can be compressed
• Highest gravitational stable densities occur in neutron stars
• Hadrons are very densely packed
• Changes the strong interaction
  • Overlap of dense particles: Deconfined
Dense: Neutron stars

- Matter can be compressed
- Highest gravitational stable densities occur in neutron stars
- Hadrons are very densely packed
- Changes the strong interaction
  - Overlap of dense particles: Deconfined
  - Mass generation changes
Dense: Neutron stars

• Matter can be compressed
• Highest gravitational stable densities occur in neutron stars
• Hadrons are very densely packed
• Changes the strong interaction
  • Overlap of dense particles: Deconfined
  • Mass generation changes
    • Similar to superconductors
    • Very enigmatic phases possible, e.g. crystals
Dense: Neutron stars

• Matter can be compressed
• Highest gravitational stable densities occur in neutron stars
• Hadrons are very densely packed
• Changes the strong interaction
  • Overlap of dense particles: Deconfined
  • Mass generation changes
    • Similar to superconductors
    • Very enigmatic phases possible, e.g. crystals
• What are the conditions inside a neutron star?
Formulating the question

• Both questions require knowledge about QCD in a thermodynamic setting
Formulating the question

- Both questions require knowledge about QCD in a thermodynamic setting
- Map QCD at different temperatures and densities
Formulating the question

- Both questions require knowledge about QCD in a thermodynamic setting
- Map QCD at different temperatures and densities
- Very hard to tackle experimentally
- Conditions in the early universe and inside neutron stars can be recreated in heavy-ion collisions
  - Performed at AGS, SPS, GSI, RHIC, LHC and in the future FAIR and NICA
  - Yield the QCD phase diagram
Formulating the question

• Both questions require knowledge about QCD in a thermodynamic setting

• Map QCD at different temperatures and densities

• Very hard to tackle experimentally

• Conditions in the early universe and inside neutron stars can be recreated in heavy-ion collisions
  • Performed at AGS, SPS, GSI, RHIC, LHC and in the future FAIR and NICA
  • Yield the QCD phase diagram

• The phase diagram should be derivable from QCD!
The QCD phase diagram – a sketch
The QCD phase diagram – a sketch

T
The QCD phase diagram – a sketch

Chemical potential (Density)
The QCD phase diagram – a sketch
The QCD phase diagram – a sketch

Vacuum
Inside a nucleus
Chemical potential (Density)
The QCD phase diagram – a sketch

Path of the early universe

Vacuum  Inside a nucleus  Chemical potential (Density)

• Interesting areas are at high temperature
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
The QCD phase diagram – a sketch

• Interesting areas are at high temperature and density
• Experiment gives lower bounds for rapid changes
• Calculate from first principles
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
The QCD phase diagram – a sketch

- Theoretically under control - crossover
- High-temperature phase

- Normal phase
- Theoretical idea of the phase boundary
- Likely a phase transition
- Experimental lower phase boundary

- Vacuum
- Inside a nucleus
- Chemical potential (Density)

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
The QCD phase diagram – a sketch

- Interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
The QCD phase diagram – a sketch

- Theoretically under control – crossover
- Critical point?
- Theoretical idea of the phase boundary
- Likely a phase transition
- High density phase (Color-)Superconducting? Crystal?
- High-temperature phase

- The interesting areas are at high temperature and density
- Experiment gives lower bounds for rapid changes
- Calculate from first principles
QCD from the bottom up
QCD from the bottom up

- First principles: QCD as a gauge theory
QCD from the bottom up

- First principles: QCD as a gauge theory
- A correlation-function-based framework
QCD from the bottom up

• First principles: QCD as a gauge theory
• A correlation-function-based framework
• Results in the vacuum
Describing a theory
QCD as a gauge theory

• QCD is a gauge theory
QCD as a gauge theory

- QCD is a gauge theory
  \[ L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \]
  \[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c \]
- Gluons \( A_\mu^a \)
- A coupling \( g \) and some numbers \( f_{abc} \)
QCD as a gauge theory

- QCD is a gauge theory
  \[ L = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a + \overline{\Psi}_i (iD_{\mu}^{ij} \gamma^\mu - m) \Psi_j \]
  \[ F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{a}_{bc} A^b_\mu A^c_\nu \]
  \[ D_{\mu}^{ij} = \delta^{ij} \partial_\mu - ig A^a_\mu t_a^{ij} \]

- Gluons \( A^a_\mu \)
- Quarks \( \Psi_i \)
- A coupling \( g \) and some numbers \( f^{abc} \) and \( t_a^{ij} \)
QCD as a gauge theory

- QCD is a gauge theory

\[ L = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a + \overline{\Psi}_i (i D_\mu^i \gamma^\mu - m) \Psi_j \]

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{bc}^a A_\mu^b A_\nu^c \]

\[ D_\mu^{ij} = \delta_\mu^{ij} \partial_\mu - ig A_\mu^a t_a^{ij} \]

- Gluons \( A_\mu^a \)

- Quarks \( \Psi_i \)

- A coupling \( g \) and some numbers \( f_{abc}^a \) and \( t_a^{ij} \)

- Invariant under gauge transformations

\[ A_\mu^a \rightarrow A_\mu^a + (\delta_b^a \partial_\mu - g f_{bc}^a A_\mu^c) \phi^b(x) \]

\[ \Psi_i \rightarrow \Psi_i + g t_a^{ij} \phi^a \Psi_j \]

with arbitrary \( \phi^a(x) \)
Describing a theory
Describing a theory

- A theory can be described in terms of correlation functions
Describing a theory

- A theory can be described in terms of correlation functions
Describing a theory

- A theory can be described in terms of correlation functions
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_\mu A^b_\nu \rangle \]

Propagators

Quantization

Methods
Describing a theory

• A theory can be described in terms of correlation functions

\[ \langle A^a \mu A^b \nu \rangle \]

Propagators

\[ \langle \overline{\Psi}_i A^a_{\mu} t^i_a \Psi_j \rangle \]

Interaction vertices

Methods
Describing a theory

- A theory can be described in terms of correlation functions

\[
\langle A^a_{\mu} A^b_{\nu} \rangle
\]

Propagators

\[
\langle \Psi_i A^a_{\mu} t^i_j \Psi_j \rangle
\]

Interaction vertices

\[
\langle \Psi_i \Psi_i \Psi_j \Psi_j \rangle
\]

Bound states

Quantization

Methods
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_\mu A^b_\nu \rangle \]

Propagators

\[ \langle \bar{\Psi}_i A^a_\mu t^i_a \Psi_j \rangle \]

Interaction vertices

\[ \langle \bar{\bar{\Psi}}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle \]

Bound states

Quantization

Methods


[Fischer JPG 06 Maas 10]
Describing a theory

- A theory can be described in terms of correlation functions.

**Propagators** $\langle A^a_\mu A^b_\nu \rangle$

**Interaction vertices** $\langle \bar{\Psi}_i A^a_\mu t^j_a \Psi_j \rangle$

**Bound states** $\langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle$

Quantization

Methods
Describing a theory

- A theory can be described in terms of correlation functions

  Propagators \( \langle A^a_\mu A^b_\nu \rangle \)

  Interaction vertices \( \langle \bar{\Psi}_i A^a_\mu t^i_j \Psi_j \rangle \)

  Bound states \( \langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle \)
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_\mu A^b_\nu \rangle \]

### Propagators

**Quantization**

\[ \langle \bar{\Psi}_i A^a_\mu t^i_a \Psi_j \rangle \]

**Interaction vertices**

**Methods**

\[ \langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle \]

**Bound states**
Describing a theory

- A theory can be described in terms of correlation functions

Quantization

\[
\begin{align*}
\uparrow \quad \text{u} \\
\downarrow \quad \text{d} \\
\rightarrow \quad \text{g}
\end{align*}
\]
Quantization and gauge fixing

- Particle fields change under gauge transformations
- So do most correlation functions
Quantization and gauge fixing

• Particle fields change under gauge transformations
  • So do most correlation functions
• Requires a fixed gauge, a condition on the gauge fields
Quantization and gauge fixing

- Particle fields change under gauge transformations
  - So do most correlation functions
- Requires a fixed gauge, a condition on the gauge fields
- Local condition in perturbation theory
  - Used here: Landau gauge $\partial^\mu A^a_\mu = 0$
  - Many other choices...
Quantization and gauge fixing

- Particle fields change under gauge transformations
  - So do most correlation functions
  - Requires a fixed gauge, a condition on the gauge fields
- Local condition in perturbation theory
  - Used here: Landau gauge $\partial^\mu A_\mu^a = 0$
  - Many other choices...
- Insufficient beyond perturbation theory [Gribov NPA 77, Singer CMP 78]
  - Gribov copies: More than one solution to $\partial^\mu A_\mu^a = 0$
  - Gribov-Singer ambiguity: Requires non-local constraints
Quantization and gauge fixing

- Particle fields change under gauge transformations
  - So do most correlation functions
- Requires a fixed gauge, a condition on the gauge fields
- Local condition in perturbation theory
  - Used here: Landau gauge $\partial^\mu A^a_\mu = 0$
  - Many other choices...
- Insufficient beyond perturbation theory \cite{Gribov NPA 77, Singer CMP 78}
  - Gribov copies: More than one solution to $\partial^\mu A^a_\mu = 0$
  - Gribov-Singer ambiguity: Requires non-local constraints
- Resolution by specifying a sampling procedure for Gribov copies \cite{Maas PLB 10/PRD 09, Fischer, Maas, Pawlowski AoP 09}
  - Generates families of non-perturbative gauges
Describing a theory

• A theory can be described in terms of correlation functions

Quantization

Methods

\[ u \quad d \quad g \]
Methods to calculate correlation functions

- Lattice
Lattice calculations

• Take a finite volume – usually a hypercube
Lattice calculations

- Take a **finite volume** – usually a hypercube
- Discretize it, and get a **finite, hypercubic lattice**
Lattice calculations

• Take a **finite volume** – usually a hypercube
• Discretize it, and get a **finite, hypercubic lattice**
• Calculate observables using path integral
  • Can be done numerically
  • Uses **Monte-Carlo methods**
Lattice calculations

- Take a **finite volume** – usually a hypercube
- Discretize it, and get a **finite, hypercubic lattice**
- Calculate observables using path integral
  - Can be done numerically
  - Uses **Monte-Carlo methods**
- Artifacts
Lattice calculations

- Take a **finite volume** – usually a hypercube
- Discretize it, and get a **finite, hypercubic lattice**
- Calculate observables using path integral
  - Can be done numerically
  - Uses **Monte-Carlo methods**
- Artifacts
  - Finite volume/discretization
Lattice calculations

• Take a finite volume – usually a hypercube
• Discretize it, and get a finite, hypercubic lattice
• Calculate observables using path integral
  • Can be done numerically
  • Uses Monte-Carlo methods
• Artifacts
  • Finite volume/discretization
  • Masses vs. wave-lengths
Methods to calculate correlation functions

- **Lattice**
  
  - Discretize space-time in a box and calculate the path-integral and expectation values explicitly
  
  ✔ Full non-perturbative dynamics correctly implemented
  
  ✗ Finite volume artifacts, disparate scales most severe obstacles
Methods to calculate correlation functions

• Lattice
  • Discretize space-time in a box and calculate the path-integral and expectation values explicitly
  ✔ Full non-perturbative dynamics correctly implemented
  ✗ Finite volume artifacts, disparate scales most severe obstacles

• Functional methods (DSE, RGE...)
(Truncated) Dyson-Schwinger Equations (DSEs)

\[ \frac{1}{\langle \bar{\Psi} \Psi \rangle(p)} = \gamma_\mu p^\mu + \int dq \gamma_\mu \langle \bar{\Psi} \Psi \rangle(q) \langle A^\mu A^\nu \rangle(p-q) \langle A^\nu \bar{\Psi} \Psi \rangle(p, q) \]

- Infinite set of coupled non-linear integral equations
(Truncated) Dyson-Schwinger Equations (DSEs)

\[
\frac{1}{\langle \bar{\Psi} \Psi \rangle(p)} = \gamma_\mu p^\mu + \int dq \gamma_\mu \langle \bar{\Psi} \Psi \rangle(q) \langle A^\mu A^\nu \rangle(p-q) \langle A_\nu \bar{\Psi} \Psi \rangle(p,q)
\]

- Infinite set of coupled non-linear integral equations
(Truncated) Dyson-Schwinger Equations (DSEs)

- Infinite set of coupled non-linear integral equations
(Truncated) Dyson-Schwinger Equations (DSEs)

- Infinite set of coupled non-linear integral equations
- Must be reduced by means of approximations
- Steady progress in relaxing approximations
Methods to calculate correlation functions

- **Lattice**
  - Discretize space-time in a box and calculate the path-integral and expectation values explicitly
  - ✔ Full non-perturbative dynamics correctly implemented
  - ✗ Finite volume artifacts, disparate scales most severe obstacles

- **Functional methods** (DSE, RGE...)
  - Coupled non-linear integral equations must be solved
  - ✔ Continuum, infinite volume, analytically accessible
  - ✗ Requires (often uncontrolled) approximations
Methods to calculate correlation functions

- Lattice
  - Discretize space-time in a box and calculate the path-integral and expectation values explicitly
    ✓ Full non-perturbative dynamics correctly implemented
    ✗ Finite volume artifacts, disparate scales most severe obstacles

- Functional methods (DSE, RGE...)
  - Coupled non-linear integral equations must be solved
    ✓ Continuum, infinite volume, analytically accessible
    ✗ Requires (often uncontrolled) approximations

- Perturbation theory included
Methods to calculate correlation functions

- **Lattice**
  - Discretize space-time in a box and calculate the path-integral and expectation values explicitly
  
  ✔ Full non-perturbative dynamics correctly implemented
  
  ❌ Finite volume artifacts, disparate scales most severe obstacles

- **Functional methods** (DSE, RGE...)
  - Coupled non-linear integral equations must be solved
  
  ✔ Continuum, infinite volume, analytically accessible
  
  ❌ Requires (often uncontrolled) approximations

- **Perturbation theory** included

- **Combination of all methods most successful!**
Describing a theory

- A theory can be described in terms of correlation functions

\[
\langle A^a_\mu A^b_\nu \rangle
\]

Quantization

Methods
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_\mu A^b_\nu \rangle \]

Propagators

\[ D^{ab}_{\mu\nu}(x-y) = \langle A^a_\mu(x) A^b_\nu(y) \rangle \]

Quantization

Methods
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_{\mu} A^b_{\nu} \rangle \]

**Propagators**

Quantization

\[ D^{ab}_{\mu\nu}(x-y) = \langle A^a_{\mu}(x) A^b_{\nu}(y) \rangle \]

\[ D^{ab}_{\mu\nu}(p) = \delta^{ab} (\delta_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}) D(p) \]
Gluon correlation function

• Calculation of correlation functions possible
Gluon correlation function

- Calculation of correlation functions possible
• Calculation of correlation functions possible
• Shows a transition from low to high energies
Gluon correlation function

- Calculation of correlation functions possible
- Shows a transition from low to high energies

Perturbative tail, measured in high-energy experiments
Gluon correlation function

- Calculation of correlation functions possible
- Shows a transition from low to high energies
Gluon correlation function

- Calculation of correlation functions possible
- Shows a transition from low to high energies
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a_\mu A^b_\nu \rangle \]

Propagators

\[ \langle \overline{\psi}_i A^a_\mu t^i_a \psi_j \rangle \]

Interaction vertices

Quantization

Methods

Vacuum – Temperature – Density – Summary
Three-gluon vertex

\[ k \quad g \quad k+q \quad g \quad q \]
Three-gluon vertex

\[ G_{\mu\nu\rho}^{abc}(x,y,z) = \langle A_\mu^a(x) A_\nu^b(y) A_\rho^c(z) \rangle \]
Three-gluon vertex

\[ G^{abc}_{\mu\nu\rho}(x, y, z) = \langle A^a_\mu(x) A^b_\nu(y) A^c_\rho(z) \rangle \]

\[ G^{A^3}(k, q, k+q) = \Gamma^{0; abc}_{\mu\nu\rho} G^{abc}_{\mu\nu\rho}(k, q, q+q) \]
Three-gluon vertex

\[ G_{\mu\nu\rho}^{abc}(x, y, z) = \langle A_{\mu}^{a}(x) A_{\nu}^{b}(y) A_{\rho}^{c}(z) \rangle \]

\[ G^{A^3}(k, q, k+q) = \Gamma_{\mu\nu\rho}^{0;abc} G_{\mu\nu\rho}^{abc}(k, q, q+q) \]
Three-gluon vertex

\[ G^{abc}_{\mu \nu \rho}(x, y, z) = \langle A^a_\mu(x) A^b_\nu(y) A^c_\rho(z) \rangle \]

\[ G^{A^3}(k, q, k+q) = \Gamma^{0;abc}_{\mu \nu \rho} G^{abc}_{\mu \nu \rho}(k, q, q+q) \]

- No emission around hadron energy scales!
Three-gluon vertex

\[ G_{\mu\nu\rho}^{abc}(x, y, z) = \langle A_\mu^a(x) A_\nu^b(y) A_\rho^c(z) \rangle \]

\[ G \vec{A}_3^i(k, q, k+q) = \Gamma^0;_{abc} G^{abc}_\mu \delta \rho \mu \nu \rho \delta (k, q, q+q) \]

- No emission around hadronic energy scales!
- Infrared enhanced: Strong emission of (non-propagating) gluons on the light-cone
Describing a theory

- A theory can be described in terms of correlation functions

\[ \langle A^a A^b \rangle \]

**Propagators**

**Interaction vertices**

\[ \langle \bar{\Psi}_i A^a_{\mu} t^i_a \Psi_j \rangle \]

**Methods**

**Bound states**

\[ \langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle \]
Describing a theory

• A theory can be described in terms of correlation functions

Propagators $\langle A^a_\mu A^b_\nu \rangle$

Interaction vertices $\langle \bar{\Psi}_i A^a_\mu t^i_a \Psi_j \rangle$

Bound states $\langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle$

Quantization

Methods

Vacuum

Temperature

Density

Summary
Meson masses

• Calculation of the meson masses
Meson masses

• Calculation of the meson masses
  • Start with the gluons

![Gluon 2-point correlation function graph]

[Blank, Krassnigg, Maas PRD 11]
Meson masses

• Calculation of the meson masses
  • Start with the gluons
  • Calculate the quark propagator
Meson masses

• Calculation of the meson masses
  • Start with the gluons
  • Calculate the quark propagator
  • Calculate the meson masses
    • Requires to fix the parameters
      • E.g. pion properties to set up and down masses
Meson masses

• Calculation of the meson masses
  • Start with the gluons
  • Calculate the quark propagator
  • Calculate the meson masses
    • Requires to fix the parameters
      • E.g. pion properties to set up and down masses
  • Calculate quantities
    • E.g. rho properties like mass and decay constant
Meson masses

• Calculation of the meson masses
  • Start with the gluons
  • Calculate the quark propagator
  • Calculate the meson masses
    • Requires to fix the parameters
      • E.g. pion properties to set up and down masses
  • Calculate quantities
    • E.g. rho properties like mass and decay constant
• Gauge invariance is recovered in the process
  • Same rho properties for input correlation functions in different gauges
The phase diagram from first principles
The phase diagram from first principles

- Hot
- Dense
The phase diagram from first principles

- Hot: Yang-Mills theory and QCD
- Dense
The phase diagram from first principles

- **Hot**: Yang-Mills theory and QCD
- **Dense**: Quantization and condensates
Describing a theory

- A theory can be described in terms of correlation functions

  Propagators \( \langle A^a \mu A^b \nu \rangle \)

  Interaction vertices \( \langle \bar{\Psi}_i A^a \mu t^i j \Psi_j \rangle \)

  Bound states \( \langle \bar{\Psi}_i \Psi_i \bar{\Psi}_j \Psi_j \rangle \)
Calculations in the phase diagram

- Calculational scheme can be transferred directly to equilibrium calculations
  - Matsubara formalism
  - More subtleties with quantization
Calculations in the phase diagram

- Calculational scheme can be transferred directly to equilibrium calculations
  - Matsubara formalism
  - More subtleties with quantization

- Gluons are vector particles
  - Can be polarized transverse or longitudinal to the heat bath
  - In general: Correlation functions get more structure
Phase diagram of Yang-Mills theory
Phase diagram of Yang-Mills theory

- **Phase line**
  - Temperature only external control parameter
Phase diagram of Yang-Mills theory

- Phase line
  - Temperature only external control parameter

T=0 MeV
Phase diagram of Yang-Mills theory

- **Phase line**
  - Temperature only external control parameter
  - Phase transition at a certain critical temperature
  - Precise temperature and order of transition depends on the number of gluons and the numbers $f^{abc}$
    - Start with second order case

$T = 0$ MeV

$T_c \sim 300$ MeV
Phase diagram of Yang-Mills theory

- **Phase line**
  - Temperature only external control parameter
  - Phase transition at a certain critical temperature
  - Precise temperature and order of transition depends on the number of gluons and the numbers $f^{abc}$
    - Start with second order case
  - **Can be investigated using (and determined from) the correlation functions**

$T = 0 \text{ MeV}$

$T_c \approx 300 \text{ MeV}$
Gluon polarized along the heat-bath
Gluon polarized along the heat-bath

- Significant dependency on the temperature
- Sensitive to finite-volume corrections

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized along the heat-bath

- Significant dependency on the temperature
- Sensitive to finite-volume corrections

Temperature dependence

[Discher, Maas, Müller EPJC 10 Maas et al., unpublished]

[Mitrushkin, Bornyakov 11]
Gluon polarized along the heat-bath

- Reaches maximum shortly before the phase transition
- Drop afterwards

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Parallel screening mass

- Phase transition manifests in the parallel screening mass
Parallel screening mass

- Phase transition manifests in the parallel screening mass $\sqrt{1/D_L(0)}$
Parallel screening mass

• Phase transition manifests in the parallel screening mass $\sqrt{1/D_L(0)}$

• How is it manifest?
Parallel screening mass

- Phase transition manifests in the parallel screening mass $\sqrt{1/D_L(0)}$
  - How is it manifest?
- Is it sufficiently sensitive to distinguish first and second order transitions?
- Critical phenomena?
Parallel screening mass

Phase transition vicinity

- Second-order-like onset, up to artifacts
Parallel screening mass

- Second-order-like onset, up to artifacts
- Susceptibility identifies phase transition
Parallel screening mass

- Second-order-like onset, up to artifacts
- Susceptibility identifies phase transition
- Can be fitted with critical behavior
Parallel screening mass

- Second-order-like onset, up to artifacts
- Susceptibility identifies phase transition
- Can be fitted with critical behavior
- Not yet clear whether this distinguishes the order

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized transverse to the heat-bath
Gluon polarized transverse to the heat-bath

- Small infrared suppression
- Mix with lattice artifacts

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized transverse to the heat-bath

- Small infrared suppression
- Mix with lattice artifacts
- Resolves the Linde problem

[Maas, Wambach, Grüter, Alkofer EPJC 04]

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized transverse to the heat-bath

- Slight increase of screening mass
Gluon polarized transverse to the heat-bath

- No response to the phase transition visible

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized transverse to the heat-bath

- No qualitative change

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
Gluon polarized transverse to the heat-bath

- No qualitative change
- Asymptotic states the same – gluons stay confined

[Fischer, Maas, Müller EPJC 10 Maas et al., unpublished]
• No qualitative change
  • Asymptotic states the same – gluons stay confined
  • But thermodynamics become Stefan-Boltzmann-like

\[D_T(p) [\text{GeV}^2]\]

\[p [\text{GeV}]\]
Towards QCD

• Quenched QCD
  • No quark backreaction
Towards QCD

• Quenched QCD
  • No quark backreaction

• Order parameters can be obtained from quark propagator [Fischer PRL 09]
Towards QCD

- Quenched QCD
  - No quark backreaction
- Order parameters can be obtained from quark propagator [Fischer PRL 09]
- Change of mass generation and confinement occur simultaneously [Fischer, Maas, Müller EPJC 10]
**Towards QCD**

- Quenched QCD
  - No quark backreaction
- Order parameters can be obtained from quark propagator [Fischer PRL 09]
- Change of mass generation and confinement occur simultaneously
- Access to all relevant order parameters

![Order parameters for quarks](image-url)
Next step: Finite density

• Large densities
Next step: Finite density

- Large densities
- Condensates of fermions
Next step: Finite density

• Large densities

• Condensates of fermions
  • May be colored
Next step: Finite density

- Large densities
- Condensates of fermions
  - May be colored
  - Color charge is gauge-dependent
Next step: Finite density

• Large densities

• Condensates of fermions
  • May be colored
  • Color charge is gauge-dependent
    • Condensates can be zero in some gauges
    [Rajagopal, Wilzcek 2000]
Next step: Finite density

- Large densities
- Condensates of fermions
  - May be colored
  - Color charge is gauge-dependent
    - Condensates can be zero in some gauges
      - [Rajagopal, Wilzcek 2000]
    - No breaking of gauge symmetry because of Elitzur's theorem
Next step: Finite density

- Large densities
- Condensates of fermions
  - May be colored
  - Color charge is gauge-dependent
    - Condensates can be zero in some gauges
      [Rajagopal, Wilzcek 2000]
    - No breaking of gauge symmetry because of Elitzur's theorem
- Requires careful quantization and gauge-fixing
Next step: Finite density

- Large densities
- Condensates of fermions
  - May be colored
  - Color charge is gauge-dependent
    - Condensates can be zero in some gauges
      [Rajagopal, Wilzcek 2000]
    - No breaking of gauge symmetry because of Elitzur's theorem
- Requires careful quantization and gauge-fixing
- Framework requires a fixed gauge
  - Contrast gauges with and without condensates
Next step: Finite density

- Large densities
- Condensates of fermions
  - May be colored
  - Color charge is gauge-dependent
    - Condensates can be zero in some gauges
      - [Rajagopal, Wilczek 2000]
    - No breaking of gauge symmetry because of Elitzur's theorem
- Requires careful quantization and gauge-fixing
- Framework requires a fixed gauge
  - Contrast gauges with and without condensates
- Understand first in a simpler model
Role model: Higgs

• Technically simpler than finite density
Role model: Higgs

- Technically simpler than finite density
- Same situation
  - Higgs condenses in the Higgs phase
  - Condensate vanishes in some gauges, e.g. non-aligned Landau gauge
Role model: Higgs

- Technically simpler than finite density
- Same situation
  - Higgs condenses in the Higgs phase
  - Condensate vanishes in some gauges, e.g. non-aligned Landau gauge
  - Analogy not complete: Not a distinct phase from the confining phase
    - Difference essentially gauge-dependent
Role model: Higgs

- Technically simpler than finite density

- Same situation
  - Higgs condenses in the Higgs phase
  - Condensate vanishes in some gauges, e.g. non-aligned Landau gauge
  - Analogy not complete: Not a distinct phase from the confining phase
    - Difference essentially gauge-dependent

- Application of the framework to determine the gluon propagator
Gluons

Gluon propagator

\[ D(p) \quad [\text{GeV}^{-2}] \]

- **No condensation**
- **Leading order condensate effect**
- **Full solution in gauge without condensate**

[Maas EPJC 2011]
Gluons

Gluon propagator

- Screening mass – full quantum (Meissner) effect
Gluons

- Screening mass – full quantum (Meissner) effect
- All correlation functions accessible
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
- Framework can be applied
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
- Framework can be applied
  - Simplifications by a suitable gauge selection
Applicability to finite density

• Condensate effects are present even when the condensate vanishes

• Framework can be applied
  • Simplifications by a suitable gauge selection
  • Physics unchanged: Just a gauge choice
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
- Framework can be applied
  - Simplifications by a suitable gauge selection
  - Physics unchanged: Just a gauge choice
- For a full description requires baryons
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
- Framework can be applied
  - Simplifications by a suitable gauge selection
  - Physics unchanged: Just a gauge choice
- For a full description requires baryons
  - Bound states included for the Higgs role model
Applicability to finite density

- Condensate effects are present even when the condensate vanishes
- Framework can be applied
  - Simplifications by a suitable gauge selection
  - Physics unchanged: Just a gauge choice
- For a full description requires baryons
  - Bound states included for the Higgs role model
  - At finite density no full lattice simulations
    - Functional methods [Nickel, Wambach, Alkofer 06-08]
    - Zero density comparison to anchor approximations
    - Two-color QCD to improve systematic reliability [Skullerud 08-10]
Summary

- Quarks and gluons make up strongly interacting matter
Summary

• Quarks and gluons make up strongly interacting matter

• Strongly interacting matter plays a crucial role in extreme astrophysical settings
Summary

- Quarks and gluons make up strongly interacting matter
- Strongly interacting matter plays a crucial role in extreme astrophysical settings
- QCD can be described from bottom up
  - A combination of methods gives systematic reliability
  - Deep field-theoretical problems have been resolved
Summary

- Quarks and gluons make up strongly interacting matter

- **Strongly interacting matter plays a crucial role in extreme astrophysical settings**

- QCD can be described from bottom up
  - A combination of methods gives systematic reliability
  - Deep field-theoretical problems have been resolved

- **Application to the phase diagram**
  - No intrinsic problems
  - The early universe
  - Framework for finite density is established
Outlook

• Self-consistent bound-state dynamics
Outlook

• **Self-consistent bound-state dynamics**
  
  • Mesons for heavy-ion collisions
    
    • Hard probes beyond factorization
Outlook

- **Self-consistent bound-state dynamics**
  - Mesons for heavy-ion collisions
    - Hard probes beyond factorization
  - Baryons for finite density
    - Self-consistent inclusion of weak/electromagnetic probes for neutron stars
Outlook

- **Self-consistent bound-state dynamics**
  - Mesons for heavy-ion collisions
    - Hard probes beyond factorization
  - Baryons for finite density
    - Self-consistent inclusion of weak/electromagnetic probes for neutron stars
  - First steps done
Outlook

• **Self-consistent bound-state dynamics**
  • **Mesons for heavy-ion collisions**
    • Hard probes beyond factorization
  • **Baryons for finite density**
    • Self-consistent inclusion of weak/electromagnetic probes for neutron stars
  • **First steps done**

• **Non-equilibrium**
  • Required for heavy-ion collisions and neutron star formation
  • Needs firm control over quantization