Exploiting Lattice Ward-Identities

Andreas Wipf, TPI Jena

"Aspects of Confinement and
Nonperturbative QCD"

Trento, 11. March, 2003

with:

Thomas Heinzl and Leander Dittmann (Jena)

OUTLINE:

- Effective Actions
- Inverse Monte-Carlo with WARD-Identities
- Testing the FADDEEV-NIEMI Conjecture
- Effective Actions for POLYAKOV-Loops
- Summary, Outlook

1. Effective Actions

given: action $S[U]$ for *microscopic* variables U

$\rightarrow$ macroscopic (composite) variables $X = X[U]$ with

$$e^{-S_{\text{eff}}[X]} = \int \mathcal{D}U \, \delta(X - X[U]) e^{-S[U]}$$

interesting cases in physics: $\int \mathcal{D}U$?

But (cp. eff. potentials, χPT, GINZBURG-LANDAU)

- $S_{\text{eff}}[X]$ inherits (global) symmetries of $S[U] \rightarrow$
- ansatz: $S_{\text{eff}}[X] = \sum_k \lambda_k \, S_k[X]$, symmetric S_k.
- systematic expansion?
  1. couplings λ_k via phenomenology
  2. couplings λ_k via simulations

MC: known $S[U] \longrightarrow \langle X[U] \rangle$

$\boxed{\text{need inverse Monte Carlo}}$

2. Inverse MC with Ward-Identities

Monte Carlo:

$S \longrightarrow$ ensemble of configurations

inverse Monte Carlo:

ensemble of configurations $\longrightarrow S$

IMC Algorithm:

- generate ensemble $\{X^{(i)}\}$
  - $\diamond$ standard MC: $S[U] \longrightarrow \{U^{(i)}\}$, U-ensemble
  - $\diamond$ $\Longrightarrow \{X(U^{(i)})\} = \{X^{(i)}\}$, X-ensemble
  - $\diamond$ $\Longrightarrow \langle O[X] \rangle = \sum O[X^{(i)}]$

- determine λ_k via SD equations (Parisi et al. '86)
  - $X \in \mathscr{X}$ = target space with metric g_{ab}
  - assume isometry $\to$ Killing vector ξ,

$$\mathcal{L}_\xi \, g_{ab}(X) = 0$$

  - invariance of $\mathcal{D}X \equiv \prod_x dX_x \sqrt{g(X_x)} \longrightarrow$

$$\int \mathcal{D}X \, \mathcal{L}_\xi \left\{ F[X] \exp(-S_{\text{eff}}[X]) \right\} = 0$$

    in simulations: $F[X] = 1, X^a, X^a X^b, \ldots$
  - ansatz $S_{\text{eff}} \to$

$$\boxed{\textstyle\sum_k \langle F \mathcal{L}_\xi S_k \rangle \lambda_k = \langle \mathcal{L}_\xi F \rangle \quad \text{SDE}}$$

  - (overdetermined) linear system for λ_k

check: $\langle O \rangle_S \sim \langle O \rangle_{S_{\text{eff}}} \Longrightarrow$ MC for $S_{\text{eff}}[X, \lambda_k]$

3. Testing the Faddeev-Niemi Conjecture

- CHO-decomposition:

$$\boldsymbol{A}_\mu = C_\mu \boldsymbol{n} - \underbrace{\boldsymbol{n} \times \partial_\mu \boldsymbol{n}}_{\to H_{\mu\nu}} + \boldsymbol{X}_\mu, \quad \boldsymbol{n}: \text{ monopoles}$$

FADDEEV and NIEMI: IR-dynamics of glue

$$S_{\mathrm{SFN}} = \int d^4x \left[m^2 (\partial_\mu \boldsymbol{n})^2 + \frac{1}{4e^2} (\underbrace{\boldsymbol{n} \cdot \partial_\mu \boldsymbol{n} \times \partial_\nu \boldsymbol{n}}_{\text{field strength } H_{\mu\nu}})^2 \right]$$

- $O(3)$-Sigma Model: $\boldsymbol{n} \cdot \boldsymbol{n} = n^a n^a = 1$

- *dynamically* generated mass scale m! FN: "*unique local and Lorentz invariant action for the unit vector* $\boldsymbol{n}$ *which is at most quadratic (!) in time derivatives ... and involves all such terms that are either relevant or marginal in the infrared*"

  S_{SFN} supports stable knot–like *solitons*

BATTYE/SUTCLIFFE '98:

- Bound on energy of static solitons: rescale x

$$\frac{2e}{m}E = \tilde{E} = \int d^3x\Big((\nabla \boldsymbol{n})^2 + \frac{1}{2}H_{ij}(\boldsymbol{n})\Big)$$

finite energy $\rightarrow$

$$\boldsymbol{n}\big(|\vec{x}| \rightarrow \infty\big) = \boldsymbol{n}_0 \quad , \quad \boldsymbol{n} : S^3 \rightarrow S^2$$

$$\pi_2(S^3) = \mathbb{Z} \quad , \quad \text{classified by } Q$$

HOPF invariant $Q =$

linking number of 2 loops C_i with $\boldsymbol{n}(C_i) = \boldsymbol{n}_i$.

> configuration space consists of topological sectors, classified by HOPF-number Q

VALENKO AND KAPITANSKY (1979):

$$\tilde{E} \geq C|Q|^{3/4}, \qquad C = 16\pi^2 3^{3/8} \sim 238$$

can be improved. BPS-bound?

Problems

- change of variables $\boldsymbol{A}_\mu \to (C_\mu, \boldsymbol{n}, \boldsymbol{X}_\mu)$?

- functional integration over $C_\mu, \boldsymbol{X}_\mu$? (H. Gies)

- $H^2 = (\boldsymbol{n}\Box\boldsymbol{n})^2 - (\boldsymbol{n}\partial_\mu\partial_\nu\boldsymbol{n})^2$

  why same coupling?

- universality class: $O(3)-$Heisenberg ?

- $S_{\mathrm{SFN}} \to$ SSB: SO(3) $\to$ S0(2)

  $\boxed{\text{2 Goldstone bosons, no mass gap???}}$

  where are the Goldstones?

- lattice test: negative! (JHEP 12, 2002, 014)

Reformulations: (P. van Baal, A. W., PLB 515 (01) 181)

- CP_1 formulation: $n_a = z^\dagger \tau_a z,\ z \in \mathbb{C}^2,\ z^\dagger z = 1$
  gauge potential = composite
  Hopf-number = Chern-Simons-action

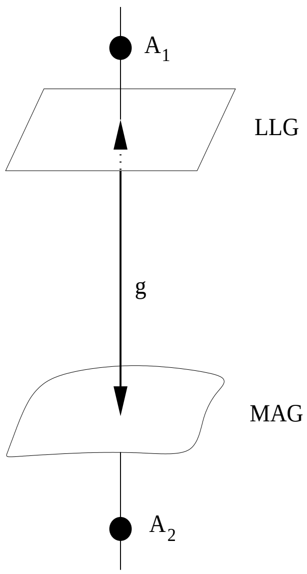
- The coset formulation: $n_a(x) = \frac{1}{2}\text{tr}[\tau_3 g^\dagger(x)\tau_a g(x)]$
  $J = g^\dagger dg$ current, flat connection
  Q = gauge field winding number
  $E \sim$ gauge fixing functional for non-linear MAG

> minima of E (Q fixed) $\sim$ gauge fixed pure gauge potentials in a sector with winding number Q.

pure gauge theory vacua $\longleftrightarrow$ knots

Calculating S_{eff}

- gauge invariant definition of $\boldsymbol{n}$:



Landau gauge:

$$\sum \mathrm{tr}\big(U_\mu(x) + \mathrm{h.c.}\big)$$

$$\downarrow g(x)$$

maximally Abelian gauge:

$$\sum \mathrm{tr}\left(\tau_3 U_\mu(x) \tau_3 U_\mu^\dagger(x)\right)$$

$$\downarrow$$

explicit SB: $O(3) \to O(2)$

- numerical 'observables':

magnetization:	$\mathfrak{M} \equiv \langle n^3 \rangle$
susceptibility:	$\chi^\perp \equiv \sum_x n_x^i n_0^i / 2$
mass gap:	M

algorithm	$\mathfrak{M}$	$a^{-4}\chi^{\perp}$	aM	M [GeV]
AI	0.438	92.57	0.61	0.95
AII	0.366	79.66	0.67	1.03

- ansatz: $S_{\text{eff}} = \sum_k \lambda_k S_k + \sum_j \lambda'_k S'_j$

  symmetric operators S_i:

$\partial\boldsymbol{n}\cdot\partial\boldsymbol{n}$	$\Box\boldsymbol{n}\cdot\Box\boldsymbol{n}$	$(\boldsymbol{n}\cdot\Box\boldsymbol{n})^2$	$(\boldsymbol{n}\cdot\partial_\mu\partial_\nu\boldsymbol{n})^2$

  non-symmetric operators S'_i:

$\boldsymbol{n}\cdot\boldsymbol{h}$	$(\boldsymbol{n}\cdot\boldsymbol{h})^2$	$(\partial\boldsymbol{n}\cdot\partial\boldsymbol{n})(\boldsymbol{n}\cdot\boldsymbol{h})$

- lattice Schwinger-Dyson-equations:

$$\mathcal{D}\boldsymbol{n} = \prod_x d\boldsymbol{n}_x \delta\big(\boldsymbol{n}^2 - 1\big) \quad \text{SO(3)-invariant}$$

Isometries of $S^2 \rightarrow$

angular momentum at x : $i\boldsymbol{L}_x = \boldsymbol{n}_x \times \frac{\partial}{\partial \boldsymbol{n}_x} \Rightarrow$

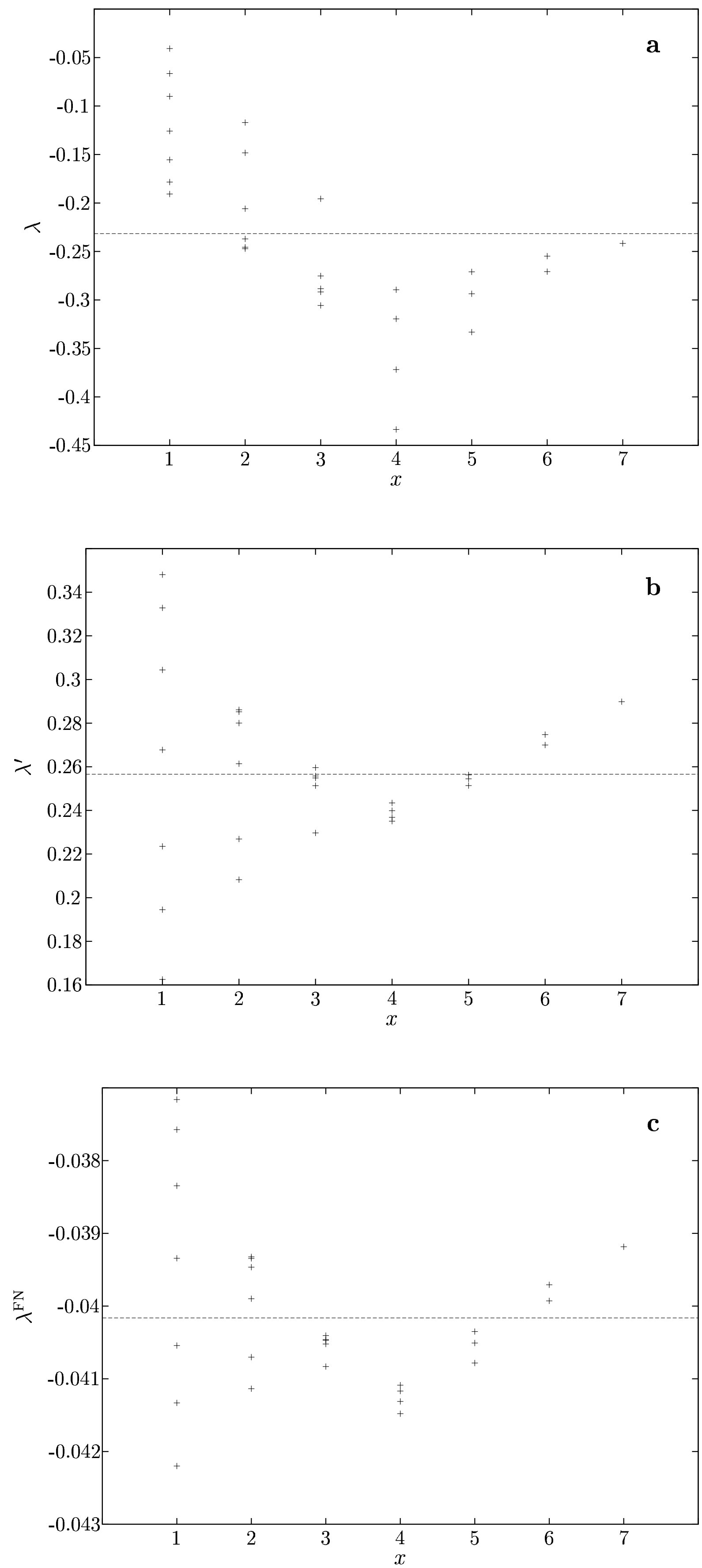
$$\sum_k \langle F \boldsymbol{L}_x S_k \rangle \lambda_k + \sum_k \langle F \boldsymbol{L}_x S'_k \rangle \lambda'_k = \langle \boldsymbol{L}_x F \rangle$$

- $\sum_x \Rightarrow \lambda_k$-independent Ward identities
  $F = \boldsymbol{n}$, minimal model (one λ') $\rightarrow$

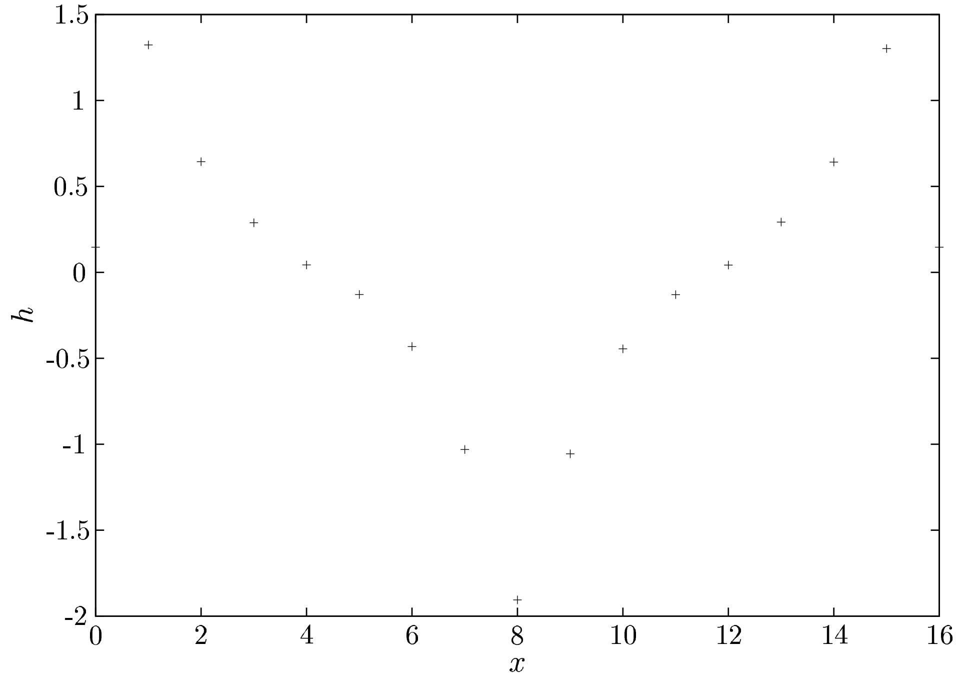
$$\lambda' = -\mathfrak{M}\chi^\perp < 0$$

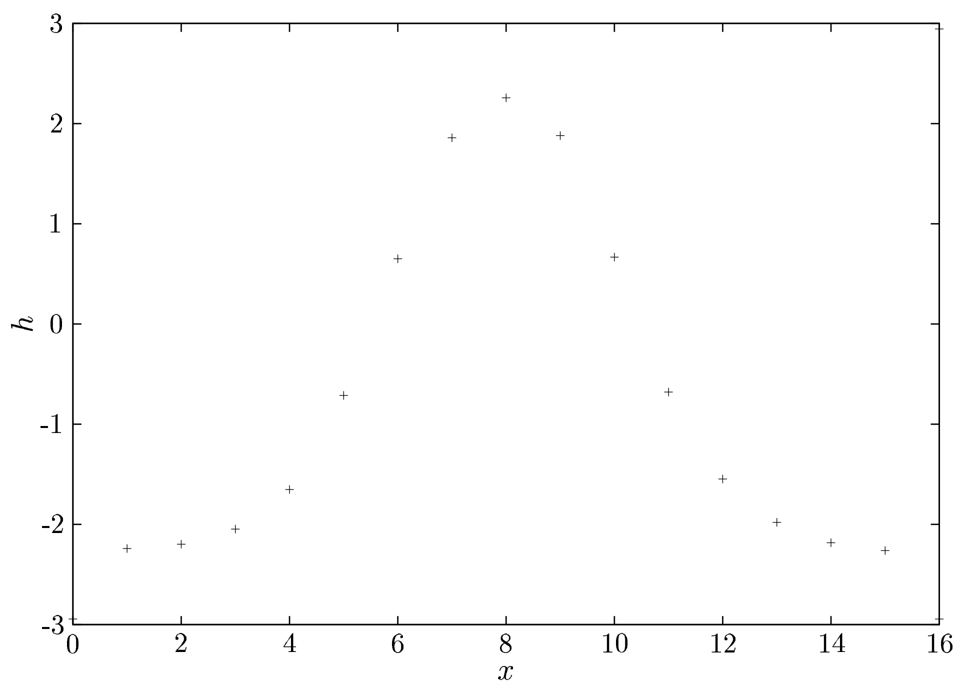
- Inverse MC:
  $\lambda' > 0$ and mass gap

M_{nYM}	$<$	M_{FN}	$<$	M_{YM}
1.0 GeV		1.2 GeV		1.5 GeV


a
b
c
λ
λ'
λ^{FN}
x

two–point functions

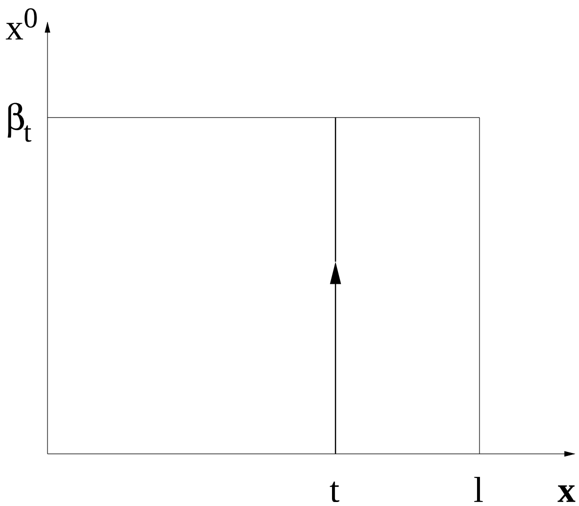




4. Effective Actions for Polyakov Loops

Polyakov loop = traced holonomy

$$L_{\boldsymbol{x}}[U] \equiv \frac{1}{N_C} \mathrm{tr}\, \mathfrak{P}_{\boldsymbol{x}}[U]\,, \quad \mathfrak{P}_{\boldsymbol{x}}[U] \equiv \prod_{t=1}^{N_t} U_{t,\boldsymbol{x};0}$$



$\beta_t \equiv N_t a$: inverse temp.

$l \equiv N_s a$: spatial size

$\boxed{L\text{: order parameter}}$

$T > T_c : \langle L \rangle \neq 0$: deconfinement

$\rightarrow \mathbb{Z}_2$ center symmetry spontaneously broken conjecture: finite temperature $SU(2)$ YM-theory in universality class of $3d\,\mathbb{Z}_2-$Ising model (Svetitsky-Yaffe, 82)

well established on lattice (critical exponents) (Karsch et al., de Forcrand et al. '01)

$\mathbb{Z}_2$-symmetric effective action: (Svetitsky '86, Pisarski '00)

$$S_{\text{eff}}[L] \equiv \lambda_0 \sum_{\langle xy \rangle} L_x L_y + \sum_{x,k} \lambda_{2k} L_x^{2k} \equiv \sum_k \lambda_{2k} S_{2k}$$

- $L_x[U]$ simple
- $\mathbb{Z}_2$ symmetry of S_{eff} *is* symmetry of S_{YM} $\rightarrow$
- S_{eff} should exhibit SSB of $\mathbb{Z}_2$

SD-equations:

- assume $S_{\text{eff}}[L] \equiv S_{\text{eff}}[\mathfrak{P}]$

$$\mathfrak{P}_{\boldsymbol{x}} = L_{\boldsymbol{x}}\mathbb{1} + P_{x}^{a}\sigma^{a} \equiv P_{\boldsymbol{x}}^{\mu}\sigma^{\mu} \in S^{3}$$

- $O(4)$ invariance of HAAR-measure

$$iM_{\boldsymbol{x}}^{\mu\nu} \equiv P_{\boldsymbol{x}}^{\mu}\frac{\partial}{\partial P_{\boldsymbol{x}}^{\nu}} - P_{\boldsymbol{x}}^{\nu}\frac{\partial}{\partial P_{\boldsymbol{x}}^{\mu}}$$

- HAAR-measure $\mathcal{D}\mathfrak{P} \equiv \prod_{\boldsymbol{x}} d^{4}P_{\boldsymbol{x}}\,\delta(P_{\boldsymbol{x}}P_{\boldsymbol{x}} - 1) \Rightarrow$

$$\int \mathcal{D}\mathfrak{P}\, M_{\boldsymbol{x}}^{\mu\nu}\left\{F[\mathfrak{P}]\exp(-S_{\text{eff}}[\mathfrak{P}])\right\} = 0$$

- non-trivial relations from M^{0a}

- choice $F_{\boldsymbol{x}}^{a} \equiv P_{\boldsymbol{x}}^{a}G \rightarrow \langle \text{gauge invariant} \rangle$

- HAAR-measure $\mathcal{D}\mathfrak{P} \longrightarrow$ reduced HAAR-measure

$$\mathcal{D}L \equiv \prod_{\boldsymbol{x}} dL_{\boldsymbol{x}} \, (1 - L_{\boldsymbol{x}}^2)^{1/2}$$

- ansatz for $S_{\text{eff}} \Longrightarrow$

$$\boxed{\begin{aligned} &\textstyle\sum_k \big\langle (1 - L_{\boldsymbol{x}}^2) G S_{2k,\boldsymbol{x}} \big\rangle \lambda_{2k} \\ &= \big\langle (1 - L_{\boldsymbol{x}}^2) G_{,\boldsymbol{x}} - 3 L_{\boldsymbol{x}} G \big\rangle \end{aligned}}$$

- overdetermined linear system for the λ_{2k}

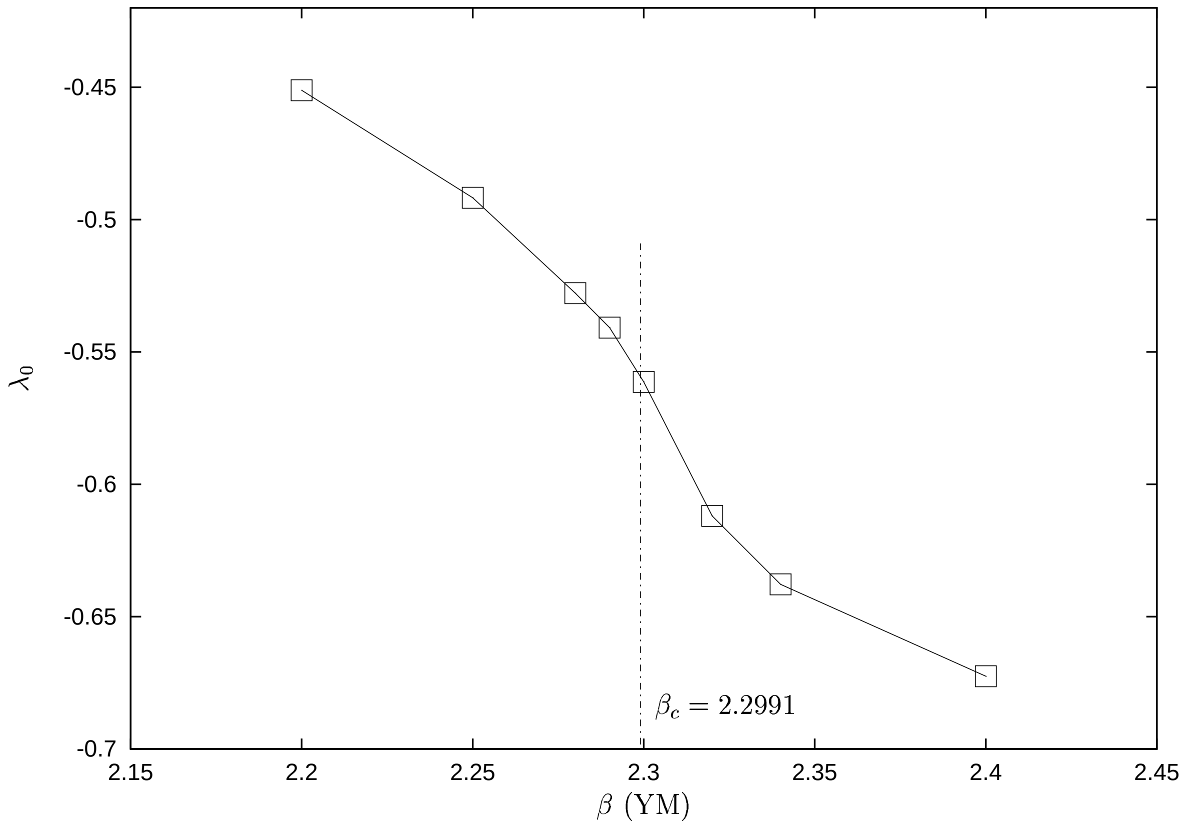
First results from inverse MC

- for 'Ising type' LO ansatz

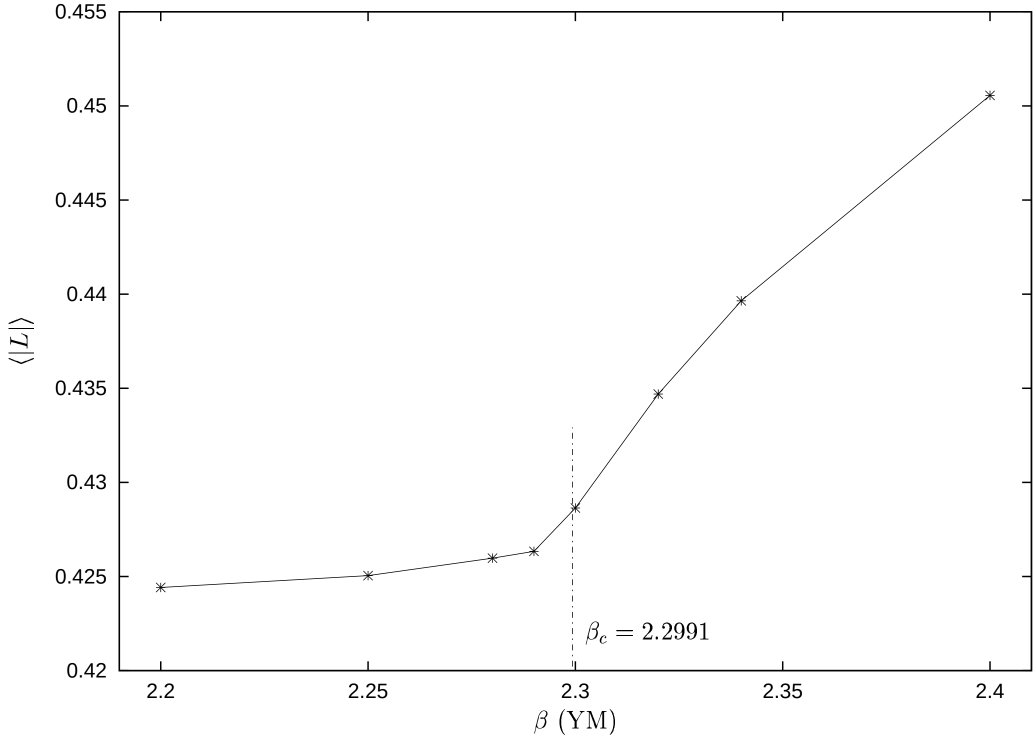
$$S_{\text{eff,LO}}[L] \equiv \lambda_0 \sum_{\langle xy \rangle} L_x L_y$$

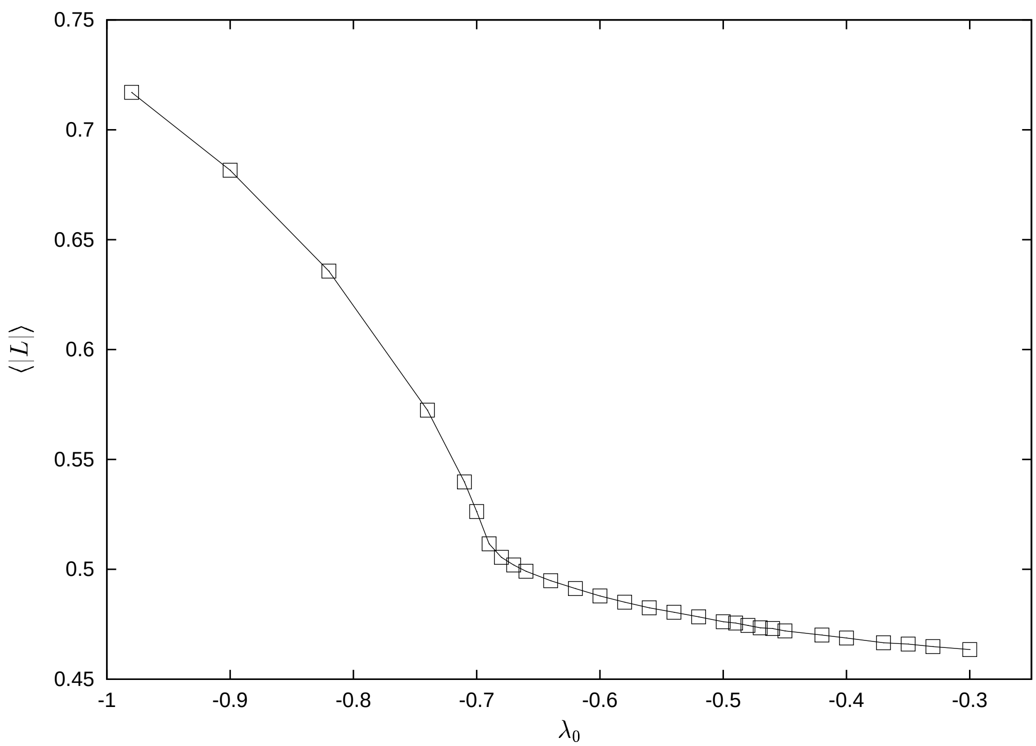
$$L_x \text{ bounded } , \mathcal{D}L \neq \prod dL_{\boldsymbol{x}}$$

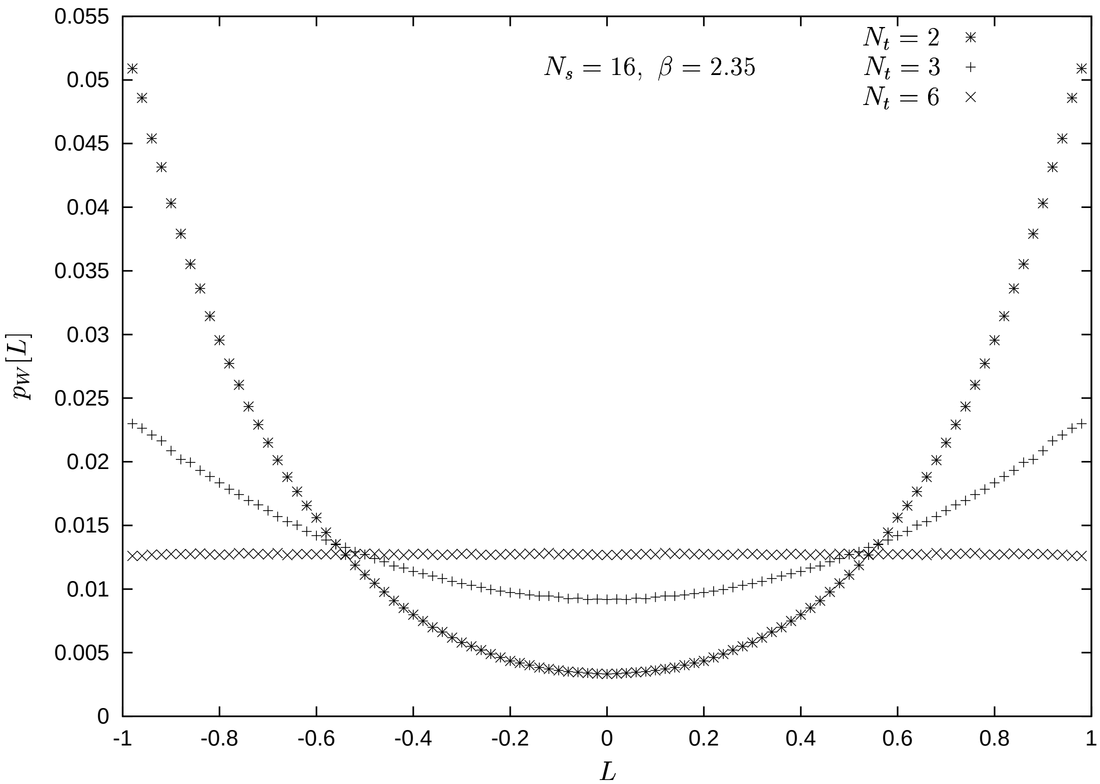
- dependency $\lambda_0(\beta_g)$



- MC simulations of $S_{\mathrm{eff,LO}}$:
  - ◇ SDE satisfied
  - ◇ SSB of $\mathbb{Z}_2 : \langle L \rangle \neq 0$ for $\lambda_0 < \lambda_{0,c} \simeq -0.6$
  - ◇ qualitative and consistency already at LO


0.455
0.45
0.445
0.44
0.435
0.43
0.425
0.42
$\langle |L| \rangle$
$\beta_c = 2.2991$
2.2
2.25
2.3
2.35
2.4
β (YM)


0.75
0.7
0.65
0.6
0.55
0.5
0.45
$\langle |L| \rangle$
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
λ_0



Example: λ_0, λ_2 and λ_4 in symmetric phase

$$N_s = 20,\ N_t = 4,\ \beta = 2.20 \quad (3.20/21/13)$$

$$\lambda_0 = -0.43721, \quad \lambda_2 = 0.37075, \quad \lambda_4 = -0.00693$$

$\sim$ 1% accuracy

4. Conclusions

- effective actions via inverse MC

- method: lattice-SDs from target space symmetries

- FN conjecture: $S_{\text{eff}}[\boldsymbol{n}],\ \boldsymbol{n} \in S^2$, ruled out

- further confirmation of SY-$\mathbb{Z}_2$ conjecture:
  'calculated' $\mathbb{Z}_2$-symmetric $S_{\text{eff}}[L]$
  useful to study confinement–deconfinement PT

Future:

- check operators contributing to $S_{\text{eff}}[L]$
- check critical exponents for $S_{\text{eff}}[L]$
- alternative effective actions (eigenvalues of $\mathfrak{P}$)
- $\mathbb{Z}_2$ variables (Ising spins, vortices?)