Exotic Phases and Phase Transitions for Interacting Fermions

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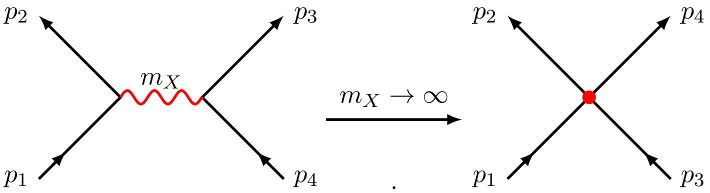
- scattering of electrons, quarks, neutrinos ...
- mediated by exchange particle X
- QED: virtual photons can be „integrated out“
  $\rightarrow$ non-local current-current four-Fermi interaction

$$S_{\text{eff}} = S_0 + \text{const} \int \mathrm{d}^4 x \mathrm{d}^4 y \, J^\mu(x) K_{\mu\nu}(x-y) J^\nu(y), \quad J^\mu = \bar{\psi}\gamma^\mu\psi$$

- X heavy $\rightarrow$ pointlike ψ^4-interaction for $q^2 \ll m_x^2$



- Lorentz-invariance → tensor-bilinears

scalar field	$S(x) = \bar\psi(x)\psi(x)$
pseudo-scalar field	$P(x) = \bar\psi(x)\gamma_5\psi(x)$
vector field	$J^\mu(x) = \bar\psi(x)\gamma^\mu\psi(x)$ etc.

- point-like ψ^4 interaction

$$\mathcal{L}_{\text{int}} \propto M^2(x), \quad M(x) = S(x),\, P(x),\, J^\mu(x), \dots$$

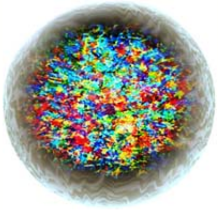
$d = 4 \to$ non-renormalizable effective theory with cutoff Λ

- weak interaction → Fermi-theory (β-decay, neutrino scattering)

$$M = \bar\psi_e\gamma^\mu(\mathbb{1} - \gamma_5)\psi_{\nu_e} + \dots \quad \text{coupling} \quad G_F$$

- „unitary limit" ($q^2 \ll m_w^2$)

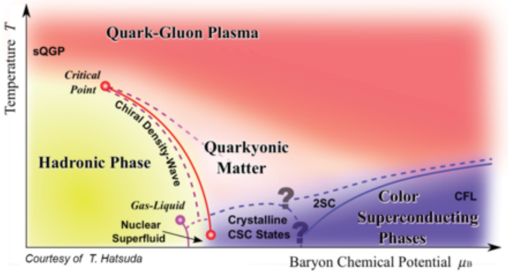
- strong interaction:
- quarks and gluons confined in hadrons and glueballs



- chiral symmetry breaking $\rightarrow$
  chiral limit: SSB of global chiral SU(2)$\times$SU(2) symmetry
- order parameter $=$ chiral condensate $\langle\bar{\psi}\psi\rangle$

dynamical mass generation (gap) $\quad m \propto \langle\bar{\psi}\psi\rangle$

- high T low μ_B: weakly-coupled quark-gluon plasma
  accessible to lattice simulations ☺
- high μ_B low T: color-superconductor expected
- intermediate regime: intense debate
  not (yet) accessible to lattice simulations ☹



- chiral properties
  $\rightarrow$ effective 4-Fermi models
- confinement not explained
  (missing gluons)

a) Nambu-Jona-Lasino model

- color-blind, in chiral limit:

$$\mathcal{L} = \bar{\psi}\mathrm{i}\gamma^\mu\partial_\mu\psi + \frac{g}{2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right], \quad \psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

- same SU(2)×SU(2) as QCD, non-renormalizable, equivalent to

$$\mathcal{L} = \bar{\psi}\mathcal{D}\psi - \frac{g}{2}\left(\sigma^2 + \vec{\pi}^2\right), \quad \mathcal{D} = \mathrm{i}\gamma^\mu\partial_\mu + g(\sigma + \mathrm{i}\gamma_5\vec{\tau}\cdot\vec{\pi})$$

(bosonization, Hubbard-Stratonovich transformation, . . .)

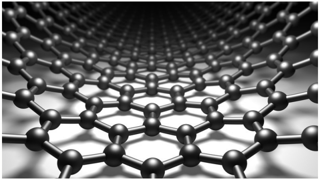
- shows SSB, gap, CSC-phase ☺

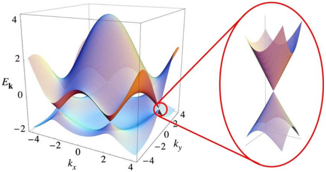
b) Quark-Meson model

$$\mathcal{L} = \bar{\psi}\mathcal{D}\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial\vec{\pi}\partial^\mu\vec{\pi}) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2$$

- similar properties as NJL, but renormalizable (Landau pole)

- conducting polymers (Trans- and Cis-polyacetylen) Su, Schrieffer, Heeger
- quasi 1-dimensional inhomogeneous superconductors Mertsching, Fischbeck
- quasi 2-dimensional Dirac materials, high-temperature SC Semenoff, Hands, Herbut,...
- description of optical lattices Cirac, ...





- modeled by relativistic ψ^4-theories in $d = 2, 3, 4$
- two Dirac points of Dirac materials $\Rightarrow$ ψ has 4 components
- particle and condensed matter physics, structures, methods, algorithms $\rightarrow$ any dimension

- boundary conditions in path integral & chemical potential

$$\mathcal{L}_E = \mathcal{L}_0 + \mathcal{L}_{\text{int}}, \quad \mathcal{L}_0 = \mathrm{i}\bar{\psi}\gamma^\mu\partial_\mu\psi + \mu\bar{\psi}\gamma^0\psi, \quad \psi \text{ anti-periodic}$$

- $\mathcal{L}_0$ flavor-blind, $\mathcal{L}_{\text{int}}$ four Fermi interaction
- admit $\mathrm{N_f}$ flavors

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_{N_f} \end{pmatrix}, \quad \bar{\psi}\gamma^\mu\partial_\mu\psi = \sum_{a=1}^{\mathrm{N_f}} \bar{\psi}_a\gamma^\mu\partial_\mu\psi_a$$

- thermodynamics: grand canonical partition function

$$Z = \mathrm{tr}\,\mathrm{e}^{-\beta(\hat{H}-\mu\hat{Q})} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\;\mathrm{e}^{-S_E[\psi,\bar{\psi}]}, \quad S_E = \int \mathrm{d}\tau\mathrm{d}\boldsymbol{x}\,\mathcal{L}_E$$

- expectation values in thermal equilibrium (T, μ)

$$\langle O[\psi,\bar\psi]\rangle = \frac{1}{Z}\int \mathcal{D}\psi\mathcal{D}\bar\psi\; \mathrm{e}^{-S_E[\psi,\bar\psi]}$$

$d = 2$: integrable systems, sometimes soluble

$d = 3$: renormalizable in large-$\mathrm{N_f}$ expansion

simple realization of asymptotic safety scenario — Braun, Gies, Scherer

$d = 4$: not renormalizable, effective theory

- lattice approach:
  - critical behavior, finite size analysis — extended FRG analysis by H. Gies et al.
  - masses of light „particles“
  - phase diagrams at finite T, finite μ
  - generic sign problem $\rightarrow$ model-dependent analysis — Schmidt, Wellegehausen, Lenz, AW
  - strong interaction $\rightarrow$ chiral lattice fermions — S. Hands et al. and Jena group

Lenz, Pannullo, Wagner, Wellegehausen, AW

- scalar-scalar interaction in Gross-Neveu model

$$\mathcal{L}_{\mathrm{GN}} = \bar\psi(\partial\!\!\!/ + \mu\gamma^0)\psi - \frac{g^2}{2\mathrm{N_f}}(\bar\psi\psi)^2$$
$$\equiv \bar\psi\mathcal{D}\psi + \frac{\mathrm{N_f}}{2g^2}\sigma^2, \qquad \mathcal{D} = \partial\!\!\!/ + \sigma + \mu\gamma^0$$

- finite temperature and density: grand canonical ensemble

$$\langle\hat{\mathcal{O}}\rangle = \mathrm{tr}(\hat\rho\,\hat{\mathcal{O}}) = \frac{1}{Z}\int \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}\sigma\,\mathrm{e}^{-S}\mathcal{O}$$
$$Z = \int \mathcal{D}\bar\psi\mathcal{D}\psi\mathcal{D}\sigma\,\mathrm{e}^{-S} = \int \mathcal{D}\sigma\,\mathrm{e}^{-\mathrm{N_f}S_{\mathrm{eff}}[\sigma]}$$

- order parameter for $\mathbb{Z}_2$ chiral symmetry

$$\langle\bar\psi(x)\psi(x)\rangle = -\frac{1}{g^2}\langle\sigma(x)\rangle$$

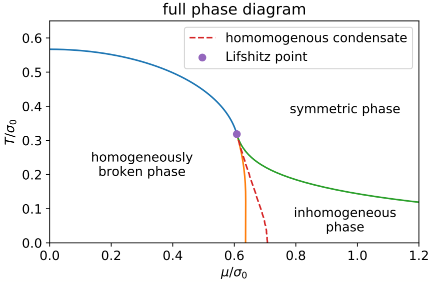
- saddle point approximation becomes exact

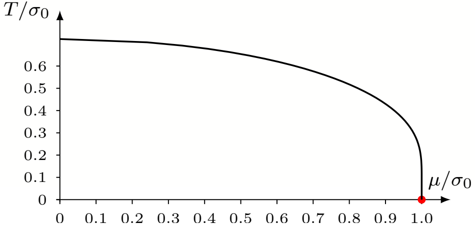
$$Z \stackrel{\mathrm{N_f}\to\infty}{\longrightarrow} \mathrm{e}^{-\mathrm{N_f}\,\min_\sigma S_{\mathrm{eff}}[\sigma]}$$

- periodic BC $\stackrel{?}{\Rightarrow}$ constant minimizing σ

$$S_{\mathrm{eff}}[\sigma] = \frac{1}{2g^2}\int \sigma^2 - \log\det\mathcal{D} = V\cdot U_{\mathrm{eff}}(\sigma)$$

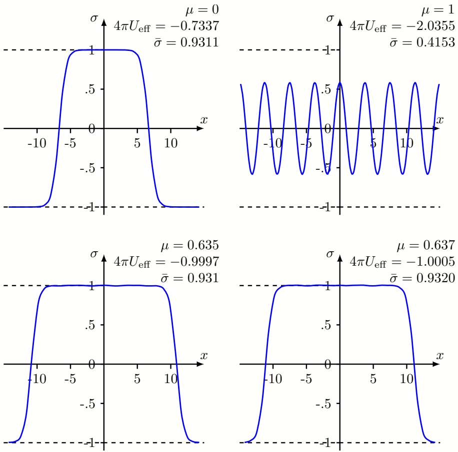
- free energy density U_{eff} known $\to$ phase diagram with hom. phases
- is equilibrium state $\hat\rho \propto \mathrm{e}^{-\beta(\hat H-\mu\hat Q)}$ translation invariant?
- inhomogeneous structures?
  - $\mathrm{N_f}\to\infty$ GN type: $d=2$ yes, $d=3$ probably no
  - $\mathrm{N_f}<\infty$ GN type: $d=2$ “yes“, $d=3$ probably no Narayanan; Buballa, Kurth, Wagner, Winstel
  - $d=4$ NJL and Quark-Meson models (cutoff dependence?)
  - large-$\mathrm{N_f}$ var. calculations Broniowski, Kotlaroz, Kutschera; Nakano, Tatsumi; Nickel; Carignano, Buballa, Schaefer; ...
  - and much more for related models ...





- $1+1$ dimensions
  second (and first) order lines
  Lifschitz-point
  $(T, \mu_0) \approx (0.318, 0.608)$
  $N_f \to \infty$: inhomogeneous phase
- $1+2$ dimensions
  second order line
  only jump at $(\mu, T) = (1, 0)$
  $N_f \to \infty$: no inhomogeneous phase
  in continuum limit

  Narayanan; Buballa, Kurth, Wagner, Winstel (2020)
- an finite lattice (crystals): inhom. condensates not excluded
- extensions (models, dimension) Nickel
- standard review Buballa, Carignano (2015)

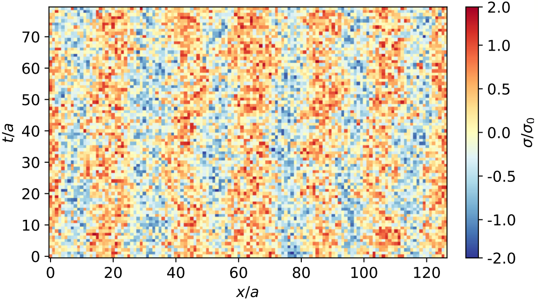


- inhomogeneous condensates at $T = 0$ for $\mu > \mu_{crit} \approx 0.636$
- $4\pi U_{eff}[\sigma] > -1 \Rightarrow$ metastable
- metastable kink-antikink (top left) $\Rightarrow$ energy(kink-antikink) $= 2/\pi$
- analytical solution in $1 + 1$d GN and cGN in large N_f limit

Thies et al.

AW, Statistical Approach to Quantum Field Theory, Lecture Notes in Physics (2021)

- inhomogeneous condensates for finite N_f
  - do they exist for finite μ?
    $\rightarrow$ breaking of translation invariance?
  - no-go theorems (Mermin-Wagner, Coleman)



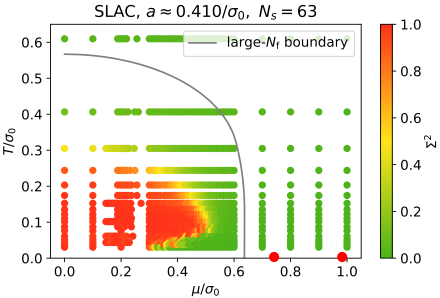
GN: Lenz, Pannullo, Wagner, Wellegehausen, AW, Phys. Rev. D101 (2020) 094512 and Phys. Rev. D102 (2020) 114501

cGN: J.Lenz, M. Mandl, AW, e-Print: 2109.05525 [hep-lat]

- discretize on quadratic or hypercubic lattice

$$S_{\text{eff}} = \frac{1}{2g^2}\sum_x \sigma_x^2 - \log\det\mathcal{D}, \quad \mathcal{D} = \gamma^\mu\partial_\mu + \mu\gamma^\mu + \sigma$$

- keep „all“ continuum symmetries
- no sign problem
  - naive fermions: $\mathrm{N_f} = 8, 16, \ldots$
  - chiral SLAC fermions: $\mathrm{N_f} = 2, 4, 8, \ldots$
- 2d: done, 3d: preliminary results
- many ensembles on grid in (T, μ)-space
- $31 \leq N_s \leq 128$ and scale setting $\rightarrow$ lattice spacing, volume
- rational HMC with $N_{\text{PF}} = 2\mathrm{N_f}$ pseudo-fermions

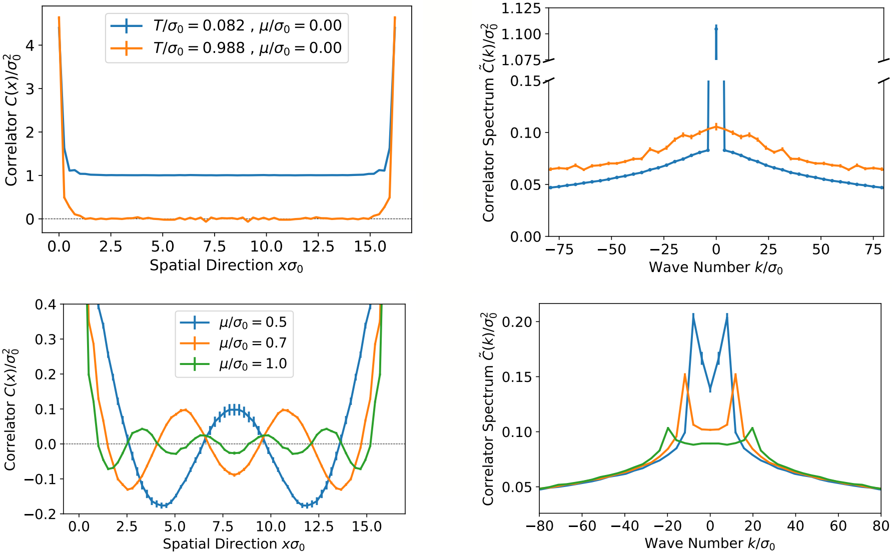


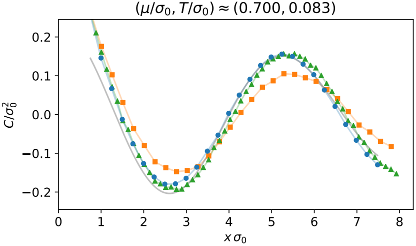
- $\langle \sigma^2 \rangle$ does not see inhom. phase
- homogeneously broken
- symmetric & inhomogeneous

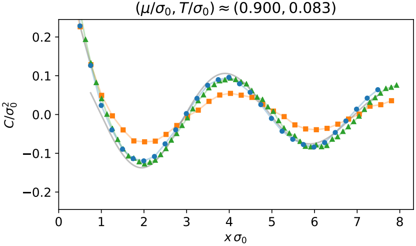
- differentiate three phases

$$C(x) = \langle \sigma(t_0, x)\sigma(t_0, 0)\rangle = \frac{1}{N_t N_s} \sum_{t,y} \langle \sigma(t, y + x)\sigma(t, y)\rangle$$

- no washing out by translations of σ
- Fourier transform $\tilde{C}(k) = \mathcal{F}_x(C)(k)$
  - symmetric phase: small amplitude
  - homogeneous broken phase: peak at $k = 0$
  - inhomomogeneous phase: peaks at $\pm q$ (dominant wavelength)


$T/\sigma_0 = 0.082$, $\mu/\sigma_0 = 0.00$
$T/\sigma_0 = 0.988$, $\mu/\sigma_0 = 0.00$
Correlator $C(x)/\sigma_0^2$
Spatial Direction $x\sigma_0$
Correlator Spectrum $\tilde{C}(k)/\sigma_0^2$
Wave Number k/σ_0
$\mu/\sigma_0 = 0.5$
$\mu/\sigma_0 = 0.7$
$\mu/\sigma_0 = 1.0$
Correlator $C(x)/\sigma_0^2$
Spatial Direction $x\sigma_0$
Correlator Spectrum $\tilde{C}(k)/\sigma_0^2$
Wave Number k/σ_0



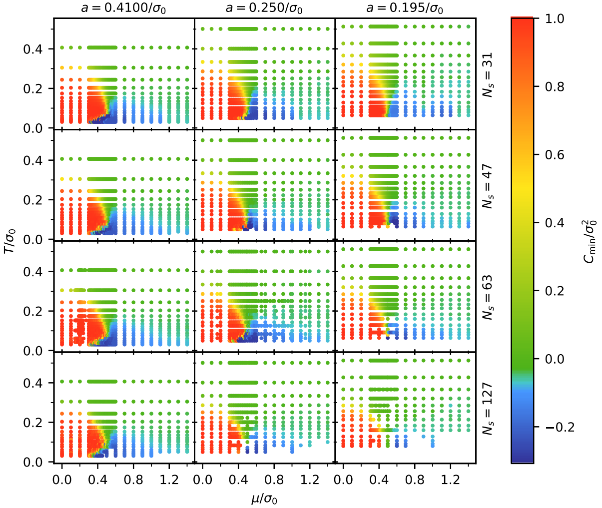


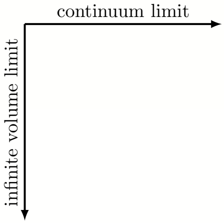
- SLAC faster convergence to continuum limit:

  SLAC $(a, N_s) = (0.250, 63)$

  naive (improved) $(a, N_s) = (0.252, 64)$

  naive (improved) $(a, N_s) = (0.126, 128)$



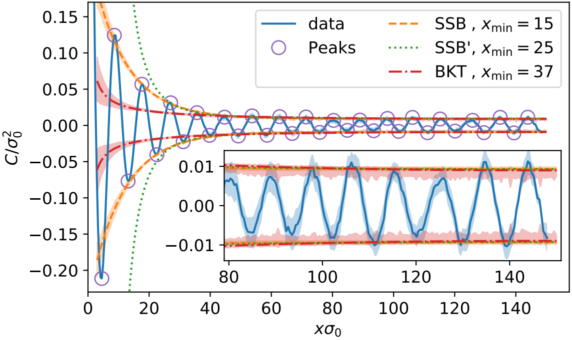


homogeneous broken

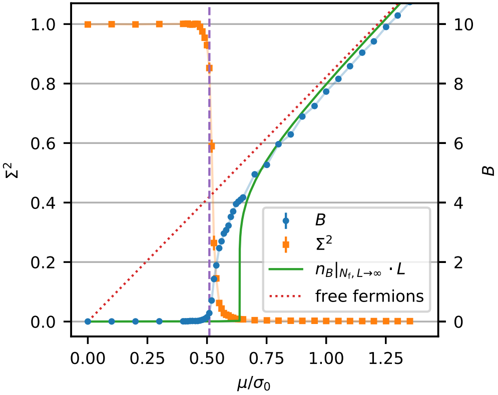
symmetric

inhomogeneous

- longe range correlations, $N_s = 725$



- inhomogeneous correlator $A(x) \cdot C_{\text{periodic}}(x)$
- BKT (Berezinskii, Kosterlitz, Thouless) $A(x) \sim |x|^{-\beta}$
- analysis of amplitude A not conclusive yet (BKT for cGN-model)

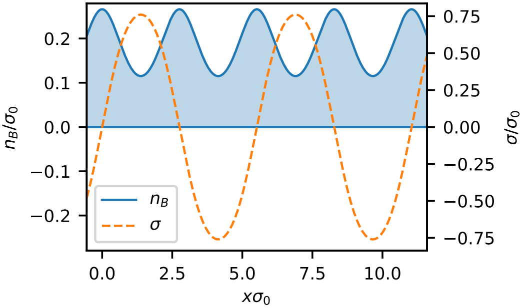


- low temperature $T = 0.076$
  - Baryon number:

$$B = \frac{i}{N_f}\left\langle \int dx\, \bar{\psi}(x)\gamma^0\psi(x)\right\rangle$$

  - homogeneous condensate
  - analytic large-N_f result
- silver blaze property
- transition symmetric $\to$ inhom. at $\mu_{crit} \approx 0.51$
- analytic large-N_f result

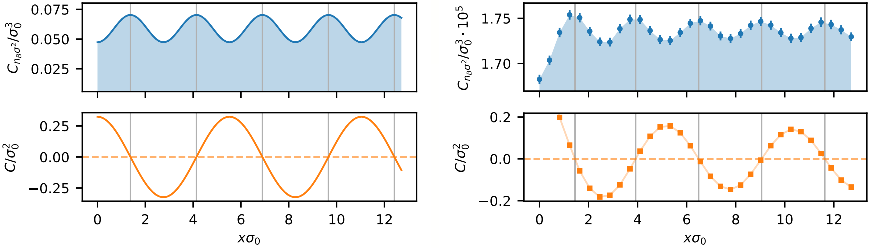
- analytic large-$\mathrm{N_f}$ result for n_B and σ at $(\mu, T) = (0.7, 0)$



- baryonic crystal: $\mathrm{N_f}$ fermions for each cycle of oscillation Schnetz, Thies, Urlichs
- fermions located at nodes of condensate field σ
  $\rightarrow$ perfect correlation $n_B(x)$ and $\sigma^2(x)$

- correlation condensate $\leftrightarrow$ baryon density

$$C_{n_B\sigma^2}(x) = \frac{\mathrm{i}}{\mathrm{N_f}} \langle n_B(0,x)\, \sigma^2(0,0) \rangle$$



- left: analytical results for $\mathrm{N_f} \to \infty$
- right: simulation results (SLAC, $\mathrm{N_f} = 8$)
- qualitative agreement $\mathrm{N_f} = 8$ and $\mathrm{N_f} \to \infty$

with J. Lenz and M. Mandl, 2021

- U(1)-invariant extension of GN-model = chiral GN-model
- scalar and pseudo-scalar channels

$$\mathcal{L}_E = \bar{\psi}\mathrm{i}\partial\!\!\!/\psi + \frac{g^2}{2\mathrm{N_f}}\left((\bar{\psi}\psi)^2 + (\bar{\psi}\mathrm{i}\gamma_*\psi)^2\right) \;,$$

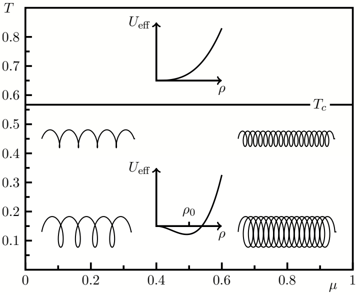
- equivalent formulation

$$\mathcal{L}_E = \bar{\psi}\mathrm{i}\mathcal{D}\psi + \frac{\mathrm{N_f}}{2g^2}\rho^2 \quad \text{with} \quad \mathcal{D} = \partial\!\!\!/ + \mu\gamma_0 + \rho\,\mathrm{e}^{\mathrm{i}\gamma_*\theta}$$

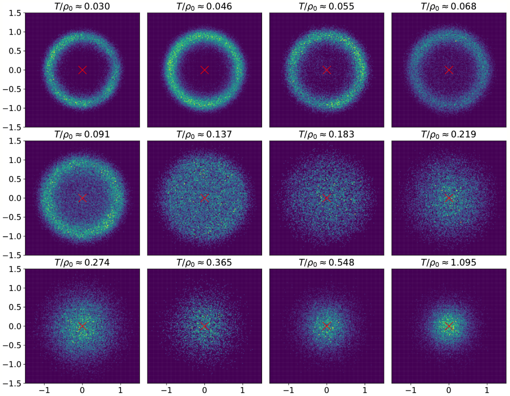
- with complex condensate field

$$\Delta = \sigma + \mathrm{i}\pi = \rho\,\mathrm{e}^{\mathrm{i}\theta}$$

- large-$\mathrm{N_f}$ solution simpler as for GN (chiral rotations)



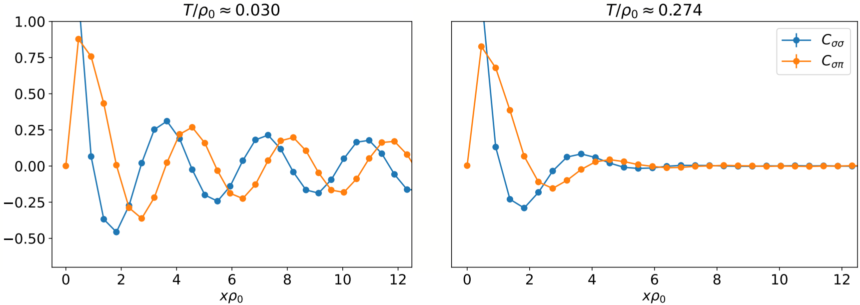
- symmetric phase $T > T_c$
- inhomogeneous phase below T_c
- condensate-field: chiral spiral $\Delta = \mathrm{e}^{iqx}$, $q \propto \mu$



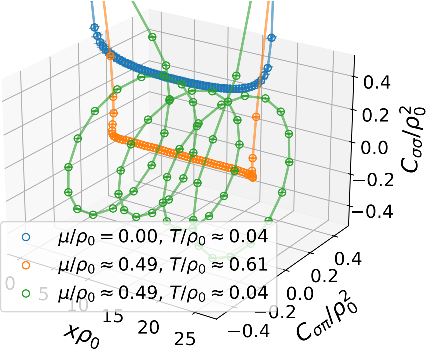
- Distributions of $\sum_{t,x} \Delta(t,x)$
- $\mu = 0,\ N_s = 63,\ a \approx 0.46$
- increasing temperature

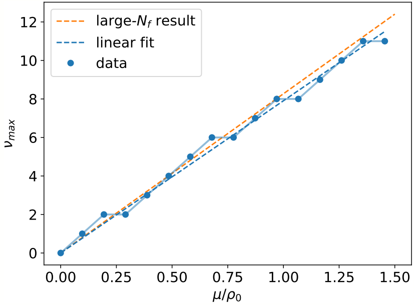
$$C_{\sigma\sigma}(x) = \frac{1}{N_t N_s} \sum_{t,y} \langle \sigma(t, y+x)\sigma(t, y)\rangle$$

$$C_{\sigma\pi}(x) = \frac{1}{N_t N_s} \sum_{t,y} \langle \sigma(t, y+x)\pi(t, y)\rangle$$



"chiral spiral" at low and higher temperature ($\mathrm{N_f} = 2$, $N_s = 63$, $\mu \approx 1.14$, $a \approx 0.46$)



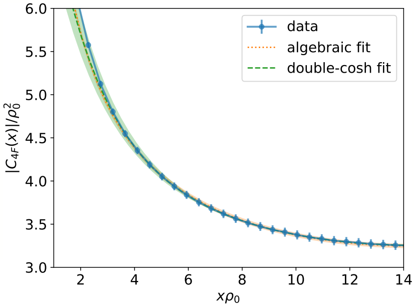


- correlators $C_{\sigma\sigma}(x)$ and $C_{\sigma\pi}(x)$
  symmetric phase
  $\mu = 0$: dominated by hom. σ
  inhomogeneous

- dominant winding number n_{max} for $T/\rho_0 \approx 0.030$
- linear fit: slope 7.91 ± 0.10
- parameters: $\mathrm{N_f} = 8$, $N_s = 63$, $a \approx 0.41$.

$$C_{4\mathrm{F}}(x) = \frac{1}{N_t N_s} \sum_{t,y} \langle \bar\psi(1+\gamma_*)\psi(t,y+x)\bar\psi(1-\gamma_*)\psi(t,y)\rangle \propto C(x)$$

$$C(x) \propto \langle \Delta^*(t,x)\Delta(t,0)\rangle \to x^{-1/\mathrm{N_f}} \quad \text{(Witten)}$$



BKT phase ($\mathrm{N_f} = 2$)

$$|C_{4F}(x) \to \frac{\alpha}{x^\beta} + \frac{\alpha}{(L-x)^\beta}$$

$$\alpha = 6.52 \pm 0.02\ , \quad \beta = 0.521 \pm 0.001$$

massive phase ($\mathrm{N_f} = 2$)

$$|C_{4F}(x) \to \sum_{i=1}^{2} \gamma_i \cosh\left[m_i\left(x - \frac{L}{2}\right)\right]$$

$$m_1 = 0.533 \pm 0.006, \quad m_2 = (5.76 \pm 0.03)\cdot 10^{-2}$$

$$\gamma_1 = (4.3 \pm 0.3)\cdot 10^{-3},\ \gamma_2 = 3.2515 \pm 4\cdot 10^{-4}$$

- current-current Thirring-interaction

$$\mathcal{L}_{\text{int}} = -\frac{g^2}{2\mathrm{N_f}}(\bar{\psi}\gamma^\mu\psi)^2, \quad \mathbb{Z}_2 \times \mathrm{U}(2\mathrm{N_f}) \text{ invariant}$$

- scalar condensate $\langle\bar{\psi}\psi\rangle$ breaks $\mathrm{U}(2\mathrm{N_f}) \to U(\mathrm{N_f})$
- pseudo-scalar condensate $\langle\bar{\psi}\gamma_4\gamma_5\psi\rangle$ breaks $\mathbb{Z}_2$ parity
- remove ψ^4-term with auxiliary vector field v_μ
- fermionic integration $\to$ fermion determinant

$$Z_{\text{Th}} = \int \mathcal{D}v_\mu \, \mathrm{e}^{-\mathrm{N_f} S_{\text{eff}}}, \quad S_{\text{eff}} = \frac{1}{2g^2}\int \mathrm{d}^3x \, v_\mu v^\mu - \log\det(\mathrm{i}\not{D})$$

- large $\mathrm{N_f} \to$ path integral localized at saddle point

$$Z \stackrel{\mathrm{N_f}\to\infty}{\longrightarrow} \mathrm{e}^{-\mathrm{N_f}\min_v S_{\mathrm{eff}}[v_\mu]}$$

- translation invariance $\Rightarrow v_\mu$ constant

$$S_{\mathrm{eff}}[v_\mu] = V \cdot U_{\mathrm{eff}}(v_\mu)$$

- effective potential ($m \neq 0$)

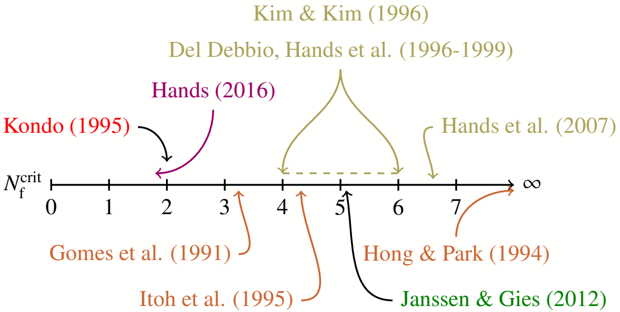
$$U_{\mathrm{eff}} = \frac{1}{2g_{\mathrm{ren}}^2} v_\mu v^\mu + U_{\mathrm{free}}(T, m^2), \quad g^2 = \frac{4\pi g_{\mathrm{ren}}^2}{4\pi + \Lambda g_{\mathrm{ren}}^2}$$

- condensate

$$\mathrm{N_f} \to \infty : \quad \langle\bar\psi\psi\rangle = \frac{1}{V}\frac{\partial}{\partial m}\log Z \stackrel{m\to 0}{\longrightarrow} 0 \quad v_\mu\text{-independent}$$

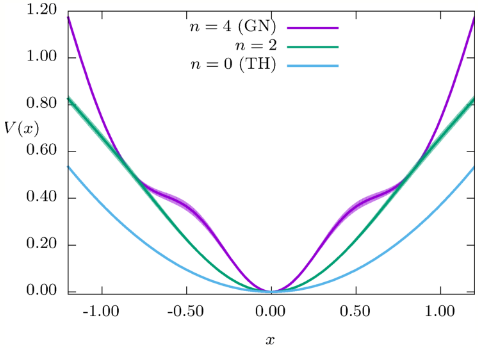
$$\mathrm{N_f} = 1/2 : \quad \langle\bar\psi\psi\rangle \neq 0 \qquad \text{equivalent to GN}$$

- exists critical flavor number $\mathrm{N_f^{crit}}$:
  there is broken phase for $\mathrm{N_f \leq N_f^{crit}}$
  only symmetric phase for $\mathrm{N_f > N_f^{crit}}$
- situation before 2017:



- SD equations
- $1/\mathrm{N_f}$-expansion
- FRG
- lattice, staggered
- lattice, domain wall
- new results change situation

- advantages of chiral SLAC fermions
  - exact $U(2N_f) \times \mathbb{Z}_2$ symmetry on hyper-cubic lattice
  - $v_\mu(x)$ site variable (not gauge field)
  - no doublers, no sign-problem for 4-component ψ
  - relatively cheap
- simulation results
  - 4-component $\psi \Rightarrow$ no SSB for $N_f = 1, 2, \ldots$ — B. Wellegehausen, D. Schmidt, AW, PRD 96 (2017)
    simulations for $0.5 \leq N_f \leq 1 \Rightarrow N_f^{crit} = 0.80(4)$ — J. Lenz, B. Wellegehausen, AW, PRD 100 (2019)
  - 2-component $\psi \Rightarrow$ breaking for $N_f^{ir} \leq 9$ — B. Wellegehausen, D. Schmidt, AW, PRD 96 (2017)
  - domain wall fermions (DWF)
    different implementations $N_f^{crit} \lessapprox 1$ or $1 < N_f^{crit} < 2$ — Hands et al. 2019 and 2020
  - recent FRG-studies compatible with $N_f^{crit} < 1$ — L. Dabelow, H. Gies, B. Knorr, PRD 99, 2019
    momentum-depended vertices



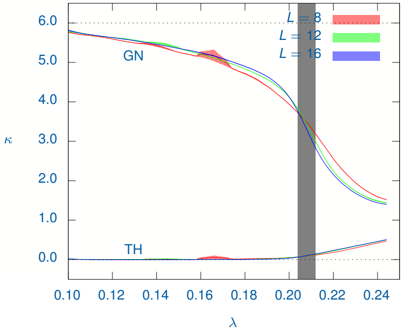
- $\mathrm{N_f} = 2$, $L = 16$
- small finite size corrections
- no SSB for $\mathrm{N_f} = 2$
- supported by
  dual formulation → filling
  strong coupling expansion

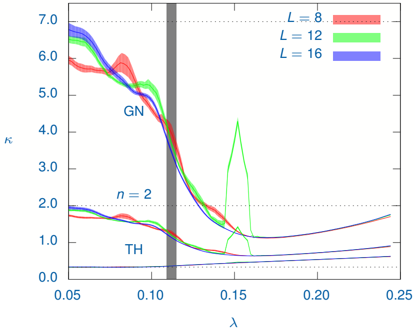
- detailed analysis:
  - effective potential in various channels (dual formulation)
  - chiral condensate $\Sigma_{L,\mathrm{N_f}}$, mean spectral density $\bar{\varrho}_{L,\mathrm{N_f}}(E)$
  - symmetry of low-lying spectrum for $\mathrm{N_f} \in [0.8, 1.0]$
  - susceptibilities

- curvature of effective potential (free energy density) at origin:

$$\text{SSB if} \quad \kappa(n) = V''_{\text{eff}}(x,n)\Big|_{x=0} < 0 \quad \text{for one } n$$

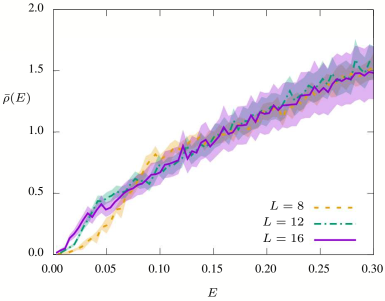
- plots: curvatures for $\mathrm{N_f} = 1$ and $\mathrm{N_f} = 2$ (dotted: strong coupling)
- physical domain: right of dark bar





- Banks-Casher: condensate from mean spectral density $\bar{\rho}$:

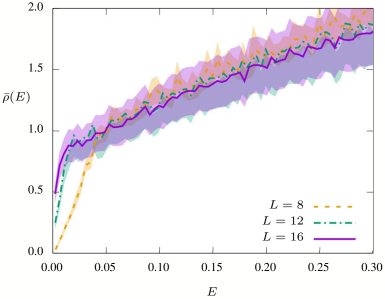
$$\langle\bar{\psi}\psi\rangle = \lim_{m\to 0}\lim_{V\to\infty}\frac{2m}{V}\int_0^\infty \frac{\mathrm{d}E}{E^2+m^2}\,\bar{\rho}(E)$$



symmetric phase

$N_f = 1.0$, $L = 8, 12, 16$

density stays small near $E = 0$



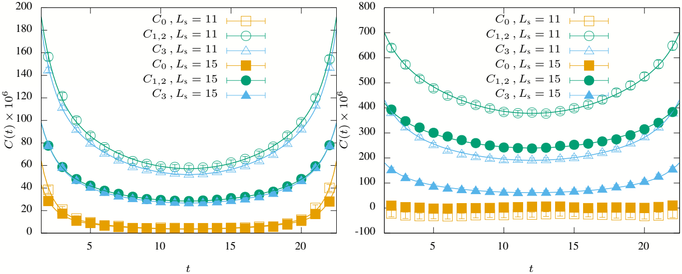
broken phase

$N_f = 0.8$, $L = 8, 12, 16$

density builds up near $E = 0$

- U(2) unbroken: expect singlet and triplet
- $U(2) \to U(1) \times U(1)$: two Goldstone modes

$N = 1.00,\ L_s^2 \times 24$ $\qquad$ $N = 0.80,\ L_s^2 \times 24$



C	$m(11)$	$m(15)$	$m^*(11)$	$m^*(15)$	N_f	Symm.
C_0	0.21(2)	0.21(2)	1.27(6)	1.22(7)	1.0	
$C_{1,2}$	0.134(3)	0.128(2)	1.03(5)	1.02(3)	1.0	U(2)
C_3	0.138(2)	0.131(2)	1.08(4)	0.98(3)	1.0	
$C_{1,2}$	0.103(2)	0.095(3)	1.04(12)	0.93(17)	0.8	U(1)× U(1)
C_3	0.109(4)	0.127(7)	0.81(7)	0.81(10)	0.8	

- first simulations of $3d$ system with fully chiral fermions

$$N_f^{crit} = 0.80(4)$$

- 2-component ψ: parity breaking PT for $N_f^{crit} = 0.5, 1.5, 3.5, 4.5$
- staggered fermions problematic: wrong universality class?
- domain wall fermions: favor $1 < N_f^{crit} < 2$
  very large extra dimension, v_μ link variable (Simon Hands)
- still discrepancy SLAC $\leftrightarrow$ DWF
- spotted new PT without order parameter $N_f \geq 0.8$ (needs clarification)

Lenz, Wellegehausen, AW

- first ψ^4-simulations with chiral fermions (finite $\mu, T, \mathrm{N_f}$)
- symmetries of lattice action relevant in $d = 3$ (staggered vs. chiral)
- recent result $\mathrm{N_f^{crit}} < 1$ (cp. domain wall fermions)
- 2*d* GN: $\mathrm{N_f} = 8$ or 16 phase diagram similar to $\mathrm{N_f} \to \infty$
- strong correlation baryon density $\leftrightarrow$ condensate
- inhomogeneous „phases" shrink with decreasing $\mathrm{N_f}$
- in progress:

  ψ^4 in magnetic fields (cascade of first order transitions) J. Lenz, M. Mandl, AW, in progress
- first simulation results for $d = 3$ GN

  coding of $d = 4$ quark-meson mode Pannullo, Wagner, Winstel; Lenz, Mandl, AW
- we are aiming at:

  behavior of condensates in rotating vessels?

  . . . . . . gauge theories

Thanks!