preprint DIAS-STP-85-21

Aharonov-Bohm effect in Presence of Superconductors

L. O'Raifeartaigh

Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland

N. Straumann

University of Zürich, Schönberggasse 9, CH-8001 Zürich, Switzerland

A. Wipf*

Institut für Theoretische Physik, Eidgenössische Technische Hochschule Hönggerberg, Zürich CH-8093, Switzerland

April 27, 2023

Abstract

The analysis of a previous paper, in which it was shown that the energy for the Aharonov-Bohm effect could be traced to the interaction energy between the magnetic field of the electron and the background magnetic field, is extended to cover the case in which the magnetic field of the electron is shielded from the background magnetic field by superconducting material. The paradox that arises from the fact that such a shielding would apparently preclude the possibility of an interaction energy is resolved and, within the limits of the ideal situation considered, the observed experimental result is derived.

Published version: Foundations of Physics, Vol. 23, No. 5, 1986, p. 703

Uploaded to arXiv to help researchers with limited library facilities

*present address: Theoretisch-Physikalisches-Institut, Friedrich-Schiller-Universität Jena, 07743 Jena, Germany; email: wipf@tpi.uni-jena.de

Contents

1 Introduction

In a recent paper [1] some of the traditional mystique surrounding the Aharonov-Bohm effect was removed by the observation that the energy $\int \boldsymbol{A} \cdot \boldsymbol{j}_e$ which produces the effect could be traced to the interaction energy $\int \boldsymbol{B} \cdot \boldsymbol{b}_e$ between the background magnetic field $\boldsymbol{B}$ and the magnetic field $\boldsymbol{b}_e$ of the electron. The puzzle posed by the fact that the effect takes place in spite of the fact that the electron does not interact locally with $\boldsymbol{B}$ is then explained by the fact that the *magnetic field* of the electron does interact locally with $\boldsymbol{B}$ and the question as to whether the gauge-potential is necessary for a description of the A-B effect is answered by the statement that it is necessary only if one requires a local description in terms of the electron. A more recent article [2] comes to similar conclusions.

As pointed out by Tassie [3], however, the $\int \boldsymbol{B} \cdot \boldsymbol{b}_e$ explanation would appear to encounter some difficulty in the case that $\boldsymbol{b}_e$ is shielded from $\boldsymbol{B}$ by a superconductor, since in that case the integral $\int \boldsymbol{B} \cdot \boldsymbol{b}_e$ is zero but there is reliable experimental evidence [4] to show that the A-B effect occurs. Indeed, due to the double-charge of the Cooper pairs, the effect occurs in this case with maximal phase-shift π. The difficulty also manifests itself in the question as to how $\int \boldsymbol{A} \cdot \boldsymbol{j}_e$ can be zero while $\int \boldsymbol{B} \cdot \boldsymbol{b}_e$ is not zero. The purpose of the present paper is to clarify this apparent paradox.

2 Recall of Original Argument

We first give a brief recall of the argument for the non-superconducting case. Stripped to its essentials the situation may be described by the diagram in Fig. 1 in which the B-field is trapped within the cylinder $r = a$ and the electron between the concentric cylinders $r = d$ and $r = e$. Here everything is assumed to be uniform in the z-direction (direction of the common axis) and axially symmetric and for the moment there is nothing between the cylinders $r = a$ and $r = d$, in particular nothing (for the moment) between the cylinders $r = b$ and $r = c$. The reason for confining the electron by the outer cylinder $r = e$ instead of just leaving it outside $r = d$ is that, following refs. [1, 5], we can then consider the

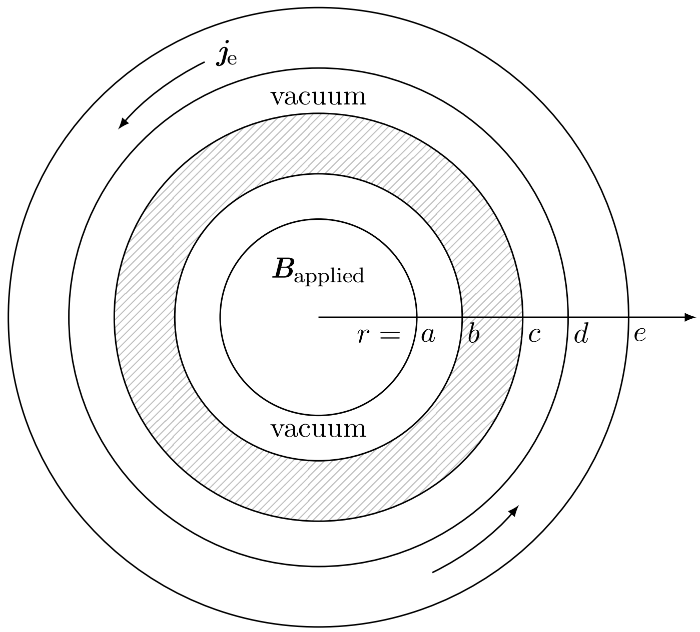

Figure 1: *An electron in $d \leq r \leq e$ is circling around a magnetic field confined in a long cylinder $r \leq a$. The superconductor is in the region $b \leq r \leq c$. In the original case the superconductor is removed and there is vacuum in this region.*

AB-Effect as an energy shift computed from the static Schrödinger equation

$$\Big(-\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} + \frac{(l+F)^2}{r^2}\Big)\psi(x) = 2mE\psi(x) \qquad \psi(d) = \psi(e) = 0, \tag{1}$$

rather than a phase shift. Here we suppressed the trivial dependence on z and F denotes the magnetic flux measured in units of the elementary flux quantum. The parameter l is an integer because, as discussed in refs. [1, 5], the electron wave-function $\psi(x) = e^{il\varphi}\psi(r)$ is single-valued. According to (1) a measurable energy shift is produced for non-integral F and this is the analogue of the usual A-B phase-shift. The potential $(l+F)^2$ in (1) which produces this energy shift derives from an interaction energy of the form

$$\int \boldsymbol{A} \cdot \boldsymbol{j}_e \quad \text{where} \quad \boldsymbol{B} = \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{j}_e = \frac{e}{2mi}\big(\bar{\psi}\boldsymbol{D}\psi - cc\big). \tag{2}$$

Here we have set $\boldsymbol{b}_e = \boldsymbol{h}_e$ since no material is present besides a possible coil. The point of our earlier paper was that, by considering the integral in (2) as over the whole 2-plane, one could use partial integration to convert it to the form

$$\int \boldsymbol{B} \cdot \boldsymbol{b}_e \quad \text{where} \quad \nabla \times \boldsymbol{b}_e = 4\pi \boldsymbol{j}_e. \tag{3}$$

In this formulation the gauge potential is eliminated and the energy which produces the energy-shift in the Schrödinger equation is traced to the interaction energy between the two magnetic fields.

3 The Superconducting Case

To analyze the superconducting case we consider the same idealized situation but with the intermediate region $b \leq r \leq c$ filled with superconducting material, which would shield the magnetic field $\boldsymbol{b}_e$ of the electron from the background field $\boldsymbol{B}$. Here (in contrast to the situation envisaged in a remark made at the conclusion of ref. [1] it is assumed that the cooling of the superconductor has been carried out *after* the field $\boldsymbol{B}$ has been switched on so that $\boldsymbol{B}$ is not expelled [6]. In this situation $\boldsymbol{B} \neq 0$ but since the magnetic field should be zero in the superconducting region $b \leq r \leq c$ it is difficult to see how the fields $\boldsymbol{B}$ and $\boldsymbol{b}_e$ could overlap and hence how the integral $\int \boldsymbol{B} \cdot \boldsymbol{b}_e$ could be non-zero. This is the paradox mentioned in the introduction.

To investigate the paradox we have to consider the field equation for the $\boldsymbol{B}$ field in more detail. This field equation has been given in [7] and may be written as

$$\frac{d}{dr}\Big(\frac{1}{r}\frac{d}{dr}A_\varphi\Big) = 4\pi j_\varphi + 4\pi j_e \qquad (\text{from} \quad \nabla \times \nabla \times \boldsymbol{A} = 4\pi \boldsymbol{J}) \tag{4a}$$

where

$$j_\varphi(r) = \frac{\beta^2}{r}\Big(A_\varphi - l\Big(\frac{\hbar}{2e}\Big)\Big)\ , \qquad \beta^2 = \frac{4\pi\rho e^2}{mc^2}, \tag{4b}$$

and ρ is the density of the Cooper pairs. Here A_φ denotes the dimensionless φ-component of the gauge potential so that the magnetic flux inside radius R is just

$$\Phi_R \equiv \int\limits_{r\leq R} B_z r dr d\varphi = \oint A_\varphi d\varphi. \tag{5}$$

As shown in [7] the Ampere equation in the London limit (4) can be solved exactly in terms of Bessel functions. But in the case that the penetration depth β^{-1} is small compared to the width of the superconductor (which we are assuming to be the case) a better insight into the solution is obtained if we approximate the

Bessel functions by exponentials and thus approximate the solution by

$$
\begin{aligned}
2\pi A_\varphi &= \Phi_a\, r^2/a^2 && r \le a \\
&= \Phi_a - \alpha\beta^2(r^2 - a^2)(\Phi_a - \Phi_q) && a \le r \le b \\
&= \Phi_q + 2\alpha\beta\sqrt{br}\,(\Phi_a - \Phi_q)e^{-\beta(r-b)} + \frac{2\pi}{\beta}\sqrt{cr}\, b_e\, e^{\beta(r-c)} && b \le r \le c \\
&= \Phi_q + \pi b_e(r^2 - c^2 + 2c/\beta) && c \le r \le d \\
&= \Phi_q + \pi b_e(r_e^2 - c^2 + 2c/\beta) && r \ge e.
\end{aligned} \tag{6}
$$

where $\alpha = (2\beta b + (\beta b)^2 - (\beta a)^2)^{-1}$. The flux

$$
\Phi_q = \frac{l}{2}\left(\frac{hc}{e}\right) \tag{7}
$$

is a halfinteger multiple of the elementary flux quantum. Here l is the integer that minimizes $(\Phi_q - \Phi_a)$, and b_e the z-component of the magnetic field due to the electron. The radius r_e in the gauge potential on the outer region belongs to the circling electron. All feedback effects are neglected and the factor one-half in Φ_q comes from the fact that the Cooper pairs have charge $2e$.

4 The Paradox

From equations (6) and (7) we see that:

a) Once the superconductor is deeply penetrated the flux becomes quantized to Φ_q.

b) There are two surface currents $\sim \alpha(\Phi_a - \Phi_q)e^{-\beta(r-b)}$ and $\sim b_e/\beta\; e^{\beta(r-c)}$.

c) Whereas the outer surface current vanishes when the electron current sustaining it is removed ($\boldsymbol{b}_e = 0$) the inner current remains even when the background field is switched off ($\boldsymbol{B} = 0$).

d) The magnetic field is zero deep within the superconductor.

e) There is a maximal A-B effect for odd l, in agreement with experiment.

So the question is: what is the source of the energy for this maximal A-B effect?

5 Toy Model

In order to solve the paradox it is instructive to remove the superconducting material from the (shaded) region $b \le r \le c$ for a moment and replace it by a particle circling like the electron, but with charge q say (or more accurately, a z-independent homogeneous charge distribution). Then we have the conventional A-B effect described in section 2 for *each* of the two particles and the source of the energy for the effect in the respective cases is

$$\int \boldsymbol{A} \cdot \boldsymbol{j}_e = \int \boldsymbol{B} \cdot \boldsymbol{b}_e \quad \text{and} \quad \int \boldsymbol{A} \cdot \boldsymbol{j}_q = \int \boldsymbol{B} \cdot \boldsymbol{b}_q. \tag{8}$$

The total energy which produces these effects is therefore

$$\int \boldsymbol{B} \cdot \left(\boldsymbol{b}_e + \boldsymbol{b}_q\right). \tag{9}$$

Suppose now that the particles are oppositely charged and their angular momenta are such that $\boldsymbol{b}_e + \boldsymbol{b}_q = 0$. Then the total energy is zero. Nevertheless there is an A-B effect for each particle, the energy shift being positive for one and negative for the other.

One arrives at the same conclusion if one puts some diamagnetic material in the region $b \leq r \leq c$. Then (8) is replaced by

$$\int \boldsymbol{A} \cdot \boldsymbol{j}_e = \int \boldsymbol{B} \cdot \boldsymbol{h}_e. \tag{10}$$

The point is that contrary to $\boldsymbol{b}_e$ the field $\boldsymbol{h}_e$ penetrates the diamagnet and even in the limit of a perfect diamagnet, $\mu \to 0$, has nonzero overlap with the external magnetic field.

6 Resolution of the Paradox

Now let us return to the superconductor. From the point of view of the toy model we see that the superconductor is nothing but a device to ensure the existence of two oppositely charged currents whose magnetic fields cancel in the interior regions. The only difference is that whereas the electron current is the usual one the q-current is replaced by the outer surface-current of the superconductor. If the fields produced by these two currents are denoted by $\boldsymbol{b}_e$ and $\boldsymbol{b}_s$ respectively, we have, as in the toy model,

$$\boldsymbol{b} = \boldsymbol{b}_e + \boldsymbol{b}_s = 0 \quad \text{for} \quad b \leq r \leq c. \tag{11}$$

Thus the total magnetic field inside the superconductor is zero, as it should be, but the individual fields $\boldsymbol{b}_e$ and $\boldsymbol{b}_s$ are not. But the individual magnetic fields $\boldsymbol{b}_e$ and $\boldsymbol{b}_s$ penetrate the superconductor and produce the A-B energy by interacting with B. It is only the *sum* of these two fields, which is the total magnetic field, that is zero inside.

Inside the superconductor we have, of course, the situation that (a) the fields $\boldsymbol{b}_e$ and $\boldsymbol{b}_s$ are only virtual, since it is only their sum that can be measured experimentally and (b) that the gauge-field is necessary because it is the only means by which information is transmitted across the superconductor. Both of these points are true, and the second is an argument in favour of the necessity of the gauge-potential in the superconducting case. It is worth observing however that (a) the fields $\boldsymbol{b}_e$ and $\boldsymbol{b}_s$ are distinct in the sense that $\boldsymbol{b}_e$ can be measured separately in the region $c< r< d$ just outside the superconductor and (b) within the superconductor the magnetic potential is no longer a true gauge-potential because, through the Higg's mechanism, it has become massive. From this point of view the gauge potential becomes necessary only when it ceases to be a true gauge-potential!

References

[1] L. O'Raifeartaigh, N. Straumann and A. Wipf, Comments Nucl. Part. Phys. **20** (1991) 15.

[2] R. Hermann, Found. of Phys., **22** (1992) 713.

[3] L. Tassie, Private Communication and Phys. Lett. **5** (1963) 43.

[4] M. Peshkin and H. Tonomura, The Aharonov-Bohm Effect, Springer, Berlin, 1989.

[5] Peshkin, Phys. Rep. **C80** (1981) 375.

[6] Orlando and Delin, Foundations of Applied Superconductivity, Addison-Wesley New York, 1991.

[7] Lipkin, M. Peshkin and L. Tassie, Phys. Rev. **126** (1962) 116.