Calculating Toroidal Figures of Equilibrium

15 October 2008

This documentation explains how to compile and run a computer program that calculates homogeneous (i.e. ε = constant), toroidal figures of equilibrium. The program is one of the electronic resources associated with the book *Relativistic Figures of Equilibrium* by R. Meinel *et al.* and can be downloaded by following links from the Cambridge University Press site `http://www.cambridge.org/9780521863834`

The notation used in this documentation complies with that of the book and equation numbers refer to those in the book.

Compiling the Program

The directory `Source` contains all the files necessary to compile an executable version of the program. Note however, that these files are not intended to be manipulated by the user and do not contain much in the way of comments.

One example of how to compile the code (using the 'gcc' compiler) is

```
gcc -lm -O2 *.c -o name_of_executable
```

Running the Program

The program **must** be run from a directory, which contains a file called `InitialData` and one called `Config`, otherwise a segmentation fault will result. `InitialData` contains a complete description of a spacetime containing a toroidal figure of equilibrium that is understandable to the program. Beginning with this spacetime, `Config` is altered by the user to direct the program toward another configuration. The program produces files as output that can be renamed and used in place of the original `InitialData`.

The `Config` File

An example of the file `Config` can be found in the subdirectory `Example`. An explanation as to the meaning of each of the entries is presented below:

test	Name_of_Sequence	The name of the main output files is chosen to be `test.000, test.001, test.002,` etc.
10	Newton_itmax	The maximum number of Newton-Raphson iterations before giving up.
1.e-8	Newton_tol	The method has converged (by definition) when the sum of the squares of the components in the solution vector from (3.24), $\mathbf{F}\tilde{\mathbf{F}}$, is smaller than this number.
10	ns_and_nt	The number of spectral coefficients in each dimension of each domain.
5	-A	This and the next line contain two integers defining which two physical parameters are to be prescribed. A list of the numbering of the parameters can be found in the file `Parameterlist`, e.g. 5 corresponds to the radius ratio $-A = \varrho_\mathrm{i}/\varrho_\mathrm{o}$ (3.46) and 8 to V_0 (see 1.27).
8	V0	See one line above.
0.5	Goal_val_Param_0	The prescribed value that the first parameter ($-A$ in this example) reaches at the and of the sequence.
-1.0	Goal_val_Param_1	The prescribed value for the second parameter (V_0 in this example).
3	#_of_Sols_in_Seq	This number of solutions will be generated, where at each step, the prescribed parameters are advanced incrementally.
50 50	nx_and_ny…	A file called `PhysQuant_CylCoord.dat` is produced in the terminating step in the sequence that contains information about the metric functions and the matter on an equidistant grid in ϱ-ζ-space. These two parameters specify how many grid points in each of the two directions are to be used.
0.4 0.0	rhomin&zetamin…	One corner of the rectangular grid mentioned above is located at the point specified by these two paramters [i.e. in this example $(\varrho, \zeta) = (0.4, 0.0)$]. Note that ϱ and ζ both have to be non-negative. One can infer the value of any quantity for negative ζ because of the reflectional symmetry with respect to the equatorial plane $\zeta = 0$.

0.7	0.15	rhomax&zetamax...	The other corner of the rectangular grid is located at the point specified by these two paramters [i.e. in this example $(\varrho, \zeta) = (0.7, 0.15)$].

Interpreting the Output

This ring program makes use of a total of four domains instead of the five depicted in Fig. 3.13. Domain 2 was removed in such a way that $s = 0$ of the new domain 1 is a section of the equatorial plane running from the coordinate origin to the point $\varrho = r_1$. The coordinate mapping in this domain reads

$$\tilde{x} = \frac{x_{\mathrm{m}} s(1-t)}{x_0 + x_{\mathrm{m}}(1-2t)} + 1 - s,$$
$$\tilde{y} = \frac{x_{\mathrm{m}} st}{x_0 + x_{\mathrm{m}}(1-2t)} + \frac{(1-s)tx_1}{x_{\mathrm{m}} - x_1}.$$

The equations for the remaining two domains are unchanged.

The program prints out a table on the screen and writes three files entitled `surface.dat`, `ergosphere.dat` and `PhysQuant_CylCoord.dat`. All numerical values are in units with $G = c = 1$ and where the third 'dimensional quantity' is the energy density ϵ.

The table printed out on the screen contains the following entries:

M	gravitational mass M calculated by applying the divergence theorem to (1.57) and choosing the surface of the star for the surface integral	J	angular momentum J from (1.57) calculated in the same manner as M

M0	baryonic mass M_0, see (1.58)	Ebind	binding energy, $M_0 - M$
Rcirc_i	circumferential radius $R_\mathrm{i}^\mathrm{circ}$ for $(\varrho = \varrho_\mathrm{i}, \zeta = 0)$, see p. 149	Rcirc_o	circumferential radius $R_\mathrm{o}^\mathrm{circ}$ for $(\varrho = \varrho_\mathrm{o}, \zeta = 0)$, see p. 149
z	relative redshift z, see (1.28)	V0	V_0, see (1.27)
rho_i	inner radius ϱ_i, see (3.44)	rho_o	outer radius ϱ_o, see (3.45)
r_ratio	$\varrho_\mathrm{i}/\varrho_\mathrm{o}$	Omega	angular velocity Ω, see (1.18)
beta_i	inner mass-shed parameter β, see (3.51)	beta_o	outer mass-shed parameter β, see (3.51)
p_max	the maximal value for p	h_max	the maximal value for h
x0	the domain parameter $x_0 = \varrho_0^2$ from Fig. 3.13	x1	the domain parameter $x_1 = \varrho_1^2$ from Fig. 3.13
Test:	Two tests of the accuracy of the solution are presented. These shows how well the identities $\lvert M_\infty/M - 1\rvert$ and $\lvert J_\infty/J - 1\rvert$ are fulfilled.		

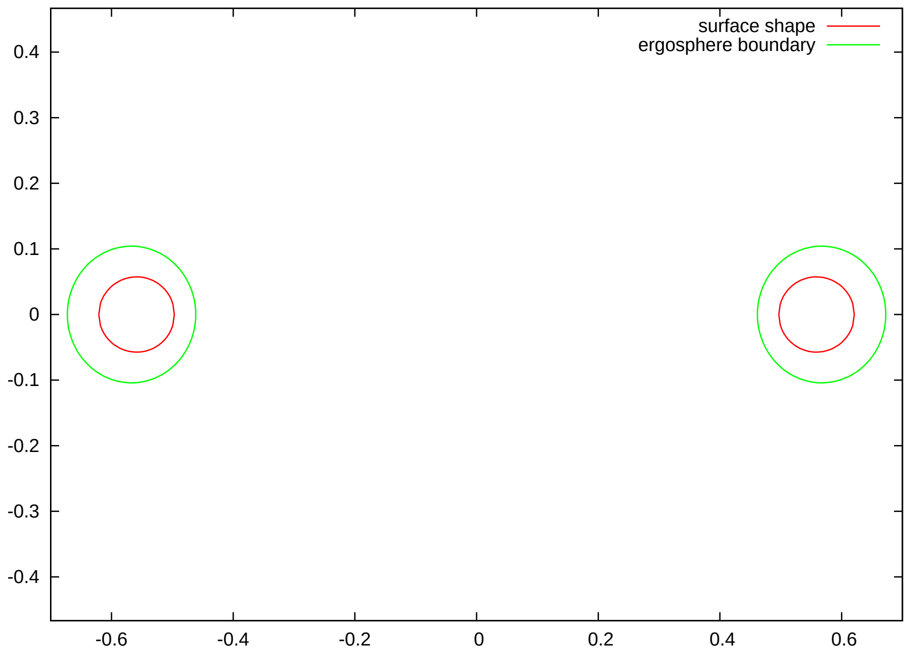

Figure 1: An example of the `gnuplot` output for the coordinate shape of a ring possessing an ergosphere.

The file `surface.dat` contains values for $x = \varrho^2$ and $y = \zeta^2$ at points on the surface of the ring beginning at the inner edge and ending at the outer one. The first two columns in `ergosphere.dat` contain x and y values for points along the boundary of the ergosphere (if one exists) and the third column contains the value of e^{2U} at that point (see Subsection 1.6.2). The fourth column indicates which domain the point lies in. By typing `gnuplot surface_and_ergo.plt` or `gnuplot surface.plt` a picture is generated showing the ring and its ergosphere (if one exists) in meridional cross-section in the ϱ-ζ plane. The picture should look something like Fig. 1 and includes 'negative ϱ values' in order to give a better impression of the three-dimensional figure.

The file `PhysQuant_CylCoord.dat` contains 16 columns of values printed out at each of the gridpoints specified by the `Config` file. The first two columns contain values of x and y. The next three contain the value for

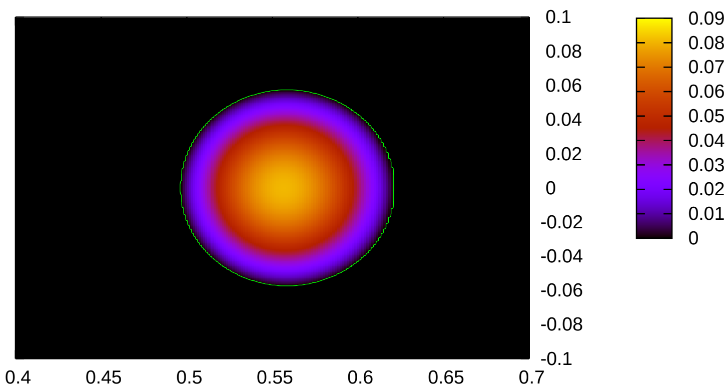

Figure 2: An example of the `gnuplot` output showing the pressure distribution of a ring.

the metric function u and the derivatives $u_{,x}$ and $u_{,y}$ respectively. The next 9 columns contain such values for the functions B, ω and α. The last two columns contain values for h, p from (1.22) and (1.23). Typing `gnuplot PhysQuant_CylCoord.plt` produces two plots similar to Figs 2 and 3 below. A colour coding showing the distribution of pressure within the ring is shown in the ϱ-ζ plane. In the second plot, isobaric surfaces beginning with the surface ($p = 0$) and increasing in increments of 0.006 can be seen. Note that the rectangle chosen does not begin at the axis.

Questions regarding this program can be addressed to David Petroff:
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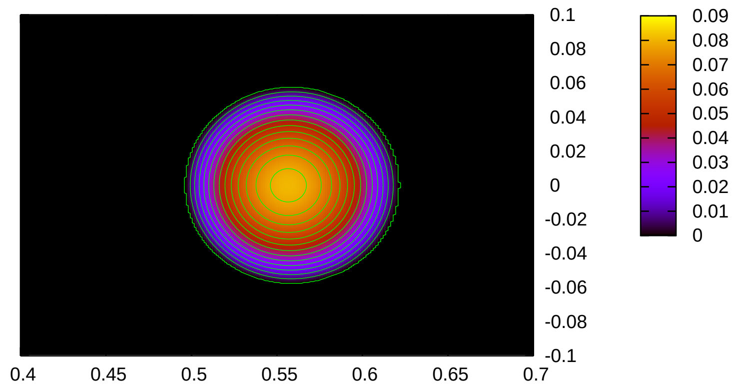

Figure 3: An example of the `gnuplot` output showing the pressure distribution of a ring together with isobaric lines.