Calculating Spheroidal Figures of Equilibrium

15 October 2008

This documentation explains how to compile and run a computer program that calculates spheroidal figures of equilibrium. The program is one of the electronic resources associated with the book *Relativistic Figures of Equilibrium* by R. Meinel *et al.* and can be downloaded by following links from the Cambridge University Press site
`http://www.cambridge.org/9780521863834`

The notation used in this documentation complies with that of the book and equation numbers refer to those in the book.

Compiling the Program

The directory `Source` contains all the files necessary to compile an executable version of the program. Note however, that these files are not intended to be manipulated by the user and do not contain much in the way of comments.

One example of how to compile the code (using the 'gcc' compiler) is

```
gcc -lm -O2 *.c -o name_of_executable
```

Running the Program

The program **must** be run from a directory, which contains a file called `InitialData` and one called `Config`, otherwise a segmentation fault will result. `InitialData` contains a complete description of a spacetime containing a spheroidal figure of equilibrium that is understandable to the program. Beginning with this spacetime, `Config` is altered by the user to direct the program toward another configuration. The program produces files as output that can be renamed and used in place of the original `InitialData`.

Initial data for each of the four equations of state available to the user can be found in each subdirectory of `Example`. In addition, the subdirectory `Homogeneous` contains the file `Nozawa.n12`, which can be used as initial data to calculate a star discussed in Table 1 of Ansorg et al. *Astron. Astrophys.* **381**, L49 (2002).

The `Config` File

An example of the file `Config` can also be found in each subdirectory of the directory `Example`. An explanation as to the meaning of each of the entries is presented below based on the file in the subdirectory `Customized`:

test	Name_of_Sequence	The name of the main output files is chosen to be `test.000, test.001, test.002,` etc.
10	Newton_itmax	The maximum number of Newton-Raphson iterations before giving up.
1.e-06	Newton_tol	The method has converged (by definition) when the sum of the squares of the components in the solution vector from (3.24), $\mathbf{F}\tilde{\mathbf{F}}$, is smaller than this number.
20	ns_and_nt	The number of spectral coefficients in each dimension of each domain.
8	V_0	This and the next line contain two integers defining which two physical parameters are to be prescribed. A list of the numbering of the parameters can be found in the file `Parameterlist`, e.g. 8 corresponds to V_0 (see 1.27) and 6 to the radius ratio.
6	r_ratio	See one line above.
-1.1	Goal_val_Param_0	The prescribed value that the first parameter (V_0 in this example) reaches at the and of the sequence.
0.6	Goal_val_Param_1	The prescribed value for the second parameter (A in this example).
0.	Goal_val_del_s	A parameter used in the coordinate transformation that places grid-points closer to the star's surface as it is increased. It is equal to $\ln \varepsilon_{\mathrm{s}}$ of (3.35b).
5	#_of_Sols_in_Seq	This number of solutions will be generated, where at each step, the prescribed parameters are advanced incrementally.

500	500	nx_and_ny...	A file called `PhysQuant_CylCoord.dat` is produced in the terminating step in the sequence that contains information about the metric functions and the matter on an equidistant grid in ϱ-ζ-space. These two parameters specify how many grid points in each of the two directions are to be used.
0.2	0.2	rhomax&zetamax...	One corner of the rectangular grid mentioned above is located at the origin and the diagonal corner at the point specified by these two paramters [i.e. in this example $(\varrho, \zeta) = (0.2, 0.2)$]. Note that ϱ and ζ both have to be non-negative. One can infer the value of any quantity for negative ζ because of the reflectional symmetry with respect to the equatorial plane $\zeta = 0$.
3		EOS:XXX	This integer indicates which of the three equation of state options is chosen.

The number '0' corresponds to the homogeneous equation of state (EOS) $\epsilon =$ constant. There are no free parameters to be chosen.

The number '1' corresponds to the polytropic EOS $p = K{\mu_{\mathrm{B}}}^{1+1/n}$. The next line contains the polytropic index n.

The number '2' corresponds to the EOS for the completely degenerate, ideal gas of neutrons. There are no free parameters to be chosen.

The number '3' corresponds to a customized EOS of the form

$$p = C \sum_{i=1}^{N} p_i H^i,$$

where H is related to h of (1.23) and $h(0)$ of (1.26) by

$$H = \frac{h}{h(0)} - 1$$

and C is a constant. It then follows that

$$\epsilon = (1 + H)\frac{\mathrm{d}p}{\mathrm{d}H} - p.$$

Note that the strange quark model described by (1.50) is a special case of the customized equation of state with

$$p = B\left[(1 + H)^4 - 1\right].$$

The syntax for the prescription of the coefficients p_i follows.

5 1.0	Goal_vals_Np_&_h0	The first number is an integer N determining the degree of the polynomial representation of p. The second number provides the value of $h(0)$.
1.	Goal_value_p[1]	These N numbers provide the numerical values for the coefficients p_i.
0.2	Goal_value_p[2]	
-0.05	Goal_value_p[3]	
0.01	Goal_value_p[4]	
0.001	Goal_value_p[5]	

Interpreting the Output

The program prints out a table on the screen and writes three files entitled `surface.dat`, `ergosphere.dat` and `PhysQuant_CylCoord.dat`. All numerical values are in units with $G = c = 1$ and where the third 'dimensional quantity' depends on the equation of state. For homogeneous matter it is the energy density ϵ, for polytropes the constant K and for the completely degenerate, ideal gas of neutrons the constant K_{n} cf. (3.57). For the customized equation of state, the quantities are given in units of the constant C from the equation $p = C \sum_{i=1}^{N} p_i H^i$.

The table printed out on the screen contains the following entries:

M	gravitational mass M calculated by applying the divergence theorem to (1.57) and choosing the surface of the star for the surface integral	J	angular momentum J from (1.57) calculated in the same manner as M

M0	baryonic mass M_0, see (1.58)	Rcirc	circumferential radius $R_{\rm circ}$, see p. 149
Ebind	binding energy, $M_0 - M$	Omega	angular velocity Ω, see (1.18)

z	relative redshift z, see (1.28)	V0	V_0, see (1.27)
rp	polar radius $r_{\rm p}$, see (3.35a)	re	equatorial radius $r_{\rm e}$, see (3.35a)
r_ratio	$r_{\rm p}/r_{\rm e}$	beta	mass-shed parameter β, see (3.51)
eps_c	the energy density ϵ at the star's centre	p_c	the pressure p at the star's centre
muB_c	the baryonic mass-density $\mu_{\rm B}$ at the star's centre	h_c	the specific enthalpy h at the star's centre, see (1.23)

eps_max	the maximal value for ϵ	p_max	the maximal value for p
muB_max	the maximal value for $\mu_{\rm B}$	h_max	the maximal value for h
u_c	the metric function u at the star's centre, see p. 115	B_c	the metric function B at the star's centre
alpha_c	the metric function α at the star's centre	om_c	the metric function ω at the star's centre

Test: Two tests of the accuracy of the solution are presented. The first shows how well the identity $|M_\infty/M - 1|$ is fulfilled, where M is calculated from (1.57) using a surface integral and M_∞ from (1.65) evaluated on the axis. The second test is based on the fact that local flatness guarantees $\alpha = -u$ along the axis, cf. (1.10). The number provided is $|u + \alpha|$ at the star's centre.

del_s the value of $\delta_c \equiv \ln \varepsilon_s$ of (3.35b).

The file `surface.dat` contains values for $x = \varrho^2$ and $y = \zeta^2$ at points on the surface of the star beginning at the north pole and ending at the equator. The first two columns in `ergosphere.dat` contain x and y values for points along the boundary of the ergosphere (if one exists) and the third column contains the value of e^{2U} at that point (see Subsection 1.6.2). The fourth column indicates if the point lies in domain 0 or 1. By typing `gnuplot surface_and_ergo.plt` or `gnuplot surface.plt` a picture is generated showing the star and its ergosphere (if one exists) in meridional cross-section in the ϱ-ζ plane. The picture should look something like Fig. 1 and includes 'negative ϱ values' in order to give a better impression of the three-dimensional figure.

The file `PhysQuant_CylCoord.dat` contains 18 columns of values printed out at each of the gridpoints specified by the `Config` file. The first two columns contain values of x and y. The next three contain the value for the metric function u and the derivatives $u_{,x}$ and $u_{,y}$ respectively. The next 9 columns contain such values for the functions B, ω and α. The last four columns contain values for h, p, ϵ and μ_B. Typing `gnuplot PhysQuant_CylCoord.plt` produces two plots similar to Figs 2 and 3 below. A colour coding showing the distribution of pressure within the star is shown in the ϱ-ζ plane. In the second plot, isobaric surfaces beginning with the surface ($p = 0$) and increasing in increments of 0.1 can be seen. Here too 'negative ϱ values' in order to give a better impression of the three-dimensional figure.

Questions regarding this program can be addressed to Marcus Ansorg:
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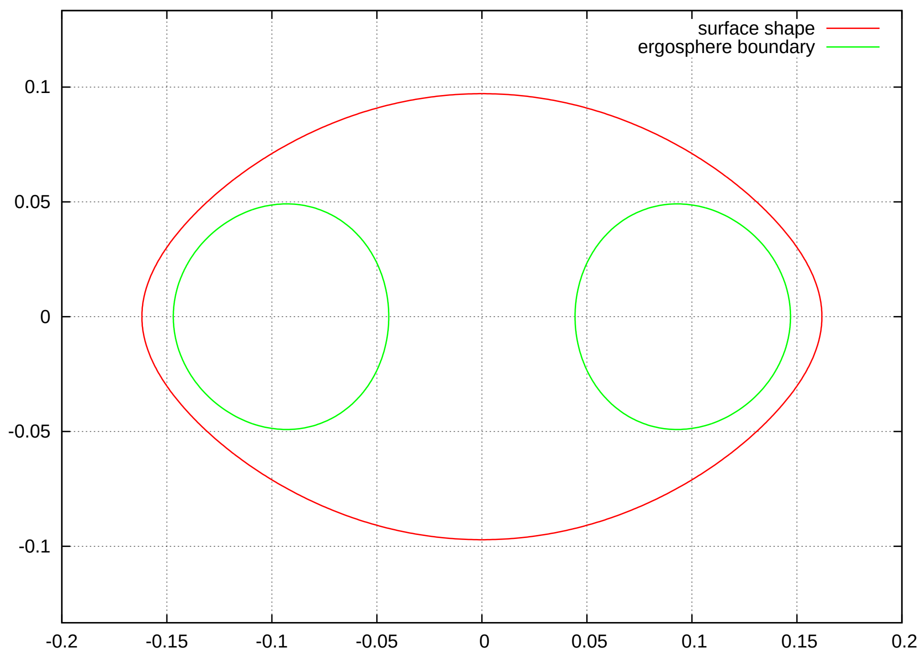

Figure 1: An example of the `gnuplot` output for the coordinate shape of a star possessing an ergosphere.

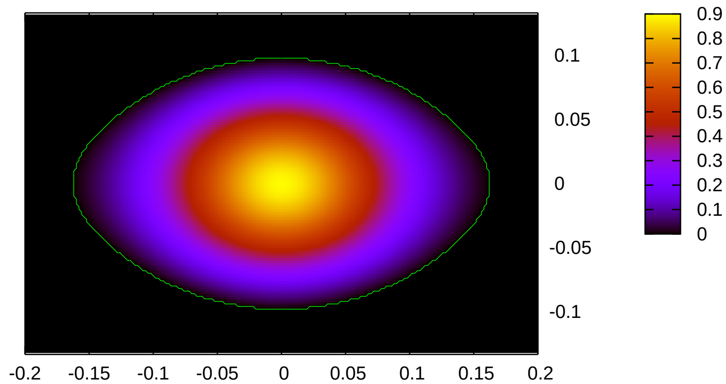

Figure 2: An example of the `gnuplot` output showing the pressure distribution of a star.

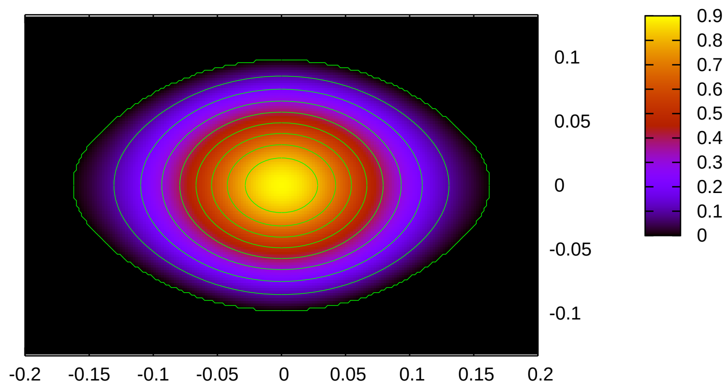

Figure 3: An example of the `gnuplot` output showing the pressure distribution of a star together with isobaric lines.