5 Correlation Functions

So far, we have studied spectral properties of the Hamiltonian or S matrix properties in QFT, similar to a canonical approach to quantum mechanics or scattering theory. In principle, all information is accessible in this manner.
Nevertheless, there is an even more comprehensive, or at least more efficient, language of QFT which is based on the concept of correlation functions; these can be viewed as generalized Green's functions of the interacting theory.
Let us start again from a Hamiltonian of the form $H = H_0 + V$, such that

$$\begin{aligned} H_0 \, |0\rangle &= 0 \\ H \, |\Omega\rangle &= E_0 \, |\Omega\rangle , \end{aligned} \tag{5.1}$$

Here, we distinguish between the ground state $|0\rangle$ of the free theory and that of the interacting theory $|\Omega\rangle$. Since we have normalized the zero-point energy of the free theory to zero, that of the interacting theory may be different.

Def.: n-point correlation function

We define the n-point correlation functions with the aid of field operators in the Heisenberg picture

$$G^{(n)}(x_1, \ldots, x_n) := \langle \Omega | \underset{\uparrow}{T}\, \phi_H(x_1) \ldots \phi_H(x_n) | \Omega \rangle \qquad (5.2)$$

time ordering

In the following, we develop a perturbative construction principle. We start with the relation of field operators in the different pictures

$$\phi_H(t) = e^{iHt} \phi_S e^{-iHt} = e^{iHt} e^{-iH_0 t} \phi_I e^{iH_0 t} e^{-iHt}$$

$$=: \Omega(t) \phi(t) \Omega^\dagger(t) \quad , \; \phi \equiv \phi_I \qquad (5.3a)$$

where we have abbreviated $\Omega(t) := e^{iHt} e^{-iH_0 t}$. (5.3b)

Now consider the combination

$$\Omega^\dagger(t) \Omega(t_0) = e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0} =: \tilde{\mathcal{U}}(t, t_0) \qquad (5.4)$$

This operator satisfies

$$\tilde{\mathcal{U}}(t, t_0)\Big|_{t=t_0} = \mathbb{1}$$

and $i\partial_t \tilde{\mathcal{U}}(t,t_0) = e^{iH_0 t} (\underbrace{-H_0 + H}_{=V}) e^{-iH(t-t_0)} e^{-iH_0 t_0}$

$$= \underbrace{e^{iH_0 t} V e^{-iH_0 t}}_{\equiv H_I(t)} \underbrace{e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0}}_{\equiv \tilde{\mathcal{U}}(t,t_0)} = H_I(t) \tilde{\mathcal{U}}(t, t_0) \qquad (5.5)$$

Because of the uniqueness of solutions to 1st order ordinary differential equations, we can conclude

$$\underline{\underline{\Omega^\dagger(t)\,\Omega(t_0)}} = \tilde{U}(t,t_0) \equiv \underline{\underline{U_I(t,t_0)}} \qquad (5.6)$$

From (5.3a), we see that $\Omega(t)$ mediates between Heisenberg- and interaction picture for operators. The same also holds for states. Eg the vacuum in the interaction picture must be related to $|\Omega\rangle$ by

$$|\Omega\rangle =: \Omega(t)\; |0,t\rangle \qquad (5.7)$$

↑ vacuum in interaction picture

As before, using the notation $\varphi_k := \varphi(t_k)$, $\Omega_k = \Omega(t_k)$, etc, we can write the n-point function (5.2) as

$$G^{(n)}(x_1,\dots,x_n) = \langle 0,t|\,\Omega_t^\dagger\, T\left[\Omega_1\varphi_1\Omega_1^\dagger\,\Omega_2\varphi_2\Omega_2^\dagger \dots \Omega_n\varphi_n\Omega_n^\dagger\right]\Omega_{t'}|0,t'\rangle \qquad (5.8a)$$

Let us choose $t > t_k$ and $t' < t_k$ for all $k=1,\dots,n$ such that $\Omega_t^\dagger$ and $\Omega_{t'}$ can be moved inside the T product. Inside the T-ordering symbol, operators can be moved around at will (since the T-ordering takes care of the correct ordering). Hence, we get

$$G^{(n)}(x_1,\dots,x_n) = \langle 0,t|\,T\Big[\varphi_1\dots\varphi_n\,\underbrace{\Omega_t^\dagger\Omega_1}_{U_I(t,t_1)}\,\underbrace{\Omega_1^\dagger\Omega_2}_{U_I(t_1,t_2)\dots}\dots\Omega_n\underbrace{\Omega_n^\dagger\Omega_{t'}}_{U_I(t_n,t')}\Big]|0,t'\rangle$$

$$= \langle 0,t|\,T\left[\varphi_1\dots\varphi_n\; U_I(t,t')\right]|0,t'\rangle \qquad (5.8b)$$

Upon relating the interaction (or Schrödinger) picture to the Heisenberg picture, we have to fix the latter with respect to a definite time t_0. For time-dependent processes like scattering, it is useful to choose asymptotic times $t_0 \to \pm\infty$, where the ground state becomes the free ground state again (in a certain sense). For the following discussion, both choices $t_0 \to \pm\infty$ lead to identical results; let us, for definiteness, choose $t_0 \to +\infty$ "the OUT vacuum".

Then

$$
\begin{aligned}
\langle 0,t| &= \langle 0,t_0|\, U_I^\dagger(t_0,t) \xrightarrow{t_0\to+\infty} \langle 0,t| = \langle 0|\, U_I^\dagger(\infty,t) \\
|0,t'\rangle &= U_I(t',t_0)\,|0,t_0\rangle \xrightarrow{t_0\to+\infty} |0,t'\rangle = U_I(t',\infty)\,|0\rangle
\end{aligned}
\tag{5.9}
$$

such that

$$
G^{(n)}(x_1,\dots,x_n) = \langle 0|\, U_I^\dagger(\infty,t)\, T\left[\varphi_1\dots\varphi_n\, U_I(t,t')\right] U_I(t',\infty)\,|0\rangle \tag{5.10}
$$

As the left-hand side is independent of the times t and t', so must the right-hand side. Hence, we can choose t and t' to our convenience. Let $t\to\infty$, $t'\to-\infty$, such that

$$
\begin{aligned}
\langle 0,t=\infty| &= \langle 0|\,\underbrace{U_I^\dagger(\infty,\infty)}_{=\mathbb{1}} = \langle 0| \\
|0,t'=-\infty\rangle &= U_I(-\infty,\infty)\,|0\rangle = U_I^\dagger(\infty,-\infty)\,|0\rangle = S|0\rangle .
\end{aligned}
\tag{5.11}
$$

We obtain

$$
\begin{aligned}
G^{(n)}(x_1,\dots x_n) &= \langle 0|\, T[\varphi_1\dots\varphi_n S]\, S^\dagger|0\rangle \\
&= \langle 0|\, T[\varphi_1\dots\varphi_n S] \sum_{n=0}^{\infty} |n\rangle \underbrace{\langle n|\, S^\dagger|0\rangle}_{=:\, S_n^*} .
\end{aligned}
\tag{5.12}
$$

Since $1 = \langle 0|0\rangle = \langle 0|SS^\dagger|0\rangle = \sum_n \langle 0|S|n\rangle\langle n|S^\dagger|0\rangle$

$$= |\langle 0|S|0\rangle|^2 + \sum_{n\neq 0} |\langle n|S^\dagger|0\rangle|^2 ,$$

the stability of the vacuum, $|\langle 0|S|0\rangle|^2 = 1$ implies

$$\sum_{n\neq 0} |\langle n|S^\dagger|0\rangle|^2 = \sum_{n\neq 0} \underbrace{S_n^* S_n}_{\geq 0} = 0$$

$$\Rightarrow \quad S_n^* = 0 \qquad \text{for } n \neq 0 \qquad (5.13)$$

and thus

$$\underline{\underline{G^{(n)}(x_1 \ldots x_n)}} = \langle 0|T(\varphi_1 \ldots \varphi_n S)|0\rangle \cdot \underbrace{\langle 0|S^\dagger|0\rangle}_{\text{pure phase}}$$

$$= \underline{\underline{\frac{\langle 0|T(\varphi_1 \ldots \varphi_n S)|0\rangle}{\langle 0|S|0\rangle}}} \qquad (5.14)$$

This formula is not only nice and compact, but also gives a constructive prescription how to evaluate n-point correlators in terms of a perturbative expansion of the S-matrix.

On the formal side, we can even write down a generating functional for all correlation functions:

$$T[J] := \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^D x_1 \dots d^D x_n \, G^{(n)}_{(x_1 \dots x_n)} \, J(x_1) \dots J(x_n) \, , \qquad (5.15)$$

not to be confused with the transition matrix

where the "source" $J(x)$ is an auxiliary variable. We obtain all $G^{(n)}$ from $T[J]$ by functional differenciation with respect to $i \frac{\delta}{\delta J}$ (and taking subsequently the limit $J \to 0$).

Using the definition (5.2) of the $G^{(n)}$, we can give a closed formula for $T[J]$:

$$T[J] = \langle \Omega | T \, e^{-i \int J \phi_H} | \Omega \rangle \, . \qquad (5.16)$$

From (5.15) as well as (5.16), it is obvious that

$$G^{(n)}(x_1 \dots x_n) = i^n \frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} \, T[J] \, . \qquad (5.17)$$

In order to also make contact with (5.14), we consider the S matrix including a source term

$$S[J] := T \left(e^{-i \int d^D x \, \mathcal{H}_I(x) \; - i \int d^D x \, J(x) \, \phi(x)} \right) , \qquad (5.18)$$

with $S[0]$ being the ordinary S matrix. The corresponding vacuum expectation value is

$$Z[J] := \langle 0 | S[J] | 0 \rangle \, , \qquad (5.19)$$

Such that

$$i^n \frac{\delta}{\delta J_1} \cdots \frac{\delta}{\delta J_n} Z[J] = \langle 0 | T\left(\varphi_1 \cdots \varphi_n\, S[J]\right) | 0 \rangle \tag{5.20}$$

Hence, we can relate the generating functional $T[J]$ based on the Heisenberg picture to the S matrix language:

$$\underline{\underline{T[J]}} = \underline{\underline{\frac{Z[J]}{Z[0]}}} = \underline{\underline{\frac{\langle 0 | S[J] | 0 \rangle}{\langle 0 | S[0] | 0 \rangle}}} \tag{5.21}$$

These compact formulas characterize a direct relation between S matrix elements and correlation functions.

More precisely, with our technology so far, we can rewrite a (perturbative) computation of correlation functions into a (perturbative) computation of S matrix elements.

We will see below that we can also determine S matrix elements from correlation functions by means of the LSZ reduction formula.

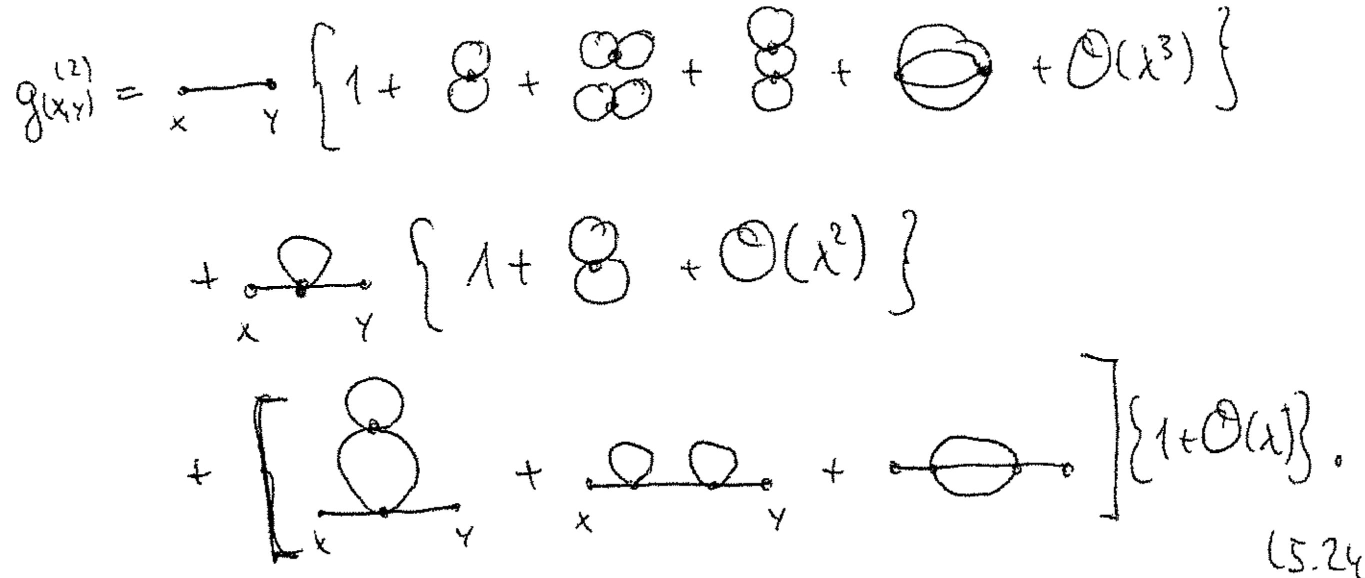
Example: computation of the 2-point function to order $\mathcal{O}(\lambda^2)$ in ϕ^4 theory

$$G^{(2)}_{(x,y)} = \frac{\langle 0|T[\phi_x\phi_y\, e^{-i\frac{\lambda}{4!}\int d^Dx_1\, \phi_1^4}]|0\rangle}{\langle 0|S|0\rangle} \equiv \frac{g^{(2)}_{(x,y)}}{\langle 0|S|0\rangle} \quad (5.22)$$

Using Wick's theorem, we first compute $g^{(2)}(x,y)$

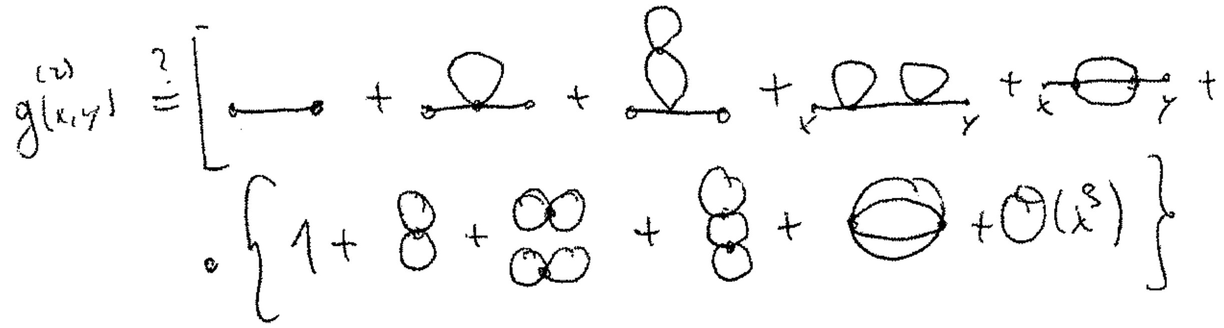
$$\begin{aligned}
g^{(2)}_{(x,y)} &= \langle 0|T[\phi_x\phi_y]|0\rangle - i\frac{\lambda}{4!}\int d^Dx_1 \langle 0|T[\phi_x\phi_y\phi_1^4]|0\rangle \\
&\quad + \frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2 \int d^Dx_1 d^Dx_2 \langle 0|T[\phi_x\phi_y\phi_1^4\phi_2^4]|0\rangle + \mathcal{O}(\lambda^3) \\
&= \underbrace{\phi_x\phi_y}_{i\Delta(x-y)} - \frac{i\lambda}{4!}\int d^Dx_1 \Big\{ 3\,\phi_x\phi_y\,(\phi_1\phi_1)^2 + 12\,\phi_x\phi_y\,\phi_1\phi_1\,\phi_1\phi_1 \Big\} \\
&\quad + \frac{1}{2!}\left(\frac{-i\lambda}{4!}\right)^2 \int d^Dx_1 d^Dx_2 \Big\{ \phi_x\phi_y \Big[(\phi_1\phi_1)^2(\phi_2\phi_2)^2\cdot 9 + (\phi_1\phi_1)\,\phi_1\phi_1\phi_2\phi_2\,(\phi_2\phi_2)\cdot 72 \\
&\qquad + 4!\,(\phi_1\phi_2)^4 \Big] \\
&\qquad + 72\,\phi_x\phi_y\,\phi_1\phi_1\,\phi_1\phi_1\,(\phi_2\phi_2)^2 \\
&\qquad + 288\,\phi_x\phi_y\,\phi_1\phi_1\,\phi_1\phi_1\,\phi_2\phi_2\,\phi_2\phi_2 \\
&\qquad + 288\,\phi_x\phi_y\,\phi_1\phi_1\,\phi_1\phi_1\,\phi_2\phi_2\,\phi_2\phi_2 \\
&\qquad + 192\,\phi_x\phi_y\,\phi_1\phi_1\,\phi_1\phi_1\,\phi_2\phi_2\,\phi_2\phi_2 \Big\}
\end{aligned} \quad (5.23)$$

which can be represented in terms of Feynman diagrams



(5.24)

The result provides evidence for the conjecture that the factors in curly $\{\}$-brackets represent the same series of vacuum diagrams,



(5.25)

If this conjecture is true, all vacuum diagrams factorize, which would correspond to the vacuum expectation value of the S matrix,

$$\{\%\} \equiv \langle 0|S|0\rangle \,, \tag{5.26}$$

such that they cancel because of $G^{(n)} = \dfrac{g^{(n)}}{\langle 0|S|0\rangle} = \dfrac{[\ldots]\cdot\{\%\}}{\langle 0|S|0\rangle} = [\ldots]$

Then we could conclude that $G^{(n)}$ consists only of connected diagrams, with the momentum flowing continuously through the diagram from x to y.

This factorization can be shown as follows:

Let

$$\frac{\Gamma_k}{k!} := \langle 0 | T[\varphi_1 \dots \varphi_n S^{(k)}] | 0 \rangle \tag{5.27}$$

denote the contribution to order k of the Dyson series for the correlator $G^{(n)}$. Γ_k is a sum of diagrams with k vertices. Each diagram in this sum is a product of subgraphs $\Gamma(p, k-p=q)$ with q out of k vertices, which belong to the vacuum diagrams,

e.g. $\Gamma_3 \supset \Gamma(1,2) =$  (5.28)

Thus, each subgraph factorizes into subgraphs

- $C(p)$ with p vertices and no vacuum diagrams
- $V(q)$ with vacuum diagrams only that contain q vertices.

For a total number of k vertices, there are $\binom{k}{q}$ possibilities to distribute these vertices among the subdiagrams $C(p)$ and $V(q)$:

$$\Gamma(p, k-q) \equiv \Gamma(p,q) = \binom{k}{q} C(p) V(q) = \binom{p+q}{q} C(p)\, V(q) \tag{5.29}$$

In total, we have

$$g = \sum_k \frac{1}{k!} \Gamma_k = \sum_k \frac{1}{k!} \sum_{p \leq k} \Gamma(p, k-p) = \sum_{p,q} \frac{1}{(p+q)!} \Gamma(p,q)$$

$$= \sum_{p,q} \frac{1}{(p+q)!} \binom{p+q}{q} C(p) V(q) = \sum_p \frac{C(p)}{p!} \sum_q \frac{V(q)}{q!} , \quad (5.30)$$

where $\sum_q \frac{V(q)}{q!} = \langle 0|S|0\rangle$ is nothing but the sum of all S matrix vacuum diagrams. ▨

The above example has been considered in coordinate space, e.g. $G^{(2)}(x,y) = i\Delta_F(x-y) + \mathcal{O}(\lambda)$. Equivalently, we would have found in momentum space

$$G^{(2)}(p) = i\Delta_F(p) = \frac{i}{p^2 - m^2 + i\varepsilon} + \mathcal{O}(\lambda) \qquad (5.31)$$

There is one important difference to the matrix elements considered so far: Since x and y are arbitrary, also p in (5.31) is arbitrary; in particular, this 4-momentum is not restricted to be on the mass shell, i.e. ($p \neq \bar{p}$ in general).
Therefore, correlation functions contain information about real particle propagation as well as about "virtual" quantum fluctuations.

We have ignored one subtlety so far: the above argument that $G^{(2)}$ consists only of connected diagrams relies on the assumption that $\langle 0|\phi|0\rangle = 0$ (which is true for the interactions we have considered so far.) In the case when field operators can also develop a vacuum expectation value, $\langle 0|\phi|0\rangle \neq 0$, the correlation function can also receive contibutions from disconnected diagrams.

5.2 Lehmann – Källen – Spektraldarstellung

Recall that the 2-point correlator in the free theory, $\langle 0 | T\, \varphi(x)\, \varphi(y) | 0 \rangle$ has a simple interpretation:

it is the Green's function for causal boundary conditions/initial conditions and thus related to the amplitude of a particle propagating from x to y. In the following, we will be interested in the question as to whether there is an analogous interpretation also for the full 2-point function $G^{(2)}(x,y) = \langle \Omega | T[\varphi_H(x)\, \varphi_H(y)] | \Omega \rangle$.

An analysis is in fact possible without any restriction to perturbation theory or specific interactions. For this, we first need a completeness relation analogous to the free case;

$$\mathbb{1}_{1\text{-particle}} = \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\vec{p}}} |\vec{p}\rangle\langle\vec{p}| \quad , \quad \text{(free theory)} \tag{5.32}$$

but now for the interacting theory. As base vectors, we choose eigenstates of the full Hamiltonian H. Assuming translational invariance, we have $[H, \vec{P}] = 0$ and thus, these base vectors can be choosen to be eigenstates of the momentum operator $\vec{P}$ as well.

Let $|\lambda_0\rangle$ be an eigenstate of H with momentum 0,

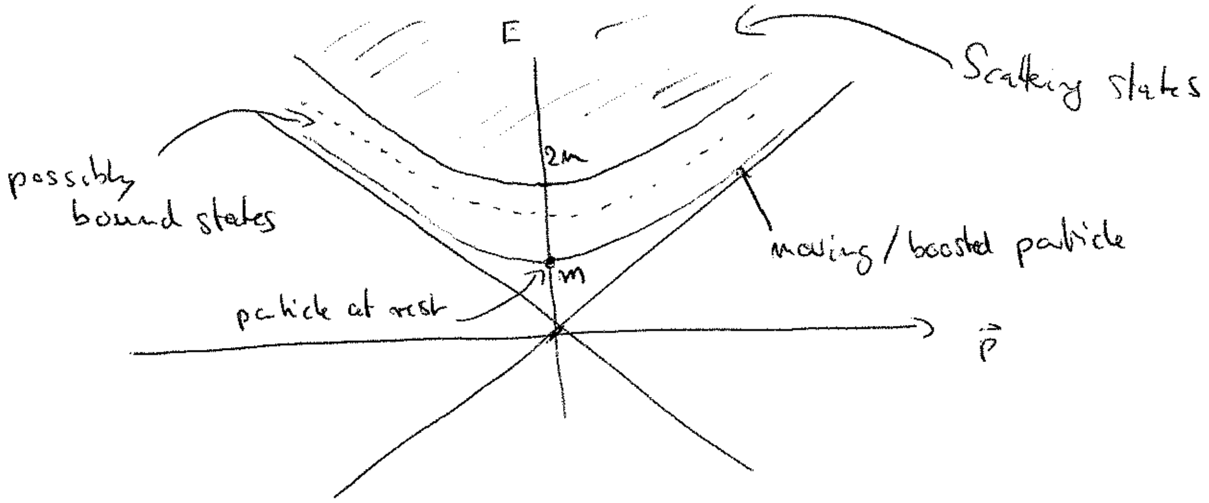
i.e. $\vec{P}\,|\lambda_0\rangle = 0$. (5.33a)

Then, the boosted state is also an eigenstate of H with a nonvanishing momentum corresponding to the momentum in the Lorentz-boosted frame:

$$\vec{P}\,|\lambda_{\vec{p}}\rangle = \vec{p}\,|\lambda_{\vec{p}}\rangle \qquad (5.33b)$$

In turn, any such eigenstate $|\lambda_{\vec{p}}\rangle$ can be viewed as a Lorentz-boosted partner state of a state $|\lambda_0\rangle$ with momentum 0.

The eigenvalues of the 4-momentum $P^\mu = (H, \vec{P})$ thus have to lie on the mass shell,



Let $|\lambda_{\vec{p}}\rangle$ be the Lorentz-boosted version of $|\lambda_0\rangle$ with momentum $\vec{p}$ and a relativistic normalization (analogous to $|\vec{p}\rangle$). Let the eigenvalue of H be

$$E_{\vec{p}}(\lambda) := \sqrt{|\vec{p}|^2 + m_\lambda^2}\,, \tag{5.34}$$

where m_λ is the "mass" of the state $|\lambda_{\vec{p}}\rangle$, i.e. the rest-mass / rest-energy of $|\lambda_0\rangle$. Then, we can write the completeness relation as

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_\lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\vec{p}}(\lambda)} |\lambda_{\vec{p}}\rangle\langle\lambda_{\vec{p}}|\,, \tag{5.35}$$

where the sum runs over all momentum 0 states $|\lambda_0\rangle$.

For $x^0 > y^0$, we obtain:

$$\langle\Omega|T[\varphi_H(x)\varphi_H(y)]|\Omega\rangle \overset{x^0>y^0}{=} \sum_\lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\vec{p}}(\lambda)} \langle\Omega|\varphi_H(x)|\lambda_{\vec{p}}\rangle\langle\lambda_{\vec{p}}|\varphi_H(y)|\Omega\rangle\,, \tag{5.36}$$

where we have assumed that the vacuum does not feature a field expectation value $\langle\Omega|\varphi|\Omega\rangle = 0$.

For the matrix element, we have

$$\begin{aligned}\langle\Omega|\varphi_H(x)|\lambda_{\vec{p}}\rangle &= \langle\Omega| e^{iPx} \varphi_H(0) e^{-iPx} |\lambda_{\vec{p}}\rangle \\ &= \langle\Omega|\varphi_H(0)|\lambda_{\vec{p}}\rangle e^{-i\bar{p}x}\,, \quad (\bar{p}^\mu = (E_{\vec{p}}(\lambda), \vec{p})) \\ &= \langle\Omega|\varphi_H(0)|\lambda_0\rangle e^{-i\bar{p}x}\,,\end{aligned} \tag{5.37}$$

where we have used the Lorentz invariance of $|\Omega\rangle$ and $\phi_H(0)$ in the last step, and the fact that $|\Omega\rangle$ doesn't carry any momentum in the first step.

$$\Rightarrow \langle\Omega|T[\phi_H(x)\phi_H(y)]|\Omega\rangle = \sum_\lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\vec p}(\lambda)} e^{-i\bar{p}(x-y)} |\langle\Omega|\phi_H(0)|\lambda_0\rangle|^2$$

$$= \sum_\lambda \int \frac{d^D p}{(2\pi)^D} \frac{i}{p^2 - m_\lambda^2 + i\varepsilon} e^{-ip(x-y)} |\langle\Omega|\phi_H(0)|\lambda_0\rangle|^2 \tag{5.38}$$

In the last step, we have introduced an additional p^0 integration analogous to the steps in the derivation of the Feynman propagator. In fact, if we had assumed $(x^0 < y^0)$ we would have arrived at (5.38) as well, by picking up the other pole of the Feynman propagator in the complex p^0 plane.

Now, the Feynman propagator occurring in (5.38) carries the mass m_λ rather than m. Writing (5.38) slightly differently, we arrive at the <u>Lehmann-Källen spectral representation</u>

$$G^{(2)}_{\Omega}(x,y) = \langle\Omega|T[\phi_H(x)\phi_H(y)]|\Omega\rangle = \sum_\lambda i\Delta_F(x-y;\lambda)|\langle\Omega|\phi_H(0)|\lambda_0\rangle|^2$$

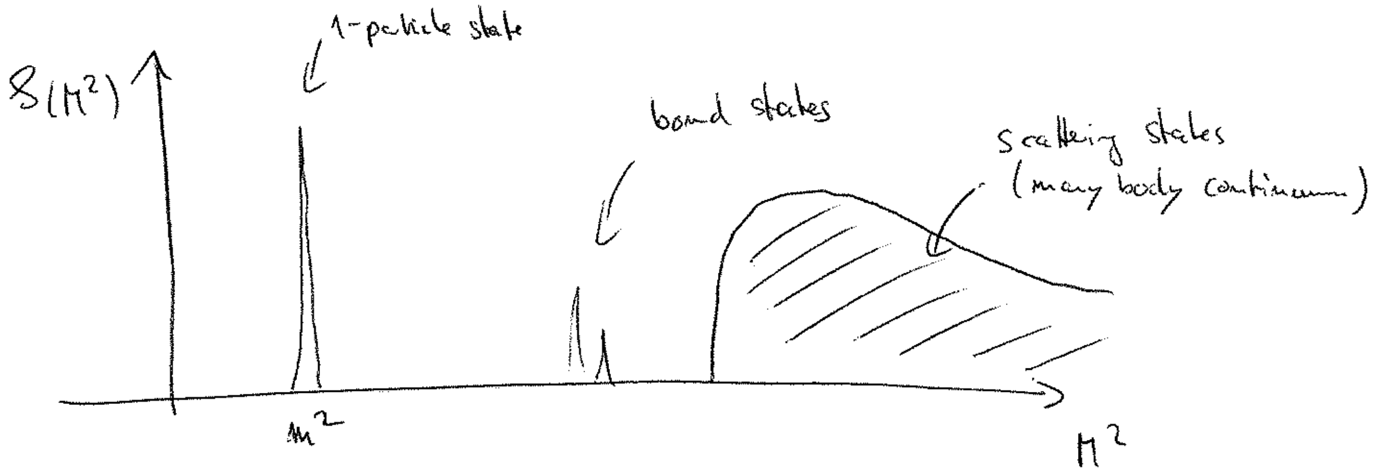
$$=: \int_0^\infty \frac{dM^2}{2\pi} \, \rho(M^2) \quad i\Delta_F(x-y;M^2) \tag{5.39}$$

where we have introduced the

Spectral density

$$\rho(M^2) = \sum_\lambda (2\pi)\, \delta(M^2 - m_\lambda^2)\, |\langle \Omega | \phi_H(0) | \lambda_0 \rangle|^2 . \quad (5.40)$$

For an interacting theory, $\rho(M^2)$ qualitatively can be expected to have the generic form



From the 1-particle state, we expect a contribution of the form

$$\rho(M^2) = 2\pi\, \delta(M^2 - m^2) \cdot Z + (\text{scattering / bound states}) \quad (5.41)$$

where we have introduced the <u>wave function renormalization</u>

$$\underline{\underline{Z}} = |\langle \Omega | \phi_H(0) | \lambda_0 \rangle|^2 \quad (5.42)$$

In (5.41), m is in fact the exact physical one-particle mass (which may be different from the mass parameter in the free Hamiltonian due to interactions).

The wave function or field strength renormalization has the obvious meaning of a probability that a the field operator $\Phi_H(0)$ creates a particle out of the vacuum.

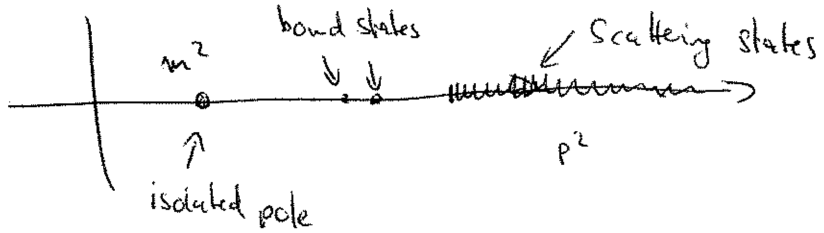
In Fourier space, we obtain

$$G^{(2)}_{(p)} = \int d^Dx \, e^{ipx} \langle \Omega | T[\Phi_H(x) \Phi_H(y)] | \Omega \rangle$$

$$= \int_0^\infty \frac{dM^2}{2\pi} \, \delta(M^2) \, \frac{i}{p^2 - M^2 + i\varepsilon}$$

$$= \frac{iZ}{p^2 - m^2 + i\varepsilon} + \underbrace{\int_{\sim 4m^2}^{\infty}}_{\text{including bond states}} \frac{dM^2}{2\pi} \, \delta(M^2) \, \frac{i}{p^2 - M^2 + i\varepsilon} \qquad (5.43)$$

In the complex p^2 plane, the one-particle state corresponds to a pole at the mass scale $p^2 = m^2$. Bond states correspond to further poles, whereas continuum scattering states lead to a branch cut in $G^{(2)}_{(p)}$



In the free theory, we obviously have $Z = 1$ and no bond- or scattering states.

We conclude that $G^{(2)}$ by means of the Lehmann-Källen spectral representation is a direct generalization of the free-theory's propagator to the interacting case.

$G^{(2)}$ is also called "full propagator".

Whereas 2-point correlations in the free case are mediated by the 1-particle states, all other states with the appropriate quantum numbers can contribute in the interacting case. This includes bound states and scattering states.