4. Basics of scattering theory

In view of its application to collider experiments, a strong emphasis in QFT is put on the description of scattering processes,

$$a_1 + a_2 \longrightarrow b_1 + b_2 + b_3 + \dots ,$$

i.e. an initial state $|i\rangle$ consisting, e.g., of two particle beams a_1 and a_2 goes into a final state $|f\rangle$ consisting of particles $b_1, b_2, b_3, \dots$

In general, we are interested in the non-trivial case $|i\rangle \neq |f\rangle$.

The scattering is in principle a time-dependent process, with $|i\rangle$ being prepared at initial time t_i and $|f\rangle$ being measured at final time t_f. Hence the physical state of the system is time-dependent. However, since the scattering process is very short, it is a very accurate description to consider the limits $t_i \to -\infty$, $t_f \to +\infty$.

(NB: in analogy to quantum mechanics, one might wonder why a QFT lecture starts with scattering processes and not with static quantities such as bound-state spectra; the reason is that computations of bound-states are much more difficult as they are not accessible through a perturbative expansion in the coupling.)

4.1 Interaction picture and S matrix

As should be familiar from quantum mechanical scattering theory it is useful to introduce a particular picture, the interaction or Dyson picture, in addition to the Schrödinger and Heisenberg picture, in order to handle the relevant part of the time evolution. Let us briefly summarize: starting from explicitly time-independent Hamiltonians, time evolution is described by the time evolution operator

$$U(t,t_0) = e^{-iH(t-t_0)} \quad , \quad \frac{\partial H}{\partial t} = 0 \quad . \tag{4.1}$$

In the Schrödinger picture, the field operators are time independent

$$\Phi_S(\vec{x}) \equiv \Phi(\vec{x}) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right) \tag{4.2a}$$

whereas the states carry the time dependence,

$$|\Psi\rangle_S \equiv |\Psi, t\rangle = U(t,t_0)\,|\Psi, t_0\rangle \, . \tag{4.2b}$$

In the Heisenberg picture, the states are time independent,

$$|\Psi\rangle_H \equiv |\Psi, t_0\rangle = U^\dagger(t,t_0)\,|\Psi,t\rangle = U^\dagger\,|\Psi\rangle_S \tag{4.3a}$$

while the operators are time dependent

$$\begin{aligned} \Phi_H = \Phi(\vec{x},t) &= U^\dagger(t,t_0)\,\Phi(\vec{x})\,U(t,t_0) \\ &= U^\dagger(t,t_0)\,\Phi_S\,U(t,t_0) \end{aligned} \tag{4.3b}$$

Eq. (4.35) does not only hold for the field operators, but is a general relation between operators in the Heisenberg and Schrödinger picture. Of course, the physics is independent of the picture. Any observable acquires the same value independently of the picture we use for its description. The two pictures are <u>unitarily equivalent</u>.

As an example, consider the transition amplitude from an initial state $|i\rangle$ at t_i to a final state $|f\rangle$ at time t_f via an elastic process such that $E_f = E_i$. Assuming that $|i\rangle$ is an eigenstate of the Hamiltonian and that the states are orthonormalized n-particle states in a free Klein-Gordon theory, we first have to recognize that the eigenstates $|\vec{p}_1 \dots \vec{p}_n\rangle$ in the Heisenberg picture depend on time (seemingly despite the rule (4.3a)), since they are eigenstates of time-dependent operators. Another way to see this is to realize that the ladder operators in the Heisenberg picture are promoted to time dependent operators,

$$a_H(\vec{p},t) := U^\dagger a_{\vec{p}} U = a_{\vec{p}}\, e^{-iE_{\vec{p}}t} \qquad (t_0=0), \quad (4.4)$$

and hence

$$|\vec{p}_1 \dots \vec{p}_n\rangle_H \equiv |\vec{p}_1, \dots, \vec{p}_n; t\rangle = U^\dagger |\vec{p}_1, \dots, \vec{p}_n\rangle = e^{it\sum_{k=1}^{n} E_{\vec{p}_k}} |\vec{p}_1 \dots \vec{p}_n\rangle_S \quad (4.5)$$

Hence, the transition amplitude reads ($t_0 = t_i$)

$$ {}_H\langle f, t_f | i, t_i \rangle_H = {}_S\langle f | U(t_f, t_i) | i \rangle_S $$
$$ = {}_S\langle f | e^{-iH(t_f - t_i)} | i \rangle_S = e^{-iE_i(t_f - t_i)} \delta_{fi} . \qquad (4.6) $$

This implies that there is no transition of an initial state into a different final state in the free Klein-Gordon theory. This is obviously the consequence of the absence of any interaction.

A paradigmatic example for an interacting generalization of free Klein-Gordon theory is the introduction of a quartic interaction: "ϕ^4" theory. The Lagrangian reads

$$ \mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!} \varphi^4 . \qquad (4.7) $$

The quartic self-interaction in fact occurs in the standard model of particle physics for the self-interaction of the so-called Higgs field. ϕ^4 theory also has a variety of further applications, e.g. in the description of magnetic systems, etc. The quartic term leads to a cubic piece in the equations of motion thus lifting the superposition principle of the free theory.

A general solution of ϕ^4 theory is not known so far. In the following, we aim at an expansion for weak coupling λ corresponding to "perturbation theory".

Our general construction principle for QFT's are:

① Locality: the Lagrangian can be formulated in terms of products of fields and their derivatives
(⇒ causality)

② Lorentz invariance
(⇒ relativistic spacetime structure

③ real action (⇒ self-adjoint Hamiltonian
⇒ unitarity)

④ perturbative renormalizability
(technical criterion, guaranteeing that perturbation theory can be constructed without further parameters
See "Physics of Scales" lecture course for details)

In order to introduce the interaction picture, we write H in the form

$$H = H_0 + V_S \qquad (4.8)$$

(V_S: S ↑ Schrödinger)

For ϕ^4 theory, V_S corresponds to $V_S = \frac{\lambda}{4!}\int d^4x\, \phi^4$, and H_0 to the free theory. In the Schrödinger picture, we would be interested in the solutions to the

Schrödinger equation

$$i\partial_t \, |\bar{\Psi}(t)\rangle_S = (H_0 + V_S) \, |\bar{\Psi}(t)\rangle_S \qquad (4.9)$$

For the interaction picture, labeled by "I", we introduce the free time evolution operator

$$U_0(t,t_0) = e^{-iH_0(t-t_0)} \, . \qquad (4.10)$$

The states of the interaction picture are introduced by "undoing" the free evolution:

$$|\Phi(t)\rangle \equiv |\bar{\Psi}(t)\rangle_I := U_0^\dagger(t,t_0) \, |\Psi(t)\rangle_S \qquad (4.11)$$

It is straightforward to check that the interaction picture states satisfy

$$i\partial_t \, |\Phi(t)\rangle = H_I(t) \, |\Phi(t)\rangle \, , \qquad (4.12)$$

where the interaction part of the Hamiltonian is defined as

$$H_I(t) = U_0^\dagger(t,t_0) V_S \, U_0(t,t_0) \qquad (4.13)$$

Hence, only the interaction part of the Hamiltonian generates the time evolution of the states in the interaction picture. Introducing the time evolution operator for the interaction part that satisfies

$$\partial_t \, U_I(t,t_0) = H_I(t) \, U_I(t,t_0) \, , \qquad (4.14)$$

We can express the solution to (4.12) in terms of the initial condition $|\phi_0\rangle \equiv |\phi(t_0)\rangle$ given at some initial time t_0:

$$|\phi(t)\rangle = U_I(t,t_0)\,|\phi_0\rangle \tag{4.15}$$

Since $H_I(t)$ is explicitly time dependent, the time evolution operator is not simply given by the exponential of $H_I(t)$, but — as we know from time dependent perturbation theory in QM — by a time-ordered exponential of the integrated Hamiltonian:

$$\begin{aligned} U_I(t,t_0) &= \mathcal{T}\exp\left[-i\int_{t_0}^{t} dt'\; H_I(t')\right] \\ &= \mathcal{T}\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{t_0}^{t}dt_1\cdots\int_{t_0}^{t}dt_n\; H_I(t_n)\cdots H_I(t_n) \end{aligned} \tag{4.16}$$

(Dyson series)

As defined above, time ordering $\mathcal{T}$ implies that operators at earlier times are sorted to the right of those at later times. $U_I(t,t_0)$ satisfies the initial condition

$$U_I(t_0,t_0) = \mathbb{1} \tag{4.17}$$

For the (idealized) description of scattering processes, we now introduce the initial state as an asymptotic

initial time in the interaction picture

$$|i\rangle \equiv |i, t_i \to -\infty\rangle = |\phi(t=-\infty)\rangle \qquad (4.18)$$

Let us for the moment assume that $|i\rangle$ satisfies the free Schrödinger equation

$$H_0 |i\rangle = E_i |i\rangle, \qquad (4.19)$$

which implicitly means the we assume the interactions to switch off away from the interaction region where the scattering takes place. (NB: we will see that this assumption is not correct, as it ignores the fact that asymptotic states still are subject to self-interactions. Though the latter are important and will have to be treated carefully, they are not related to the scattering process, which we are focussing on here.)

Once $|i\rangle$ is specified, we can also determine the state at asymptotic future times:

$$\begin{aligned} |i'\rangle &\equiv |i, t_f \to +\infty\rangle = |\phi(t\to+\infty)\rangle \\ &= U_I(\infty, -\infty)|i\rangle =: S|i\rangle \end{aligned} \qquad (4.20)$$

Here, we have introduced the S matrix as the asymptotic limit of the time evolution operator,

$$S = T\, e^{-i\int_{-\infty}^{\infty} dt\, H_I(t)} = T\, e^{-i\int d^D x\, \mathcal{H}_I(x)}, \qquad (4.21)$$

which can also be written in terms of the interaction part of the Hamiltonian density $\mathcal{H}_I \left(\overset{\phi^4:}{=} \frac{\lambda}{4!} \phi_I^4 \right)$.

Now let $|f\rangle$ be the asymptotic final state (that is measured by the detector; note that there is a difference between $|i'\rangle$ which "arrives" at the asymptotic detector and the state $|f\rangle$ as a result of the measurement process).

$|f\rangle$ also should satisfy the free Schrödinger equation

$$H_0 |f\rangle = E_f |f\rangle \tag{4.22}$$

(with the same caveats as above). The probability to measure the transition $|i\rangle \to |f\rangle$ is then given by

$$\langle f | i' \rangle = \langle f | S | i \rangle =: S_{fi} \tag{4.23}$$

the <u>elements</u> of the <u>S matrix</u>.

For the non-interacting system, we already found the trivial result

$$S = \mathbb{1} \quad , \; S_{fi} = \langle f | i \rangle = \delta_{fi} \; . \tag{4.24}$$

4.2 Properties of the S matrix

The S matrix together with its series representation in (4.16) serves as the starting point of perturbation theory to be further developed below. The expansion in $\mathcal{H}_I$ can be viewed as an expansion in the coupling parameter λ. Let us first concentrate on the general properties of the S matrix:

Lorentz invariance

The n-th term in the Dyson series (4.16) reads

$$S^{(n)} = \frac{(-i)^n}{n!} \int d^D x_1 \dots \int d^D x_n \, T \, \mathcal{H}_I(x_1) \dots \mathcal{H}_I(x_n). \qquad (4.25)$$

The D-dimensional integrations over densities are Lorentz invariant. Also the time ordering inside the forward light cone is an invariant property, closely related to causality. However, at first sight the time ordering outside the light cone seems problematic, since, for each spacelike distance of two events x_i^μ and x_j^μ with $(x_i - x_j)^2 < 0$, there is a Lorentz transformation such that $(x_i - x_j)^\mu \to -(x_i - x_j)^\mu$. This also changes time ordering of the events $(t_i - t_j) \to -(t_i - t_j)$.

Nevertheless, causality saves the day, because it

guarantees that

$$\left[\mathcal{H}_I(x_i), \mathcal{H}_I(x_j) \right] = 0 \quad \text{for } (x_i - x_j)^2 < 0 \qquad (4.26)$$

such that time ordering beyond the light cone becomes irrelevant.

We conclude that $S^{(n)}$ is Lorentz invariant, as also is the complete S matrix.

Unitarity:

Since the S matrix is an asymptotic limit of a unitary time evolution operator, it is unitary by construction:

$$S^\dagger S = S S^\dagger = \mathbb{1} \qquad (4.27)$$

Given two orthonormalized initial states $|i\rangle$, $|j\rangle$, also the time evolved states satisfy

$$\langle i' | j' \rangle = \langle i | S^\dagger S | j \rangle = \langle i | j \rangle = \delta_{ij} . \qquad (4.28)$$

As is familiar from QM, unitarity guarantees that probabilities are conserved in the course of a scattering process.

As discussed above, we have $S = \mathbb{1}$ in the absence of interactions. Hence, the interesting part of the

Scattering process is actually contained in the deviations of the S matrix from the identity.

For this, we define the transition matrix T

$$S = \mathbb{1} + iT \,. \tag{4.29}$$

Unitarity implies:

$$\mathbb{1} = S^\dagger S = (\mathbb{1} - iT^\dagger)(\mathbb{1} + iT)$$

$$= \mathbb{1} + i(T - T^\dagger) + T^\dagger T$$

$$\Rightarrow \quad \frac{1}{i}(T - T^\dagger) = T^\dagger T \tag{4.30}$$

This is nothing but the operator version of the optical theorem. More concretely, let us study its expectation value with respect to an initial state,

$$\frac{1}{i}\left(\langle i|T|i\rangle - \langle i|T^\dagger|i\rangle\right) = \langle i|T^\dagger T|i\rangle = \sum_f \langle i|T^\dagger|f\rangle\langle f|T|i\rangle$$

$$\Rightarrow 2\,\mathrm{Im}\, T_{ii} = \sum_f |T_{fi}|^2 \tag{4.31}$$

If $\mathrm{Im}\, T_{ii} > 0$, the real part of

$$\mathrm{Re}\, S_{ii} = \mathrm{Re}\langle i|S|i\rangle = \mathrm{Re}\langle i|\mathbb{1} + iT|i\rangle < 1$$

is reduced, generically reducing the total $i \to i$ amplitude.

On the other hand, the right-hand side corresponds to the total probability to find nontrivial final states. Heuristically, (4.31) tells us that anything that is scattered out of the initial state must be scattered into another state.
An interesting aspect of the optical theorem is that it interconnects different orders of perturbation theory with each other, because $T \sim \lambda$.

4.3 Wick's theorem

From a technical viewpoint, the computation of S matrix elements corresponds to working out expectation values or transition amplitudes of time-ordered products of field-operators. Though straightforwardly possible, this is not efficient in practice. A better efficiency is provided by Wick's theorem. Let us first introduce some technical concepts:

Def: normal ordering

Let us decompose a field operator into positive and negative frequency components,

$$\phi(x) = \phi^+(x) + \phi^-(x), \qquad (4.32)$$

where $\phi^+ \sim a_{\vec{p}}\, e^{-iE_{\vec{p}}t}$ and $\phi^- \sim a^\dagger_{\vec{p}}\, e^{iE_{\vec{p}}t}$.

We define a normal-ordered product of field operators by sorting all annihilation operators to the right and all creation operators to the left,

$$
\begin{aligned}
N[\mathbb{1}] &= \mathbb{1} \\
N[\varphi] &= \varphi = \varphi^+ + \varphi^- \\
N[\varphi(x_1)\varphi(x_2)] &\equiv N[\varphi(x_2)\varphi(x_1)] = N[\varphi_1\varphi_2] \\
&= \varphi_1^+\varphi_2^+ + \varphi_1^-\varphi_2^+ + \varphi_2^-\varphi_1^+ + \varphi_1^-\varphi_2^- \\
&\vdots \qquad\qquad \varphi_1 := \varphi(x_1)\,,\ \varphi_2 := \varphi(x_2)
\end{aligned}
\tag{4.33}
$$

This immediately implies that the vacuum expectation value of a normal-ordered product vanishes

$$
\langle 0|\, N[\ldots]\,|0\rangle = 0 \quad , \text{ (as long as } (\ldots) \neq c\cdot\mathbb{1}\text{)}
\tag{4.34}
$$

Def. Contraction

We define a contraction of two field operators by the difference between time-ordered and normal-ordered product:

$$
\overbrace{\varphi(x_1)\,\varphi(x_2)} := T[\varphi(x_1)\,\varphi(x_2)] - N[\varphi(x_1)\varphi(x_2)]
\tag{4.35}
$$

The result can be computed straightforwardly:

Let $x_1^0 > x_2^0$:

$$T[\varphi_1 \varphi_2] = \varphi_1^+ \varphi_2^+ + \varphi_1^+ \varphi_2^- + \varphi_1^- \varphi_2^+ + \varphi_1^- \varphi_2^-$$

$$= \varphi_1^+ \varphi_2^+ + \varphi_2^- \varphi_1^+ + \varphi_1^- \varphi_2^+ + \varphi_1^- \varphi_2^- + [\varphi_1^+, \varphi_2^-]$$

$$= N[\varphi_1 \varphi_2] + [\varphi_1^+, \varphi_2^-] \qquad (4.36)$$

Analogously for $x_1^0 < x_2^0$, we have: $T[\varphi_1 \varphi_2] = N[\varphi_1 \varphi_2] + [\varphi_2^+, \varphi_1^-]$

Hence, we have to work out the commutators, e.g. with $x_1 = x$, $x_2 = 0$

$$[\varphi^+(x), \varphi^-(0)] = \int \frac{d^d p}{(2\pi)^d} \frac{1}{\sqrt{2E_{\vec{p}}}} \int \frac{d^d q}{(2\pi)^d} \frac{1}{\sqrt{2E_{\vec{q}}}} e^{-ipx} [a_{\vec{p}}, a_{\vec{q}}^+]$$

$$= \int \frac{d^d p}{(2\pi)^d} \frac{1}{2E_{\vec{p}}} e^{-ipx} = i\Delta^+(x) \qquad (4.37)$$

Similarly $[\varphi^+(0), \varphi^-(x)] = i\Delta^-(x)$

We conclude:

$$\overbrace{\varphi(x_1)\, \varphi(x_2)} = T[\varphi(x_1) \varphi(x_2)] - N[\varphi(x_1) \varphi(x_2)]$$

$$= \theta(x_1^0 - x_2^0)\, i\Delta^+(x_1 - x_2) + \theta(x_2^0 - x_1^0)\, i\Delta^-(x_1 - x_2)$$

$$= i\, \Delta_F(x_1 - x_2) \qquad (4.38)$$

A contraction thus gives us a Feynman propagator!

Wick's theorem generalizes this relation between time ordering, normal ordering and propagators to the case of higher operator products. The theorem can most conveniently formulated in terms of a generating functional.

Wick's theorem:

Let $J(x)$ be an auxiliary function ("source term") and $\Phi(x)$ a field operator in spacetime (spanned by ladder operators), then we have

$$T\, e^{-\int J\Phi} = N\left[e^{-i\int J\Phi}\right] e^{-\frac{1}{2}\iint J\, i\Delta_F J} \qquad (4.39)$$

where $\int J\Phi \equiv \int d^Dx\, J(x)\,\Phi(x)$, $\iint J\Delta_F J \equiv \int d^Dx \int d^Dy\, J(x)\,\Delta_F(x-y)\,J(y)$. (4.40)

Proof:

We need the Baker-Campbell-Hausdorff formula for the case

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]} \qquad \text{for } [A,B] \sim \mathbb{1}. \qquad (4.41)$$

We define a (pseudo)-Hamilton operator $H(t) = \int d^dx\, J(x)\Phi(x)$ and construct the time ordering by discretization in t with step size $\Delta t = t_{i+1} - t_i$. Apart from discretization errors being higher order in Δt, we find

$$T\, e^{-i\int J\varphi} = T\, e^{-i\int dt\, H(t)} = e^{-i\Delta t H(t_n)}\, e^{-i\Delta t H(t_{n-1})} \cdots e^{-i\Delta t H(t_1)}$$

$$= e^{-i\Delta t \sum_i H(t_i) \;-\; \frac{1}{2}\Delta t^2 \sum_{k>j} \underbrace{[H(t_k), H(t_j)]}_{\sim \mathbb{1}}}$$

$$= e^{-i\int J\varphi}\; e^{-\frac{1}{2}\int d^Dx\, d^Dy\, J(x) J(y)\, \underbrace{\theta(x^0-y^0)[\varphi(x),\varphi(y)]}_{= \theta(x^0-y^0)\, i\Delta(x-y) \;=:\; i\Delta_R(x-y)}}$$

$$= e^{-i\int J\varphi}\; e^{-\frac{1}{2}\iint J\, i\Delta_R\, J} \tag{4.41}$$

where we have used the retarded propagator in the last step (c.f. exercises). Next, we study the normal ordered product

$$N\left[e^{-i\int J\varphi}\right] = e^{-i\int J\varphi^-}\, e^{-i\int J\varphi^+}$$

$$\overset{BCH}{=} e^{-i\int J(\varphi^- + \varphi^+) \;-\; \frac{1}{2}\iint J\,[\varphi^-, \varphi^+]\, J}$$

$$\overset{(4.37)}{=} e^{-i\int J\varphi}\; e^{+\frac{1}{2}\iint J\, i\Delta^-\, J} \tag{4.42}$$

In summary, we have

$$T\, e^{-i\int J\varphi} \overset{(4.41)}{=} e^{-i\int J\varphi}\; e^{-\frac{1}{2}\iint J\, i\Delta_R\, J} \overset{(4.42)}{=} N\left[e^{-i\int J\varphi}\right] e^{-\frac{1}{2}\iint J\,(i\Delta_R + i\Delta^-)\, J} \tag{4.43}$$

Since $\Delta_R(x) + \Delta^-(x) = \theta(x^0)\underbrace{\Delta(x)}_{=\Delta^+ - \Delta^-} + \Delta^-(x) = \theta(x^0)\,\Delta^+(x) + \theta(-x^0)\,\Delta^-(x)$

$$= \Delta_F(x) \tag{4.44}$$

which completes the proof.

(4.39) now is a generating functional for expressing arbitrary time ordered products in terms of normal ordered ones and the Feynman propagator:

$$T[\varphi(x_1)\varphi(x_2)\dots\varphi(x_m)] = i^m \frac{\delta}{\delta J(x_1)}\frac{\delta}{\delta J(x_2)}\cdots\frac{\delta}{\delta J(x_m)} T\, e^{-i\int J\varphi}\Big|_{J=0} \qquad (4.45)$$

Using (4.39) and applying the differentiation to the right-hand side yields, e.g. ($\varphi_i = \varphi(x_i)$)

$$T[\varphi_1\varphi_2] = N[\varphi_1\varphi_2] + \overbrace{\varphi_1\varphi_2} = N[\varphi_1\varphi_2] + i\Delta_{F12}$$

$$T[\varphi_1\varphi_2\varphi_3] = N[\varphi_1\varphi_2\varphi_3] + \overbrace{\varphi_1\varphi_2}\varphi_3 + \overbrace{\varphi_1\varphi_2\varphi_3} + \varphi_1\overbrace{\varphi_2\varphi_3}$$

$$T[\varphi_1\varphi_2\varphi_3\varphi_4] = N[\varphi_1\varphi_2\varphi_3\varphi_4] + \overbrace{\varphi_1\varphi_2}N[\varphi_3\varphi_4] + \overbrace{\varphi_1\varphi_3}N[\varphi_2\varphi_4]$$

$$+ \text{4 more terms}$$

$$+ \overbrace{\varphi_1\varphi_2}\,\overbrace{\varphi_3\varphi_4} + \overbrace{\varphi_1\varphi_2\varphi_3\varphi_4} + \overbrace{\varphi_1\varphi_2\varphi_3\varphi_4} \qquad (4.46)$$

Arbitrary T-ordered products can be computed along this line in a purely algebraic manner. This is also easy to automatize using computer algebra.

For expectation values with respect to the free vacuum, the results simplify further:

$$\langle 0|T[\varphi_1\varphi_2\varphi_3\varphi_4]|0\rangle = -(\Delta_{F12}\Delta_{F34} + \Delta_{F13}\Delta_{F24} + \Delta_{F14}\Delta_{F23}) \qquad (4.47)$$

(NB: as long as $\langle 0|\varphi|0\rangle = 0$, i.e. no nonzero field expectation value)