G_2-QCD at Finite Density

A. Wipf

Theoretisch-Physikalisches Institut, FSU Jena

collaboration with

Axel Maas (Jena)

Lorenz von Smekal (Darmstadt/Gießen)

Bjoern Wellegehausen (Gießen)

Christian Wozar (Jena)

Swansea, 07.06.2014

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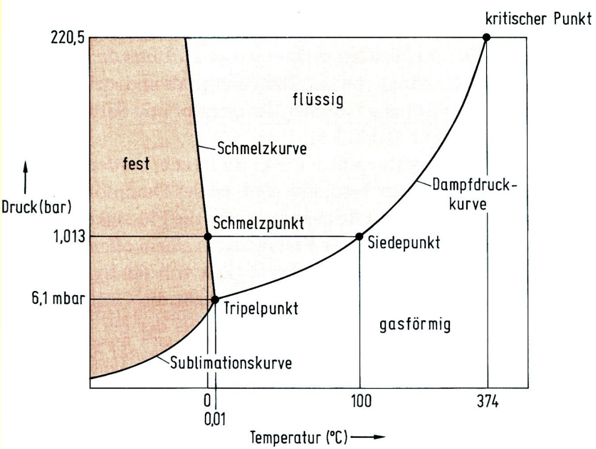
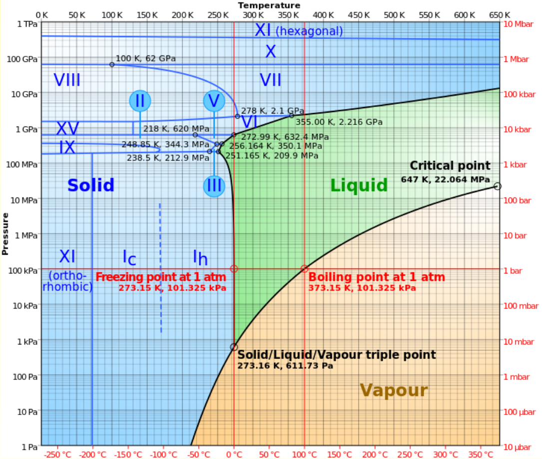
Water



different phases, transition lines, critical point, triple point, ...

that is not all:



- crystalline and amorphous phases
- order of H-binding
- critical point
- triple points
- structural transitions
- small p: hex/cub ice, ice XI
- high p: ice VII, VIII, X

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Some facts about exceptional Lie group G_2

- smallest exceptional Lie group
- rank $=2$, dimension $=14$
- real group (not only pseudo-real!)
- subgroup of SO(7)
- $G_2/SU(3)\sim S^7 \to$ efficient parametrization
- fundamental representations $\{7\},\ \{14\}$ (= adjoint)
  7 quarks instead of 3 (cp. GUTS)
- can be broken to SU(3) with scalars in $\{7\}$
  fermions: $\{7\}\to\{3\}+\{\bar{3}\}+\{1\}$, gauge bosons: $\{14\}\to\{8\}+X$
- smallest (simply connected) Lie group with trivial center

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- singlet representation colorless states

$$
\begin{aligned}
\{7\} \otimes \{7\} &= \{1\} \oplus \{7\} \oplus \{14\} \oplus \{27\} \\
\{7\} \otimes \{7\} \otimes \{7\} &= \{1\} \oplus 4 \cdot \{7\} \oplus 2 \cdot \{14\} \oplus \dots \\
\{14\} \otimes \{14\} &= \{1\} \oplus \{14\} \oplus \{27\} \oplus \dots, \\
\{14\} \otimes \{14\} \otimes \{14\} &= \{1\} \oplus \{7\} \oplus 5 \cdot \{14\} \oplus \dots, \\
\{7\} \otimes \{14\} \otimes \{14\} &= \{1\} \oplus \dots
\end{aligned}
$$

- branching $G_2 \to SU(3)$

$$
\begin{aligned}
\{7\} &\longrightarrow \{3\} \oplus \{\bar{3}\} \oplus \{1\}, \\
\{14\} &\longrightarrow \{8\} \oplus \{3\} \oplus \{\bar{3}\}.
\end{aligned}
$$

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Same facts about G_2 gauge theory

- asymptotically free
- first order confinement/deconfinement PT — Pepe et al.; Greensite; Cossu et al.
- no center, no order parameter
- chiral restoration in quenched theory at same T_c — Graz group
- qualitatively similar glueball spectrum as SU(3) — Wellegehausen, Wozar, AW
- Casimir scaling to high accuracy — Wellegehausen, Wozar, AW; Liptat et al.; Greensite et al.
- topological properties, instantons — Maas, Olejnik, Ilgenfritz

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Why consider G_2 Theories

- G_2 has trivial center:
  confinement models based on center?
- G_2 contains SU(3) as subgroup:
  smooth interpolation between G_2- and SU(3)-gauge theories
- particle spectrum comparable to QCD (mesons and baryons + . . .)
- G_2 has no sign problem:
  simulations at finite T and finite possible μ possible
- G_2-QCD has baryons and mesons:
  can build a "neutron star"

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- understanding of G_2 under extreme conditions
- phases and phase-transition at finite T and n_B
- distinguish phases:
  densities, pressure, energy density, condensates, symmetries, ...

  order parameters ⇔ symmetries
- vary control parameters:

  temperature, chemical potentials, fields,
- physics question:

  what are the relevant degrees of freedom in a given phase?

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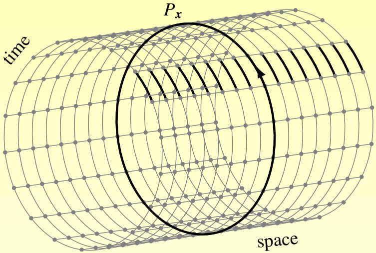
Lattice simulations for SU(3)

- large baryon density:
  sign problem
  $\Rightarrow$ not accessible to simulations based on important sampling
- effective models, functional methods, ...
- proposals/speculations on exotic phases of cold dense matter
- relevant of n^*?
- simulations of theories without sign problem
- even better: solve sign problem?
- here: fast implementation of (local)HMC, no low acceptence rate,

Confinement in pure G_2 gauge theory

finite $T \Rightarrow$
lattice = cylinder with
circumference
$\beta_T = 1/kT = aN_0$.



- approximate order parameter: Polyakov loop

$$P(x) = \mathrm{tr}\,\mathcal{P} \exp\left[i \int_0^\beta d\tau A_0(\tau, x)\right]$$

- static potential

$$\langle P(x)\rangle_\beta = e^{-\beta F(x)}\,, \quad \langle P(x)P^\dagger(y)\rangle_\beta = e^{-\beta V_{q\bar{q}}(R)}$$

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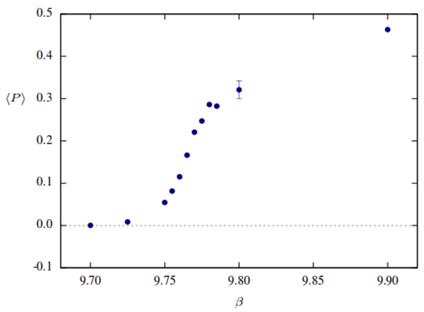
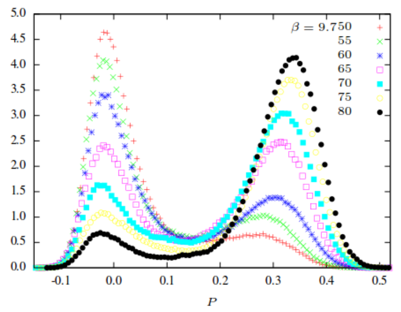
Polyakov loop in fundamental representation





rapid change with $\beta = 1/g^2$ histogramm in vicinity of β_c

- Polyakov loop approximate order parameter
- first order PT as in SU(3) gluodynamics

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- $V_{q\bar{q}}(R)$ static potential

confinement: $\langle P\rangle = 0$, de-confinement: $\langle P\rangle \neq 0$

- confinement: $V \to \sigma R \Rightarrow \langle P(\boldsymbol{x})P^\dagger(\boldsymbol{y})\rangle_\beta \propto e^{-\sigma\cdot\mathrm{Area}}$
- $e^{\sigma\cdot\mathrm{Area}}$ varies over 100 orders of magnitude
- brute force approach does not work
- Lüscher and Weisz method: exponential error reduction
- split lattice in time slices
- calculate $\langle\ldots\rangle$ with fixed bc on each slice
- full result: integral over boundary conditions
- iteration $\to$ *multilevel algorithm*

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- static potential for charges in representation $\mathcal{R}$:

$$V_{\mathcal{R}}(R) = \gamma_{\mathcal{R}} - \frac{\alpha_{\mathcal{R}}}{R} + \sigma_{\mathcal{R}} R$$

- Casimir scaling hypothesis for string tensions:

$$\frac{\sigma_{\mathcal{R}}}{c_{\mathcal{R}}} = \frac{\sigma_{\mathcal{R}'}}{c_{\mathcal{R}'}}$$

- from ratios of Wilson(Polyakov) loops

$$V_{\mathcal{R}}(R) = \frac{1}{\tau} \ln \frac{\langle W_{\mathcal{R}}(R,T)\rangle}{\langle W_{\mathcal{R}}(R,T+\tau)\rangle}.$$

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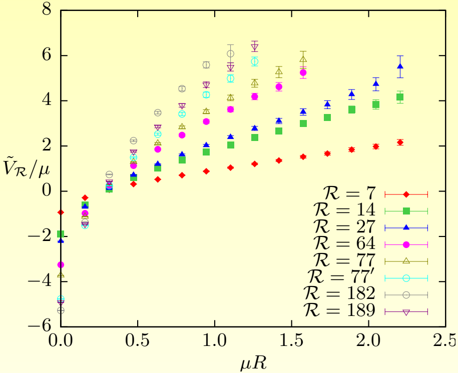

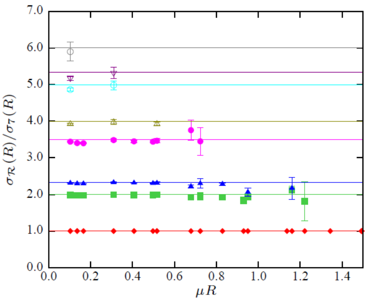

Scaled local string Tension with $\beta = 9.7, 10$ on $14^4, 20^4$

- linear potential for static quarks in different G_2 representations

Wellegehausen, AW., Wozar

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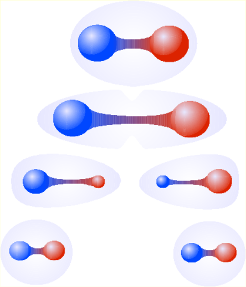
String-breaking

- meson, diquark $\bar{q}q \to$ 2 mesons, diquarks



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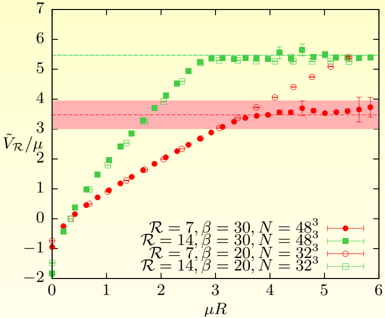
- energy scale = 2 m_{glueball}
- decay products: glue-lumps



Wellegehausen, AW., Wozar (2011)

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G_2 Yang-Mills-Higgs theory

- breaking $G_2 \to SU(3)$
- lattice action with normalized Higgs $\varphi = (\varphi_1, \dots, \varphi)^T$ in $\{7\}$

$$S_{\text{YMH}}[\mathcal{U}, \varphi] = -\frac{1}{g^2}\sum \mathcal{U}_\Box - \kappa \sum \varphi_x^T \mathcal{U}_{x,\mu}\varphi_{x+\mu}$$

- Higgs-mechanism for $v = \langle\varphi\rangle \neq 0$:
- $\{14\} \longrightarrow \{8\} \oplus \{3\} \oplus \{\bar{3}\}$

  $\{8\}$: SU(3) gluons

  $\{3\} + \{\bar{3}\}$: massive Vector bosons
- scalars $7 \to 1$

- $\kappa = 0$: pure G_2 gauge theory:
  first order deconfinement transition
- $\kappa = \infty$: 6 vector bosons decouple, pure $SU(3)$
  first order deconfinement transition
- first order transition line connecting two theories?
- calculate
  Polyakov loop and plaquette actions, susceptibilities
  on grid in $\beta \propto 1/g^2$, κ-plane ($\beta = 5 \ldots 10$, $\kappa = 0 \ldots 10^4$) $\Rightarrow$

Phase diagram of G_2 YMH theory

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average actions and susceptibilities (small lattice)

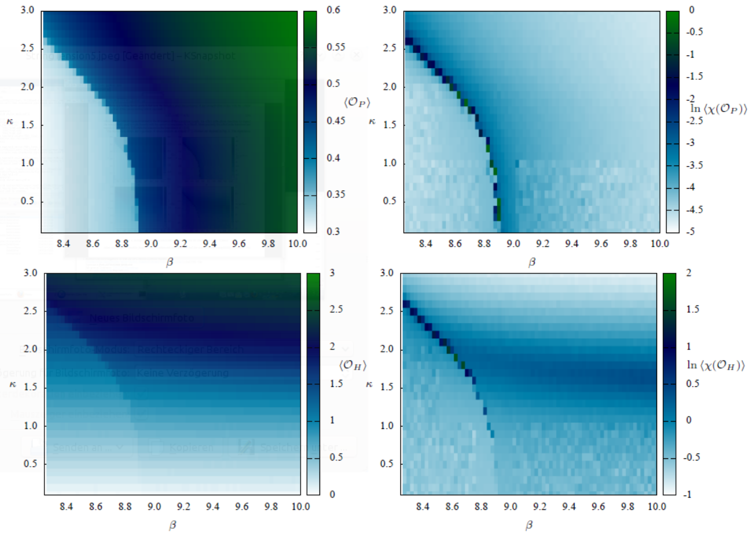

FIG. 4: Average plaquette, Higgs action and susceptibilities near the critical point on $6^3 \times 2$ lattice.

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Results

- average plaquette action, Higgs action and Polyakov loop
- susceptibilities (and higher derivatives) and finite size analysis
- large $\beta \propto 1/g^2 \Rightarrow$ Higgs transition line
  cluster algorithm for SO(7) nonlinear sigma model
  line of second order PT $O(7) \to O(6)$
- line of first order PT $G_2 \to$SU(3) with small gap in between
- triple point

$$\beta_{\text{crit}} = 9.55(5) \quad , \quad \kappa_{\text{crit}} = 1.50(4)$$

- first order (almost?) line hits second order line
- confining phase meets two deconfining phases

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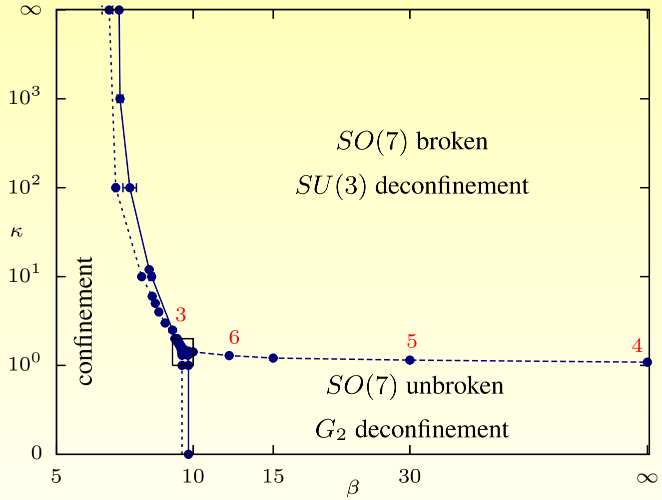

phase diagram ($16^3 \times 6$): global picture

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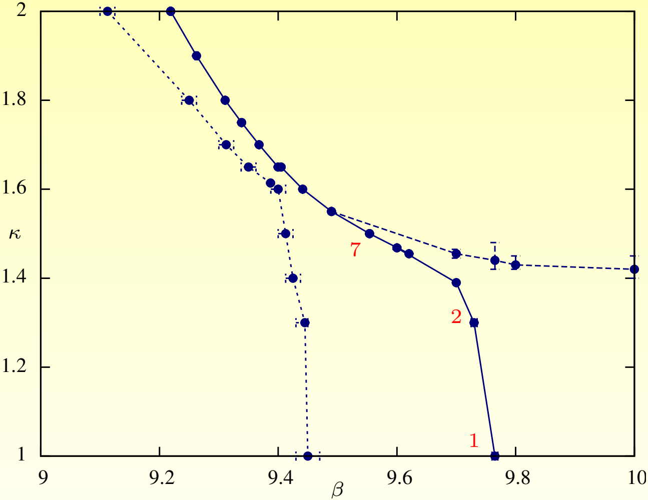

phase diagram ($16^3 \times 6$): where the lines almost meet

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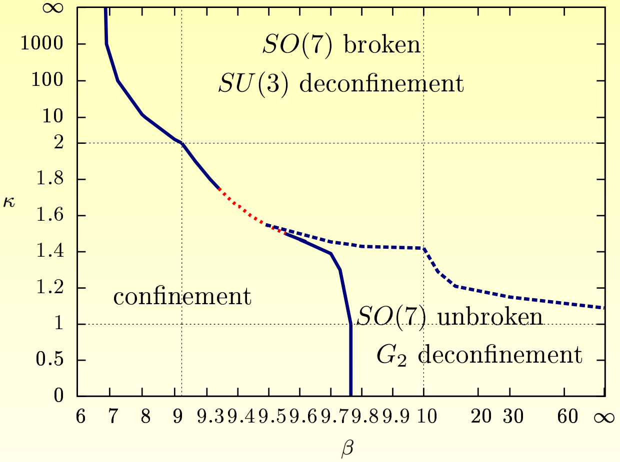

phases of G_2 YMH-theory

Wellegehausen, Wozar, AW

G_2 QCD with dynamical fermions

collaboration with Axel Maas, Lorenz von Smekal und Bjoern Wellegehausen

- fermionic determinant real and positive
- no sign problem: simulations at finite T and μ
- expected particle spectrum:

  glueballs

  bosonic quark-quark bound states (mesons, diquarks)

  fermionic 3 quark states (baryons)

  fermionic 1 quark - 2 gluon bound states (fermion hybrids)

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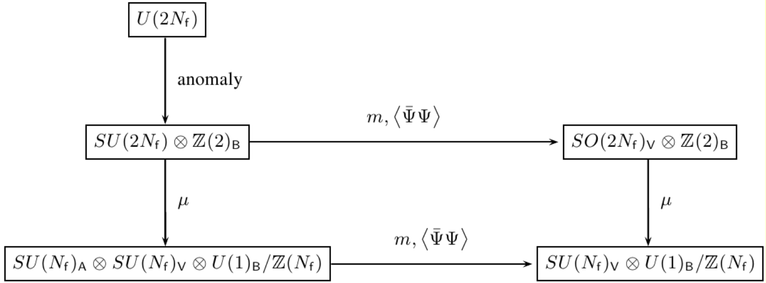
- confinement-deconfinement transition (Polyakov loop)
- chiral symmetry breaking (chiral condensate)
- quenched: same critical temperatures — Maas, Gattringer
- G_2 has fermionic baryons

  degenerate Fermigas at large ρ_B

  relevant for physics of compact 'stellar objects'
- Bose-condensates of diquarks, ...
- results from (expensive) numerical simulations

G_2-QCD: global symmetries and breaking



- anomalous
- spontaneous $\langle \bar{\psi}\psi \rangle$
- explicit m, μ

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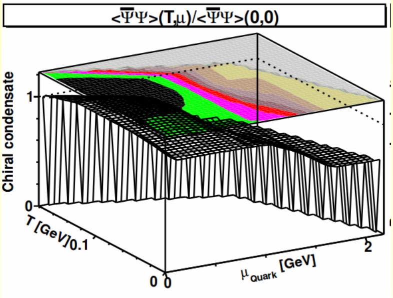
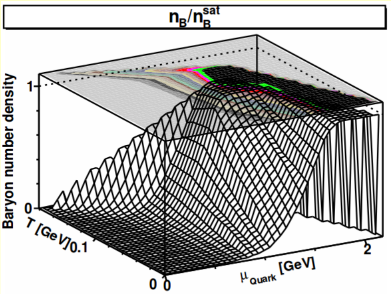
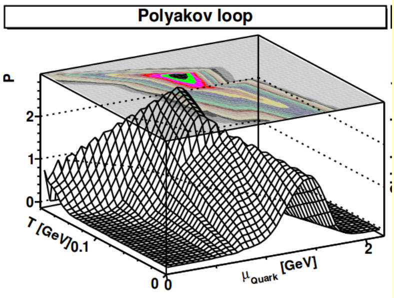
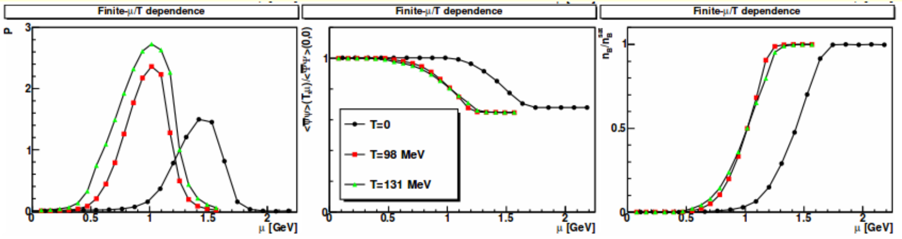
first results on $\langle\bar{\psi}\psi\rangle$, n_B and $\langle P\rangle$







Polyakov loop
P
2
1
0
T [GeV]
0.1
0 0
μ_{Quark} [GeV]
2



Finite-μ/T dependence
P
3
2
1
0
0.5
1
1.5
2
μ [GeV]
Finite-μ/T dependence
$<\bar{\Psi}\Psi>(T,\mu)/<\bar{\Psi}\Psi>(0,0)$
1
0
0.5
1
1.5
2
μ [GeV]
T=0
T=98 MeV
T=131 MeV
Finite-μ/T dependence
n_B/n_B^{sat}
1
0.5
0
0.5
1
1.5
2
μ [GeV]

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Spectroscopy I

mesons (baryon number 0)

Name	$\mathcal{O}$	T	J	P	C
π	$\bar{u}\gamma_5 d$	SASS	0	-	+
η	$\bar{u}\gamma_5 u$	SASS	0	-	+
a	$\bar{u}d$	SASS	0	+	+
f	$\bar{u}u$	SASS	0	+	+
ρ	$\bar{u}\gamma_\mu d$	SSSA	1	-	+
ω	$\bar{u}\gamma_\mu u$	SSSA	1	-	+
b	$\bar{u}\gamma_5\gamma_\mu d$	SSSA	1	+	+
h	$\bar{u}\gamma_5\gamma_\mu u$	SSSA	1	+	+

exotic particles (baryon number 1)

Name	$\mathcal{O}$	T	J	P	C
N'	$T^{abc}(\bar{u}_a\gamma_5 d_b)u_c$	SAAA	1/2	$\pm$	$\pm$
Δ'	$T^{abc}(\bar{u}_a\gamma_\mu u_b)u_c$	SSAS	3/2	$\pm$	$\pm$
Hybrid	$\epsilon_{abcdefg}u^a F^{bc}_{\mu\nu}F^{de}_{\mu\nu}F^{fg}_{\mu\nu}$	SSSS	1/2	$\pm$	$\pm$

T: (x, s, C, F)

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diquarks (baryon number 2)

Name	$\mathcal{O}$	T	J	P	C
$d(0^{++})$	$\bar{u}^C\gamma_5 u + c.c.$	SASS	0	+	+
$d(0^{+-})$	$\bar{u}^C\gamma_5 u - c.c.$	SASS	0	+	-
$d(0^{-+})$	$\bar{u}^C u + c.c.$	SASS	0	-	+
$d(0^{--})$	$\bar{u}^C u - c.c.$	SASS	0	-	-
$d(1^{++})$	$\bar{u}^C\gamma_\mu d - \bar{d}^C\gamma_\mu u + c.c.$	SSSA	1	+	+
$d(1^{+-})$	$\bar{u}^C\gamma_\mu d - \bar{d}^C\gamma_\mu u - c.c.$	SSSA	1	+	-
$d(1^{-+})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{d}^C\gamma_5\gamma_\mu u + c.c.$	SSSA	1	-	+
$d(1^{--})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{d}^C\gamma_5\gamma_\mu u - c.c.$	SSSA	1	-	-

baryons (baryon number 3)

Name	$\mathcal{O}$	T	J	P	C
N	$T^{abc}(\bar{u}_a^C\gamma_5 d_b)u_c$	SAAA	1/2	$\pm$	$\pm$
Δ	$T^{abc}(\bar{u}_a^C\gamma_\mu u_b)u_c$	SSAS	3/2	$\pm$	$\pm$

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spectroscopy II

- diquark masses are degenerate
- contain only connected contributions (as pions in QCD)
- $m_\eta - m_{\text{diquark}} =$ disconnected contributions
- tree level improved Szymanzik action
- Wilson fermions (chiral properties?)
- no sources for diquarks needed
- N_F complex-valued pseudo-fermions plus RHMC
- two time-scale integration (Sexton-Weingarten and leapfrog)
- further optimizations (preconditioning, adaptive mesh, . . .)

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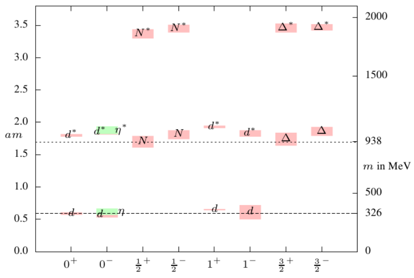
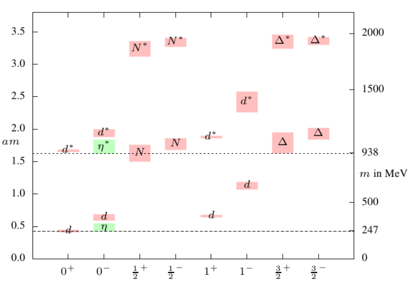
masses of mesons, diquarks, baryons

Ensemble	β	κ	$m_{d(0+)}a$	$m_N a$	$m_{d(0+)}$ [MeV]	a [fm]	a^{-1} [MeV]	MC
Heavy	1.05	0.147	0.59(2)	1.70(9)	326	0.357(33)	552(50)	7K
Light	0.96	0.159	0.43(2)	1.63(13)	247	0.343(45)	575(75)	5K



heavy ensemble



light ensemble

Wellegehausen, Maas, Smekal, AW (2013)

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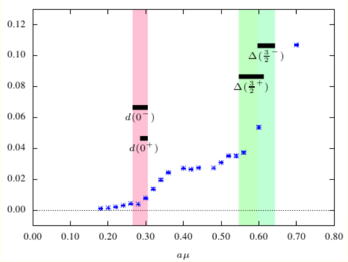
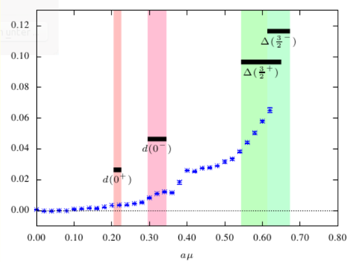
zooming in in: baryon density vs. chemical potential





- n_q grows rapidly at half of (0^-) mass (silver blaze)
- plateaus visible for larger n_q
- three transitions
- phase between $\mu_q = 300 - 600$ MeV: hadronic phase

  $\rightarrow$ quasi particle picture

Wellegehausen, Maas, Smekal, AW (2013)

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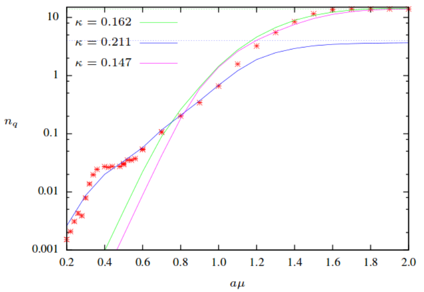
comparison with fermi gas of (Wilson) fermions



- fit above $a\mu = 1 \Rightarrow \kappa = 0.162,\ n_f^{\text{sat}} = 14.4$

  cp. with free quarks: $\kappa = 0.147,\ n_f^{\text{sat}} = 14 \Rightarrow$ saturation regime
- fit below $a\mu = 1 \Rightarrow \kappa = 0.211,\ n_f^{\text{sat}} = 4.02$

  cp. lattice gas of freee $\Delta-$baryons: $n_f^{\text{sat}} = 4$
- $0.6 \leq a\mu \leq 1$: n_q due to fermionic baryons (same as spectroscopy)

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(preliminary) interpretation

- low density: in accordance with silver blaze
  clean signal (no diquark sources)
- two small jumps at diquark thresholds
  $\Rightarrow$ two (probably) second order PT?
- two plateaus after thresholds
- Bose-condensates of diquarks?
  admixture with gas of diquarks?
- one (probably) first order PT at $\approx \Delta$ threshold
  simulations slow down near PT
- hadronic phase for higher n_q (under investigation)
- $a\mu \gtrsim 1$: lattice artifacts, e.g. saturation effects

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Summary

- G_2 QCD is a useful laboratory
- accessible at finite density by lattice methods
- phases and transition at high densities and temperatures
- condensates, access to hadronic phase
- access to dense baryonic matter (as in n^*)
- shares many features with real QCD
- interpolation G_2-QCD $\rightarrow$ QCD with Higgs-mechanism possible
- full phase diagram in principle accessible to simulations

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Outlook

- clarify further nature of dense and cold phases
- include finite temperature effects beyond rough overview
- follow first order transition line (critical end point?)
- break G_2-QCD $\to$ QCD with quarks via Higgs-mechanism
- deformation vs. sign problem?
- testbed for model building
- testbed for alternative approaches (eg. renormalization group)?

Thanks for your attention

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