Supersymmetric Flows for Supersymmetric Field Theories

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Supersymmetry (susy) and ERGE

- particle physics beyond Standard Model $\Longrightarrow$ supersymmetry

  bosons $\Longleftrightarrow$ fermions
- susy-breaking $\sim$ collective condensation phenomena
  $\Longrightarrow$ non-perturbative methods
- lattice simulations:

  susy (partially) broken by spacetime lattice, dynamical fermions, ...

  $\Longrightarrow$ need complement to lattice studies
- exact renormalization group?

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Challenges

- manifest supersymmetric renormalization flow

  e.g. $m_{\text{boson}} = m_{\text{fermion}}$
- dynamical susy breaking $\Longrightarrow$ phase transitions
  fixed-point structure, temperature effects, equation of state
- non-renormalization theorems — talk of Synatschke-Czerwonka
- relevant dof at low energies? (cp. Veneziano-Yankielowicz)

related (mostly structural) investigations by:

Sonoda; Bonini & Vian; Falkenberg & Geyer;

Arnone & Yoshida; Arnone& Guerrieri & Yoshida;

Rosten, Sonoda & Ulker; Horiskoshi & Aoki & Taniguchi & Terao

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Supersymmetric Yukawa (Wess-Zumino) models

- minimal supersymmetry $\Rightarrow$ real fields (ϕ, ψ, F): Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}F^2 + \frac{1}{2}W''(\phi)\bar{\psi}\gamma_*\psi - W'(\phi)F$$

- action invariant under susy transformation:

$$\delta\phi = \bar{\varepsilon}\gamma_*\psi, \quad \delta\psi = (F + i\gamma_*\partial\!\!\!/\phi)\varepsilon, \quad \delta F = i\bar{\varepsilon}\partial\!\!\!/\psi$$

- eliminate auxiliary field $F \Longrightarrow$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}W'^{\,2}(\phi) + \frac{1}{2}W''(\phi)\,\bar{\psi}\gamma_*\psi$$

- classical Yukawa-model determined by superpotential W
- $W(\phi) \sim \phi^m$: m even: susy always unbroken
  m odd: susy breaking possible

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- need supersymmetric regulator for flow equation (Wetterich 93)

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \partial_k R_k \right\}$$

- superspace formulation $\Longrightarrow$
  general cutoff R_k depends on two functions $r(p^2)$ and $s(p^2)$
- choose $s(p^2) = 0 \Longrightarrow$

$$\Delta S_k = \frac{1}{2} \int d^d x \left(\phi\, p^2 r(p^2)\, \phi - \bar{\psi}\, \not{p} r(p^2)\, \psi - r(p^2)\, F^2 \right)$$

- susy relates 3 cut-off functions
- here: 2 and 3 dimensions
- model with extended supersymmetry — talk of Synatschke-Czerwonka

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Flow of super potential

- this talk: mainly local potential approximation — NLO: Synatschke, Gies, Wipf

$$\Gamma_k = \int d^d x \left(\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \tfrac{1}{2}F^2 + \tfrac{1}{2}W_k''(\phi)\bar{\psi}\gamma_*\psi - W_k'(\phi)F \right)$$

- project flow to $F \Longrightarrow$ flow equation for $W_k(\phi)$:

$$\partial_k W_k(\phi) = -\frac{k^{d-1}}{A_d}\frac{W_k''(\phi)}{k^2 + W_k''(\phi)^2}, \qquad A_2 = 4\pi,\ A_3 = 8\pi^2$$

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Fixed point structure in 2 dimensions

- dimensionless quantities $k \to t,\; W_k(\phi) = k w_t(\phi)$

$$\partial_t w_t(\phi) + w_t(\phi) = -\frac{1}{4\pi}\frac{w_t''(\phi)}{1 + w_t''(\phi)^2}$$

- allow for susy-breaking: even w_t'

$$w_t'(\phi) = \lambda_t(\phi^2 - a_t^2) + b_{4,t}\phi^4 + b_{6,t}\phi^6 + \dots$$

$\Longrightarrow$ system of coupled ODE's

- a_t does not enter equations for higher order couplings $\lambda_t, b_{2i,t}$
- fixed point analysis: $(a^*)^2 = 1/2\pi$, a_t^2 : always IR-unstable

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$2d$-fixed points continued

- keep terms up to $b_{2n,t}\phi^{2n} \Longrightarrow$ $2n$ non-Gaussian fixed points

$$\pm\left(\lambda_p^*, b_{4,p}^*, \dots, b_{2n,p}^*\right), \quad p = 1, \dots, n$$

- ordering $\lambda_n^* > \lambda_{n-1}^* > \dots \Longrightarrow$

  λ_n^* : 1 IR-unstable direction a_t^2

  λ_{n-1}^* : 2 IR-unstable directions

  λ_{n-2}^* : 3 IR-unstable directions ...

- root belonging to IR-stable fixed point $\lambda_n^* \stackrel{n\to\infty}{\longrightarrow} \lambda_{\text{crit}} = 0.9816$

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λ^*	$\Re(\theta^I)$ of non-Gaussian fixed points, truncation at 2n=16							
$\pm.9816$	-1.54	-7.43	-18.3	-37.3	-68.9	-120	-204	-351
$\pm.8813$	6.16	-1.64	-9.82	-25.6	-52.5	-96.9	-170	-300
$\pm.7131$	21.4	4.37	-1.57	-11.1	-30.1	-63.3	-120	-223
$\pm.5152$	28.7	13.3	3.33	-1.39	-11.6	-32.8	-71.7	-145
$\pm.3158$	20.0	20.0	8.40	2.57	-1.14	-11.6	-34.3	-80.4
$\pm.1437$	11.2	11.2	8.63	5.19	1.95	$-.842$	-11.1	-35.7
$\pm.0322$	4.20	4.20	2.86	2.72	2.72	1.47	$-.540$	-10.5
$\pm.0003$	1.57	1.57	1.43	1.43	1.14	.542	.542	-0.221

- odd solutions of nonlinear ODE for $u(\phi) = w_*''(\phi)$:

$$(1-u^4)u'' = 2u'^2(3-u^2)u - (1-u^2)^3 4\pi u$$

- periodic solutions for $u'(0) \leq 2\lambda_{\text{crit}},\ \lambda_{\text{crit}} = 0.982$
  previous polynomials converge to periodic solution
- $u'_{\text{crit}}(0) = 2\lambda_{\text{crit}}$: IR-stable fixed point, finite for $|\phi| \leq 0.5823$

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next-to leading-order flows

- wave function renormalization $\Longrightarrow \eta = -\partial_t \log Z_k^2$
- θ^0 critical exponent of relevant direction a_t^2 (related to W)
- new superscaling relation (exact in NLO)

$$\nu_w = \frac{1}{\theta_0} = \frac{d-\eta}{2}$$

superscaling relation at maximally IR-stable fixed point (d=2)

$2n$	2	4	6	8	10	12	14
η	0.3284	0.4194	0.4358	0.4386	0.4388	0.4387	0.4386
$1/\nu_W$	0.8358	0.7903	0.7821	0.7807	0.7806	0.78065	0.7807

- number of IR-unstable direction $=$ number of nodes of u plus 1

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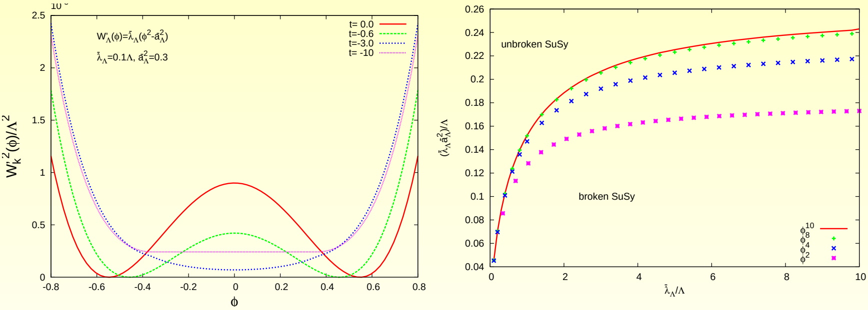
Supersymmetry breaking

- supersymmetric phase: $\min_\phi V_{k=0}(\phi) = \min_\phi W'^2_{k=0}(\phi) = 0$
- susy broken: $W'_{k=0}(\phi)$ has no node



left: flow of a potential $V = W'^2$ with susy breaking, $W'_\Lambda(\phi) = \bar\lambda_\Lambda(\phi^2 - \bar a^2_\Lambda)$
right: phase diagram for couplings specified at Λ, different truncations.

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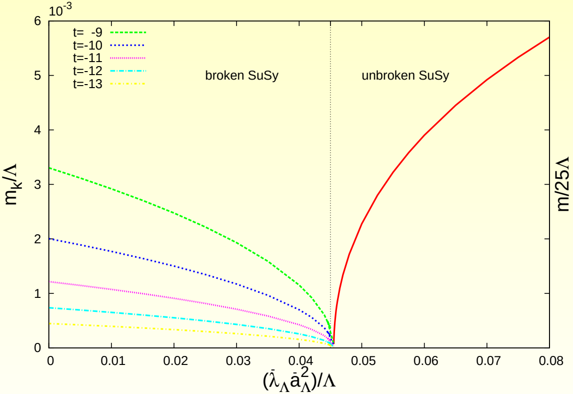
masses of bosons and fermions

- supersymmetric phase: $Z_k^4 m^2_{k,\mathrm{boson}} = W_k''^{\,2}(\chi_{\min}/Z_k) = Z_k^4 m^2_{k,\mathrm{fermion}}$
- broken phase:(superscaling) $Z_k^4 m^2_{k,\mathrm{boson}} = W_k'(0)\,W_k'''(0) \sim k^{1+\eta/2}$



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Wess-Zumino model in 3 dimensions

with J. Braun and F. Synatschke-Czerwonka

- one Wilson-Fisher fixed point for Yukawa-model
- LPA, polynomial expansion

Wilson-Fisher fixed point from polynomial expansion

$2n$	$\pm\lambda^*$	$\pm b_4^*$	$\pm b_6^*$	$\pm b_8^*$	$\pm b_{10}^*$	$\pm b_{12}^*$
4	1.546	2.305				
6	1.590	2.808	6.286			
8	1.595	2.873	7.150	13.41		
10	1.595	2.873	7.155	13.48	1.212	
12	1.595	2.870	7.118	12.90	-8.895	-183.3

- rapid convergence (contrary to 2 dimensions)
- a_t^2 defines the only IR-unstable direction

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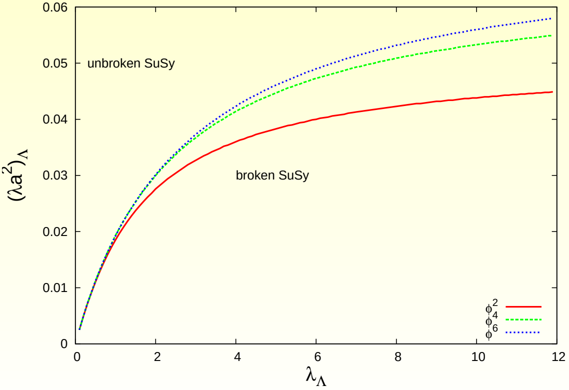
$2n$	critical exponents for different truncations								
6	−0.799	−5.92	−20.9						
8	−0.767	−4.83	−14.4	−38.2					
10	−0.757	−4.35	−11.5	−26.9	−60.8				
12	−0.756	−4.16	−9.94	−21.4	−43.8	−89.0			
14	−0.756	−4.10	−9.13	−18.3	−35.1	−65.4	−123		
16	−0.756	−4.08	−8.72	−16.4	−29.9	−52.9	−91.9	−163	
18	−0.756	−4.08	−8.54	−15.2	−26.4	−45.0	−75.0	−124	−209

- phase diagram from parameter study of $W'_{k\to 0}$



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Finite temperature

- $\int dp_0 \longrightarrow$ summation over Matsubara frequencies
- sums can be calculated explicitly $\Longrightarrow$ two flow equations

$$\partial_k W_k'^{\,\mathrm{bos}} = -\frac{k^2}{8\pi^2} W_k''' \frac{k^2 - W_k''^2}{(k^2 + W_k''^2)^2} \times F_{\mathrm{bos}}(T,k)$$

$$\partial_k W_k'^{\,\mathrm{ferm}} = -\frac{k^2}{8\pi^2} W_k''' \frac{k^2 - W_k''^2}{(k^2 + W_k''^2)^2} \times F_{\mathrm{ferm}}(T,k)$$

- susy breaking by thermal fluctuations (bosons $\neq$ fermions)
- $T = 0$: susy broken $\longleftrightarrow$ $\mathbb{Z}_2$ unbroken

  $\Longrightarrow$ study $\mathbb{Z}_2$ breaking at finite T

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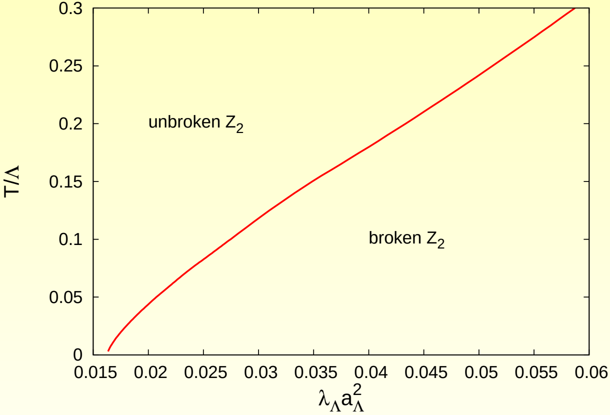
Phase diagram



finite-temperature phase diagram for fixed $\lambda_\Lambda = 0.8$

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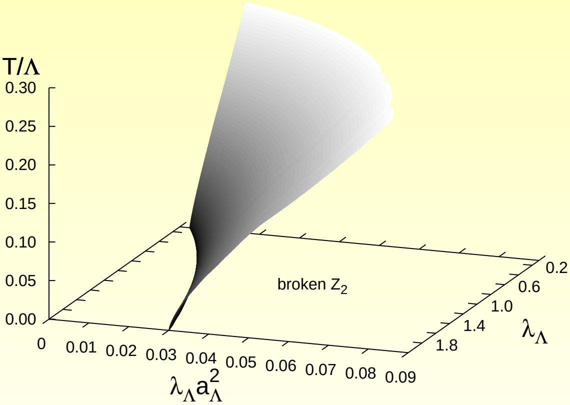
Phase diagram, continued



finite-temperature phase diagram

Future

- supersymmetric $O(n)$ and $CP(n)$ models
  lattice $O(3) \sim CP(1)$ — R. Flore, D. Körner, C. Wozar
  flow equation for large n — M. Masthaler, F. Synatschke-Czerwonka
- supersymmetric gauge theories – first studies
  lattice — B. Wellegehausen
  flow equation — F. Synatschke-Czerwonka

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