On SU(2N) and G2 – Gauge Theories

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Graz, May 13th, 2009

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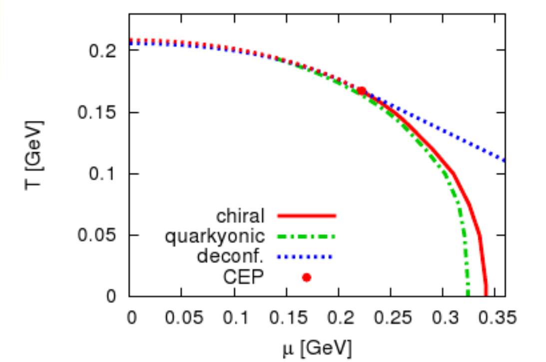
Fermionic BC in SU(N) gauge theories

- finite μ and T:
  large N_c-expansion suggests:
  phase with confinement and chiral symmetry?
- quarkionic phase: Fermi sphere of quarks



Phase diagram for 3 colors
(McLerran et al.)
Solid: first order
dashed: cross over

Fermi surface vs. boundary conditions

- Fermi surface for quasi-free quarks crucial
- $SU(N)$ QCD-like theories different, depending on N even/odd
- Dirac fermions: $\psi(x^0 + \beta, \boldsymbol{x}) = -\psi(x^0, \boldsymbol{x})$

$$\rho_{a,\mathrm{free}}(E) \propto \frac{1}{e^{\beta(E-\mu)}+1}$$

$\longrightarrow$ Fermi surface

- charged bosons: $\phi(x^0 + \beta, \boldsymbol{x}) = +\phi(x^0, \boldsymbol{x})$

$$\rho_{p,\mathrm{free}}(E) \propto \frac{1}{e^{\beta(E-\mu)}-1}$$

$\longrightarrow$ no Fermi surface

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Fermionic determinant

- partition function

$$\mathcal{Z} = \int d\mathcal{Q}\, \mathcal{Q}\, P_a(\mathcal{Q})$$

- $P_a(\mathcal{Q})$ probability distribution for quark determinant

$$P_a(\mathcal{Q}) = \int \mathcal{D}U_\mu\, \delta(\mathcal{Q} - \det_{\mathrm{a}} M[U])\, e^{-S_{\mathrm{YM}}[U]}$$

- non-periodic 'gauge transformation'

$$\begin{aligned} U_\mu^\Omega(x) &= \Omega(x) U_\mu(x) \Omega^\dagger(x+\mu) \\ q^\Omega(x) &= \Omega(x) q(x) \\ \Omega(x_0 + N_t a, \boldsymbol{x}) &= Z\Omega(x_0, \boldsymbol{x}), \quad Z \in \text{center} \end{aligned}$$

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Center transformation

- U_μ periodic $\rightarrow$ U_μ^Ω periodic
- $S_{\rm YM}[U]$ and $\mathcal{D}U_\mu$ invariant
- Fermionic determinant only covariant

$$M[U]q_n = \lambda q_n \quad \Longrightarrow \quad M[U^\Omega]q_n^\Omega = \lambda q_n^\Omega$$
$$q_n \text{ anti-periodic} \quad \Longrightarrow \quad q_n^\Omega(x_0 + N_t a) = -Z q_n^\Omega(x_0)$$

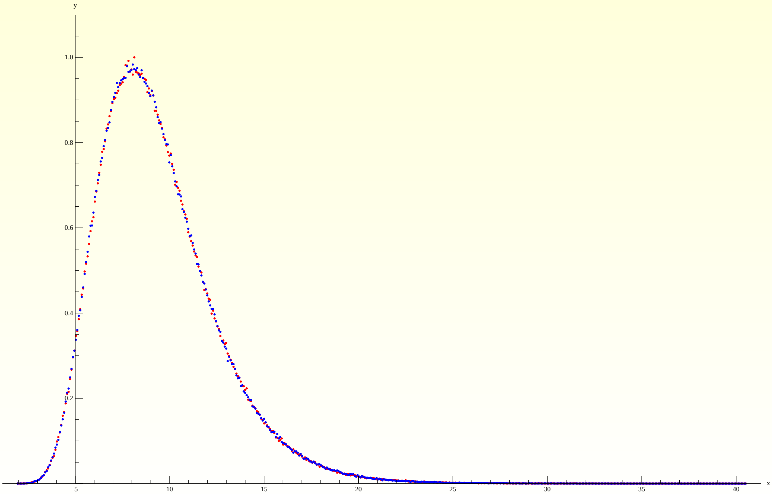
- gauge group $SU(N_c)$ with N_c even $\Rightarrow$ can choose $Z = -\mathbb{1}$

$$\det M_a[U] = \det M_p[U^\Omega]$$

$$P_a(\mathcal{Q}) = \int \mathcal{D}U_\mu \, \delta(\mathcal{Q} - \det_p M[U^\Omega]) \, e^{-S_{\rm YM}[U]} = P_p(\mathcal{Q})$$

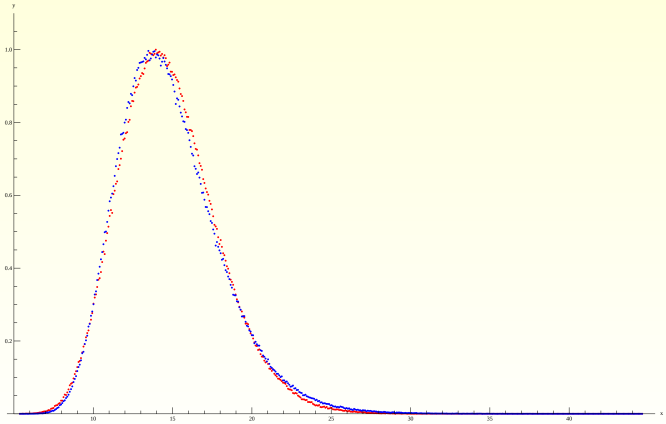
Distribution of det $\not{D}$ for SU(2), 6^3-lattice, 10^6 configurations

red = periodic, blue = antiperiodic



Distribution of det $\not{D}$ for SU(3), , 6^3-lattice, 10^6 configurations

red = periodic, blue = antiperiodic



Distribution of det $\not{D}$ for SU(4), , 6^3-lattice, 10^6 configurations

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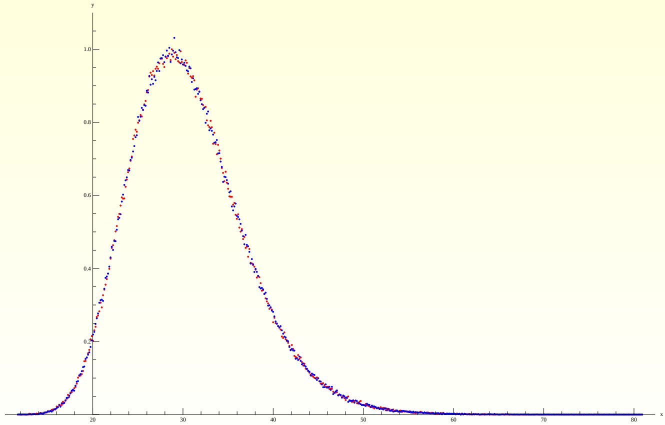

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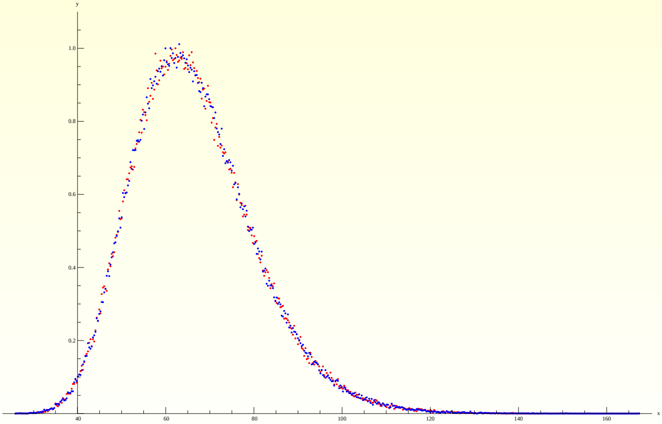
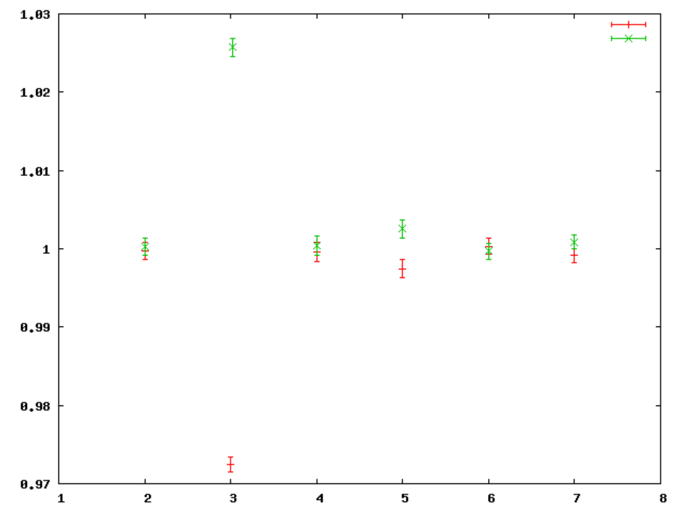
Distribution of det $\not{D}$ for SU(5), , 6^3-lattice, 10^6 configurations

red = periodic, blue = antiperiodic



asymmetries for SU(N), , small-lattice, 10^6 configurations

red $= \langle \det M_p \rangle / \Sigma$, green $= \langle \det M_a \rangle / \Sigma$, $\quad \Sigma = \frac{1}{2}(\langle \det M_p \rangle + \langle \det M_a \rangle)$



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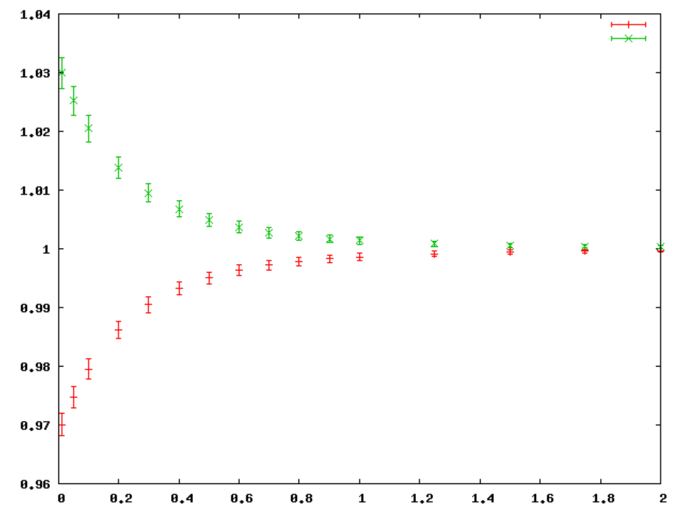
asymmetries for SU(3), , small-lattice, 10^6 configurations

red $= \langle \det M_p \rangle / \Sigma$, green $= \langle \det M_a \rangle / \Sigma$, dependence on fermion mass



Gauge group SU(N_c), with N_c even

- finite volume $\Rightarrow \mathcal{Z}$ same for periodic/antiperiodic quark b.c.
- still true for infinite volume, confining phase
- antiperiodic b.c. $\Rightarrow$ ingredient for Fermi sphere in dense phase
- SU(2N) QCD-like theories:

  quark Fermi surface unlikely to exist in confining phase
- supported by exact solutions of Thirring-model (twisted b.c.)

$$\int \mathcal{D}(\text{fields})\, e^{-S[A,\bar{\psi},\psi]}, \quad \text{const}(A_0) \in [0, 2\pi]$$

- similar reasoning for μ-independence in $U(N)$-gauge theories

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Deconfinement in G_2 Gauge Theory: Holland, Minkowski, Pepe, Wiese

- smallest simply connected gauge group with trivial center
- rank = 2, dimension = 14, subgroup of $SO(7)$
- fundamental representations $\{7\}$, $\{14\}$ (= adjoint)
- Weyl-group is D_6, order 12
- role of center symmetry for confinement/deconfinement?
- instantons, monopoles, center vortices, ...
- evidence for first-order deconfinement PT at T_c (Bern group)
- chiral restoration at same T_c (Graz group)

G_2-representations

- lowest representations:

V	[1, 0]	[0, 1]	[2, 0]	[1, 1]	[0, 2]	[3, 0]	[4, 0]	[2, 1]	[0, 3]
dim	7	14	27	64	77	77′	182	189	273
C_2	12	24	28	42	60	48	72	64	108

- confinement, string breaking

$$\{7\} \otimes \{7\} = \{1\} \oplus \{7\} \dots \quad \text{mesons}$$

$$\{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus 4 \cdot \{7\} + \dots \quad \text{baryons}$$

$$\{14\} \otimes \{14\} = \{1\} \oplus \{14\} \dots$$

$$\{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \{7\} \dots \quad \text{string breaking}$$

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Spontaneous symmetry breaking $G_2 \to SU(3)$

- G_2 gauge-Higgs model

$$\mathcal{L}_{\rm GH}[A,\varphi] = \mathcal{L}_{\rm YM}[A] + \frac{1}{2} D_\mu\varphi D_\mu\varphi + V(\varphi)$$

- $\varphi = (\varphi_1, \dots, \varphi)^T$ in $\{7\}$

$$V(\varphi) = \lambda(\varphi^2 - v^2)^2$$

- Higgs-mechanism for $v = \langle\varphi\rangle \neq 0$: $G_2 \longrightarrow SU(3)$
- $\{14\} \longrightarrow \{8\} \oplus \{3\} \oplus \{\bar{3}\}$ $\{8\}$: SU(3) gluons
  $\{3\} + \{\bar{3}\}$: massive, play similar role as SU(3) quarks

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Phases

- lattice action, $\lambda \to \infty$ and rescaling $\Longrightarrow |\varphi_x| = 1$

$$S_{\rm GH}[\mathcal{U},\varphi] = -\frac{1}{g^2}\sum \mathcal{U}_\Box - \kappa \sum_{x,\mu} \varphi_x^T \mathcal{U}_{x,\mu}\varphi_{x+\mu}$$

- lattice-parameter $1/g^2$ (effective temperature)
  hopping parameter κ
- $\kappa = 0$: G_2 Yang-Mills theory
- $\kappa = \infty$: SU(3) gauge theory
- $m_{\{3\}}$ and $m_{\{\bar{3}\}}$ increase with κ
- $1/g^2 = \infty$: SO(7) and SO(6) spin model

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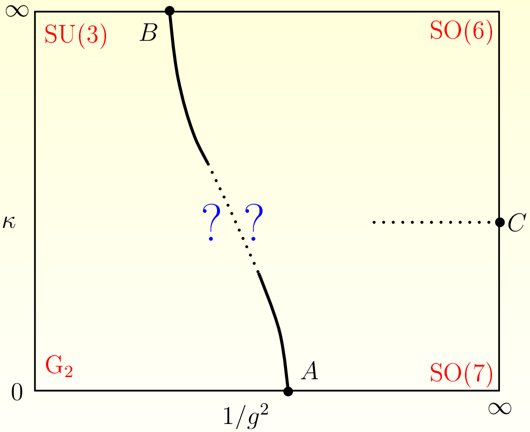
Expected phase diagram



symmetry-driven, number of degrees of freedom

Polyakov loop

- Polyakov loop

$$P(\boldsymbol{x}) = \exp\left(-\beta F\right) = \operatorname{tr}\mathcal{P}\exp\left(\int_0^\beta dt A_0(t,\boldsymbol{x})\right)$$

- $\longrightarrow$ free energy of static quark. On lattice

$$P_{\boldsymbol{x}} = \operatorname{tr}\mathcal{P}_{\boldsymbol{x}}\,,\quad \mathcal{P}_{\boldsymbol{x}} = \prod_t \mathcal{U}_{(t,\boldsymbol{x}),0}$$

- SU(3): P order parameter for $\mathbb{Z}_3$
  symmetry reason for deconfinement PT
- G_2: P not order parameter
  no symmetry reason for deconfinment PT

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- pure SU(3):
  $\langle P \rangle = 0$ in confining phase
  linear rising potential, no string breaking
- SU(3) with quarks or G_2 Yang-Mills-Higgs with $\kappa < \infty$:
  $\langle P \rangle$ small in confining phase
  phase transition gets weaker for decreasing m_q or κ
  string breaking for large separation of static 'quarks'
- similar for pure G_2:
  $\langle P \rangle$ small in confining phase, jumps at first order PT
  string breaking $(\{3\}, \{\bar{3}\})$
  effective models for $\mathcal{P}(x)$ constructed
- what happens for $\kappa \neq 0$ and $\kappa \neq \infty$?
  transition gets weaker away from pure SU(3) and pure G_2

HMC for pure G_2 gauge theory

- local hybrid Monte-Carlo: more Lie algebra, less group
- Lagrangian: $\mathcal{U}_{x,\mu}(t) \in G_2$ (not easy):

$$L = \frac{1}{2}\mathrm{tr}\sum_{x,\mu}\left(i\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}\right)^2 - S_{\mathrm{YM}}[\mathcal{U}]$$

- conjugated momentum in Lie algebra:

$$\mathfrak{P}_{x,\mu} = i\frac{\partial L}{\partial(\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1})} = i\mathcal{U}\frac{\partial L}{\partial\dot{\mathcal{U}}_{x,\mu}} = -i\dot{\mathcal{U}}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}$$

- Hamiltonian

$$H = \frac{1}{2}\mathrm{tr}\,\mathfrak{P}_{x,\mu}^2 + \frac{\beta}{2N_c}\mathrm{tr}\sum_{x,\mu\nu}\left(2N_c - \mathcal{U}_{x,\mu\nu} - \mathcal{U}_{x,\mu\nu}^{\dagger}\right)$$

HMC equations

- variation $\Rightarrow$ staple $R_{x,\mu}$:

$$\begin{aligned}\delta H &= \operatorname{tr}\sum_{x,\mu}\left(\mathfrak{P}_{x,\mu}\delta\mathfrak{P}_{x,\mu}\right)\\ &- \frac{\beta}{2N_c}\operatorname{tr}\sum_{x,\mu}\left(\delta\mathcal{U}_{x,\mu}\mathcal{U}_{x,\mu}^{-1}\right)\left(\mathcal{U}_{x,\mu}R_{x,\mu} - R_{x,\mu}^{-1}\mathcal{U}_{x,\mu}^{-1}\right)\end{aligned}$$

- equation of motion

$$\dot{\mathfrak{P}}_{x,\mu} = \frac{i\beta}{2N_c}\left(\mathcal{U}_{x,\mu}R_{x,\mu} - R_{x,\mu}^{\dagger}U_{x,\mu}^{\dagger}\right) - G_{x,\mu} \equiv F_{x,\mu} - G_{x,\mu}$$

- final HMC-equation of motion:

$$\dot{\mathcal{U}}_{x,\mu} = i\,\mathfrak{P}_{x,\mu}\mathcal{U}_{x,\mu} \quad \text{and} \quad \dot{\mathfrak{P}}_{x,\mu} = \sum_a \operatorname{tr}(F_{x,\mu}T_a)T_a$$

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Implementing HMC

- SU(3) subgroup of G_2 : $\mathcal{U} = \mathcal{S} \cdot \mathcal{V}$ with $\mathcal{V} \in$ SU(3)

$$\mathcal{U} = e^{\delta\tau \boldsymbol{u}} = e^{\delta\tau \boldsymbol{s}} \cdot e^{\delta\tau \boldsymbol{v}} = \mathcal{S} \cdot \mathcal{V}$$

- $[\boldsymbol{v}, \boldsymbol{v}'] = \boldsymbol{v}''$, $[\boldsymbol{v}, \boldsymbol{s}] = \boldsymbol{s}'$, $[\boldsymbol{s}, \boldsymbol{s}'] = \boldsymbol{u} + \boldsymbol{s}''$
- $\boldsymbol{s} \to \mathcal{S}$ and $\boldsymbol{v} \to \mathcal{V}$ simple to calculate
- depending on order of symplectic integrator: keep corresponding order in $\delta\tau$ in

$$\delta\tau \boldsymbol{u} = \delta\tau(\boldsymbol{s} + \boldsymbol{v}) + \tfrac{1}{2}\delta\tau^2[\boldsymbol{s}, \boldsymbol{v}] + \ldots$$

- use $\boldsymbol{s}$ and $\boldsymbol{v}$ in calculations: this is time reversible

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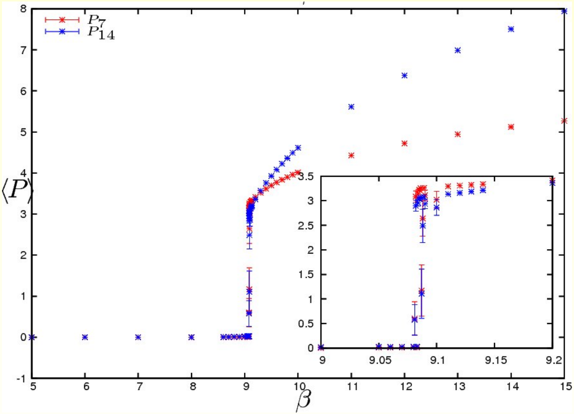
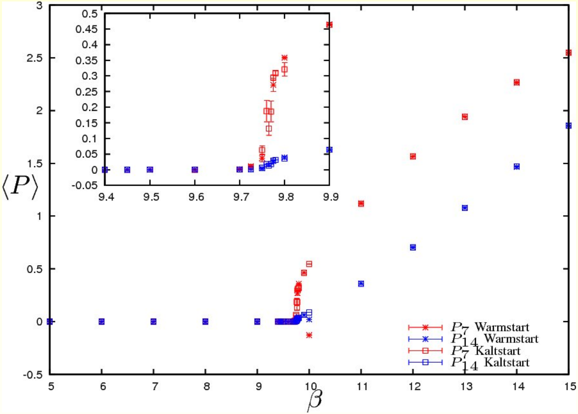
Expectation values $\chi_{\rm rep}$(untraced Polyakov loop)



Polyakov loop in $\{7\}$ and $\{14\}$, characters χ_7 and χ_{14}
$10^3 \times 2$ lattice, $10\,000 - 50\,000$ configurations

Expectation values χ_{rep}(untraced Polyakov loop)



Polyakov loop in $\{7\}$ and $\{14\}$, characters χ_7 and χ_{14}

$16^3 \times 6$ lattice, 100 000 configurations

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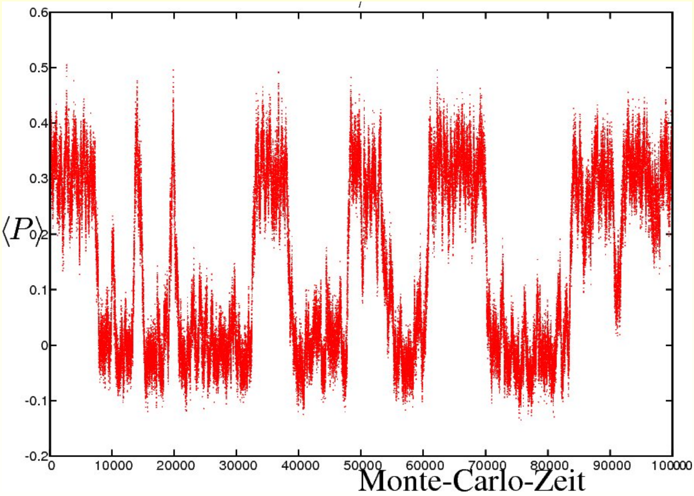
tunneling near β_c, after approximately 10 000 configurations



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Effective Theories

- G_2 has two fundamental representation $\{7\}$ and $\{14\}$ $\Rightarrow$

$$\text{class function } f(\mathcal{U}) = f\left(\chi_7(\mathcal{U}), \chi_{14}(\mathcal{U})\right)$$

- strong coupling expansion for

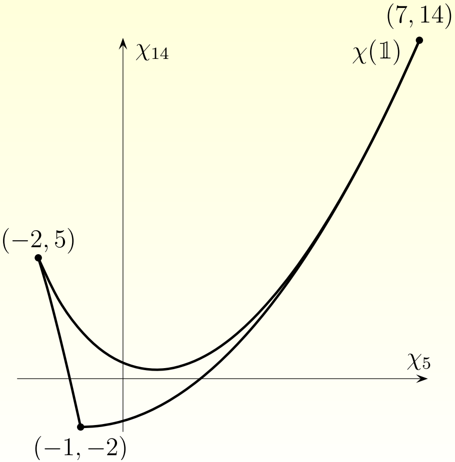
$$e^{-S_{\text{eff}}[\mathcal{P}]} = \int \mathcal{D}\mathcal{U}\, \delta\left(\mathcal{P}_{\boldsymbol{x}} - \prod_t \mathcal{U}_{(t,\boldsymbol{x}),0}\right) e^{-S_{\text{YM}}[\mathcal{U}]}$$

- leading order: basic model

$$S_{\text{eff}} = \lambda_7 \sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} \chi_7(\mathcal{P}_{\boldsymbol{x}})\chi_7(\mathcal{P}_{\boldsymbol{y}}) + \lambda_{14} \sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} \chi_{14}(\mathcal{P}_{\boldsymbol{x}})\chi_{14}(\mathcal{P}_{\boldsymbol{y}})$$

- six more terms in next order

possible values of the characters χ_7 and χ_{14}



Reduction to discrete Potts-type model

- $\mathcal{P}_{\boldsymbol{x}}$ in corners (for SU(N) $\longrightarrow$ Potts models)
  same critical exponents $S \leftrightarrow AF$, similar phase structure
- two-component spin

$$\sigma_{\boldsymbol{x}} = \begin{pmatrix} \chi_7(\mathcal{P}_{\boldsymbol{x}}) \\ \chi_{14}(\mathcal{P}_{\boldsymbol{x}}) \end{pmatrix} \in \left\{ \begin{pmatrix} 7 \\ 14 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \end{pmatrix} \right\}$$

- energy of 3-dimensional effective Potts-type spin model

$$S_{\text{Potts}} = \sum_{\langle \boldsymbol{x}, \boldsymbol{y} \rangle} \sigma_{\boldsymbol{x}}^T \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \sigma_{\boldsymbol{y}}$$

- minimize $\longrightarrow$ phases of classical model

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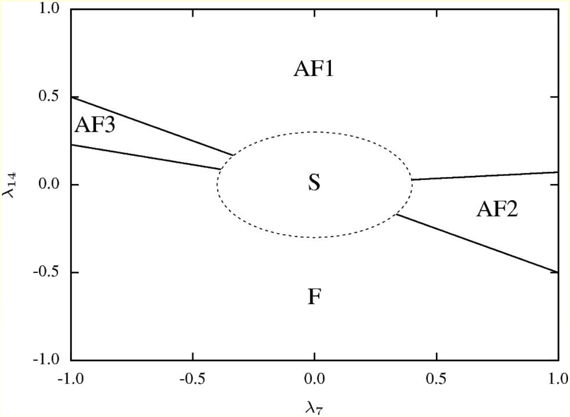
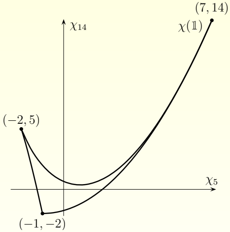
classical phases of discrete effective spin model





classical phases of effective Polyakov loop theory

- constant Polyakov loop on even and odd sublattices

$$\Gamma_e = \{\boldsymbol{x} | x_1 + x_2 + x_3 \text{ even}\}, \quad \Gamma_o = \{\boldsymbol{x} | x_1 + x_2 + x_3 \text{ odd}\}$$

- action of (anti)ferromagnetic phases

$$S_{\text{eff}}/6V = \lambda_7 \chi_7(\mathcal{P}_e)\chi_7(\mathcal{P}_o) + \lambda_{14}\chi_{14}(\mathcal{P}_e)\chi_{14}(\mathcal{P}_o)$$

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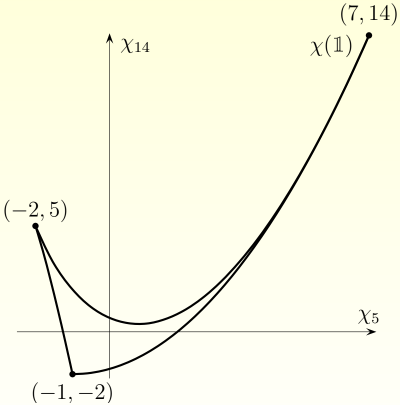
fix $\chi(\mathcal{P}_o) = (7, 14)$ on odd sublattice

phase	$\chi(\mathcal{P}_e)$
F	$(7, 14)$
AF2	$(-2, 5)$
AF1	$(-1, -2)$
F$\longrightarrow$ AF1	$(-2, 5)$



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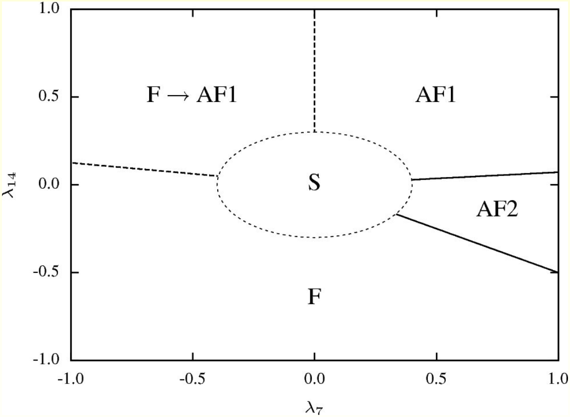

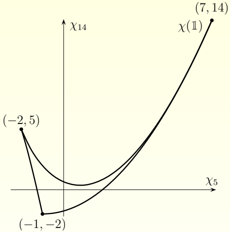

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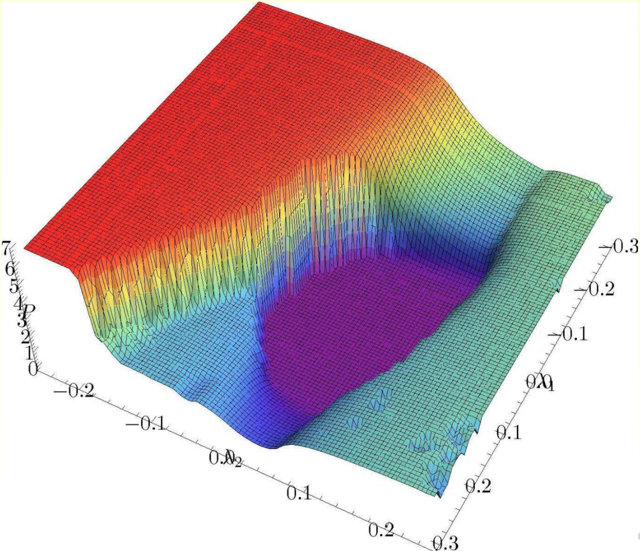
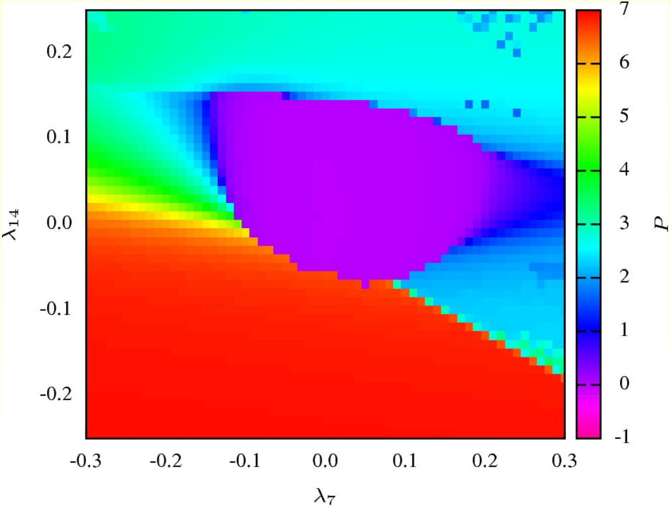
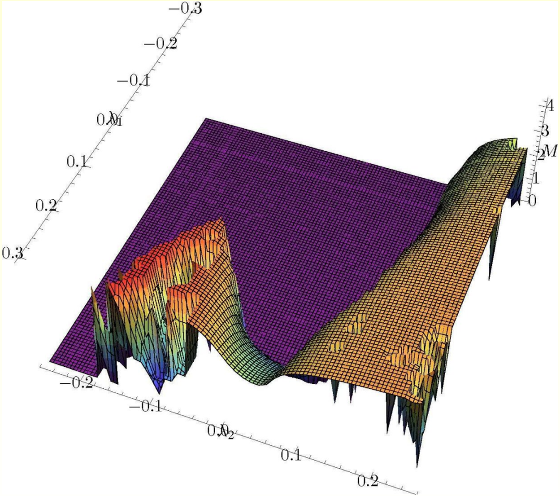
effective Polyakov-loop theory: Polyakov loop



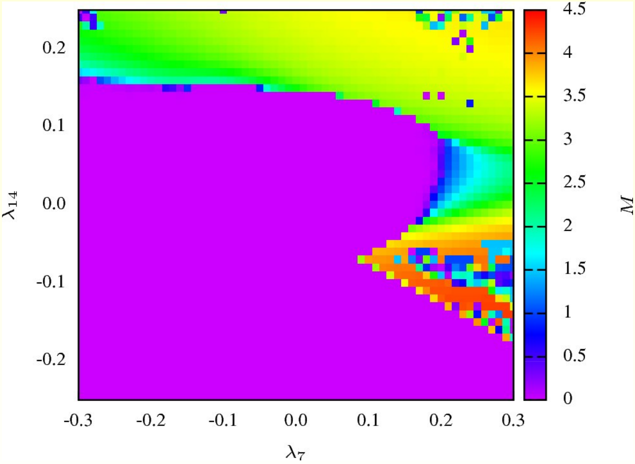
effective Polyakov loop theory: Polyakov loop, 8^3 lattice



effective Polyakov loop theory: staggered Polyakov loop



effective Polyakov-loop theory: AF order parameter, 8^3



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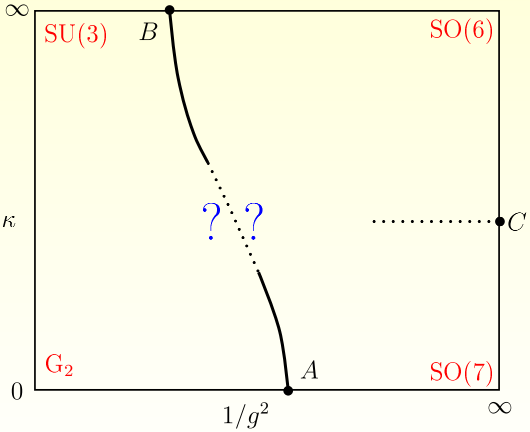
Expected phase diagram



Back to G_2 gauge-Higgs model

- symmetric phase: 7 massive scalars, 14 massless vector bosons
- broken phase: 1 massive scalar, 6 massive and 8 massless VB
  $\sim$ QCD with adjoint massive quarks
- for $1/g^2 \to \infty$
  $U_{x,\mu} = \mathbb{1}$ and O(7), O(6) sigma models
- $\beta_c(G_2) \geq 7/6 \cdot \beta_c(SU_3)$
- LHMC for G_2-gauge-Higgs system still efficient

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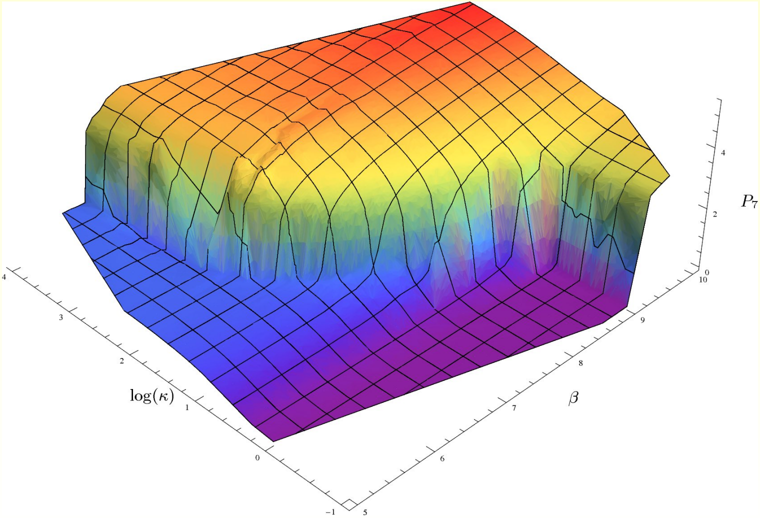
Phases of the G_2-Higgs model, $12^3 \times 2$ lattice, 10 000 configurations



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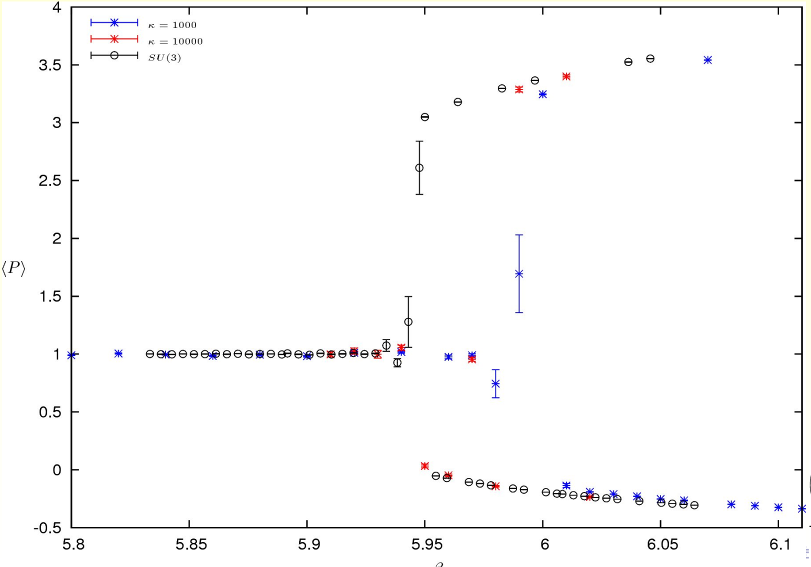

(Preliminary) conclusions

- finite temperature and density SU(2N) gauge theory

$$Z_{\mathrm{aper}}(\beta, V, \mu) = Z_{\mathrm{per}}(\beta, V, \mu)$$

- Fermi surface unlikely to exist in confining phase
- G_2 Polyakov-loop dynamics with efficient local HMC algorithm
- strong coupling expansion for eff. Polyakov loop action
- analysis of effective models (discrete and continuous) symmetric, ferromagnetic and antiferromagnetic phases
- AF phases not directly relevant for G_2 gauge theory

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- PT to AF phases useful for critical exponents?
- Casimir scaling with Lüscher-Weiss (lowest reps)
  so far no string breaking seen ($14 \otimes 14 \otimes 14$)
- next: inverse Monte-Carlo, intermediate κ (endpoints?)
- full mean field analysis for all phases

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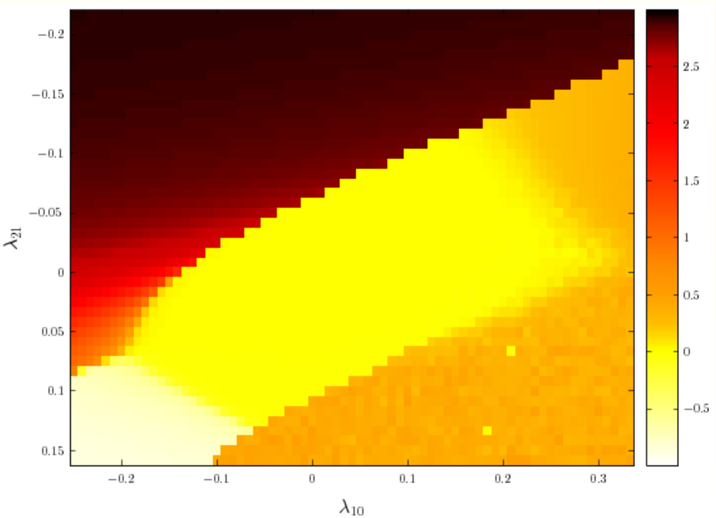
Phases of SU(3)-PLM: MC simulations



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Phases of SU(3)-PLM: mean field analysis