Magnetic Monopoles and Instantons in QCD

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See:
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Setting: pure gauge theory, gauge group G:

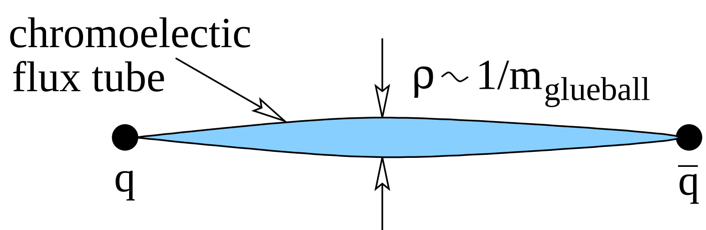
$$S \sim \int F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

strong coupling/low energies: confinement

$$V_{q\bar{q}} \sim \sigma L + c_1 - \frac{c}{L} + O(1/L)$$

σ: string tension, $c > 0$ Lüscher term

Conjecture: QCD-vacuum $\sim$ dual superconductor (Mandelstam, Parisi)



String model $\longrightarrow$ Lüscher term (Casimir energy)

Needs: Condensation of magnetic monopoles!!

QCD: no Higgs field (in adjoint): which monopoles??

ANSWER ('t Hooft):

Fix gauge symmetry only partially
ABELIAN GAUGE FIXINGS

$$G \longrightarrow U(1)^r \quad \text{e.g.} \quad SU(3) \longrightarrow U(1) \times U(1)$$

identify monopoles in EFF. ABELIAN GAUGE THEORY

$$A_\mu \longrightarrow A_\mu^{\|} + A_\mu^{\perp}$$

$A^{\|}$: neutral, massless, long range Abelian gauge field
$A^{\perp}$: charged, massive, short range matter field

Abelian Projections and monopole dominance

$$\begin{aligned}
\langle O[A] \rangle &= \int \mathcal{D}A^{\|}\mathcal{D}A^{\perp} e^{-S[A^{\|}+A^{\perp}]}\, O[A^{\|}+A^{\perp}] \\
&\sim \int \mathcal{D}A^{\|}\mathcal{D}A^{\perp} e^{-S[A^{\|}+A^{\perp}]}\, O[A^{\|}] \\
&= \int \mathcal{D}A^{\|} e^{-S_{\text{eff}}[A^{\|}]}\, O[A^{\|}], \quad S_{\text{eff}} = ??
\end{aligned}$$

Lattice MC results:

ABELIAN DOMINANCE

$$\sigma_{\rm proj} \sim 0.92\sigma_{\rm full} \qquad SU(2), \beta \sim 2.5$$

- maximal Abelian gauge (MAT)
- Laplacian Abelian gauge (LAG)

finds: monopole condensates for ALL Abelian gauges.

MONOPOLE DOMINANCE

$$A^{\|} = A^{\|}_{\rm smooth} + A^{\|}_{\rm mon}$$

keep only $A^{\|}_{\rm mon}$ in $O[A^{\|}]$:

$$\sigma_{\rm mon} \sim 0.95 \cdot \sigma_{\rm proj}$$

- Which Abelian gauges and projections?
- gauge dependency?
- Gribov copies (mostly lattice copies)?
- smoothing, cooling
- effective theories: $S_{\rm eff}, \; S_{\rm mon}$??

confinement/deconfinement: magnetic monopoles
chiral symmetry breaking: instantons

$$T_c^{\mathrm{conf}} \sim T_c^{\mathrm{CSB}}$$

EXPECT: relation monopoles $\leftrightarrow$ instantons
monopole condensate $\rightarrow$ fermionic zero modes

Abelian gauge fixings

- Polyakov: analytic results, lattice?
- MAG
- LAG

(MAG, LAG: few analytic results, lattice!!)

Polyakov gauge:
finite temperature $T = 1/\beta$, IR-cutoff $\rightarrow$ torus $\mathbb{T}^4$
fields periodic up to gauge transformations

$$\begin{aligned} \mathcal{P}(x^0, \vec{x}) &= \mathcal{P}\exp\left[i\int_0^{x^0} d\tau A_0(\tau, \vec{x})\right] \in G \\ \mathcal{P}(\beta, \vec{x}) &= \exp\left[i\phi(\vec{x})\right], \quad \phi \sim \text{adj. Higgs field} \end{aligned}$$

ORDER PARAMETER FOR CONFINEMENT

$$\langle \mathrm{tr}\left(\mathrm{e}^{\mathrm{i}\phi(\vec{\mathrm{x}})}\right)\rangle_\beta = \mathrm{e}^{-\beta \mathrm{F}(\vec{\mathrm{x}})} = \begin{cases} 0 & \text{low } T \\ \neq 0 & \text{high } T \end{cases}$$

$F(\vec{x})$ free energy of static colored source

fixing: A_0 diagonal, x^0-independent $\to \Phi, \mathcal{P}$ diagonal
residual diagonal $U(1)^r$ gauge freedom

RESULTS:

- consistent BC in all instanton sectors

$$\begin{aligned} A_\mu(x+b_\nu) &= {}^{U_\nu(x)}A_\mu(x) \\ U_\mu(x)U_\nu(x+b_\mu) &= U_\nu(x)U_\mu(x+b_\nu)z_{\mu\nu} \\ z_{\mu\nu} &= z_{\nu\mu}^{-1} \in \text{center}(G) \end{aligned}$$

U_μ :	θ-functions
$\omega_i\omega_j = \omega_j\omega_i z_{ij}$:	Heisenberg doubles

- GT singular if $\mathcal{P}(\vec{x}_0) = \pm 1\!\!1$ or $\phi(\vec{x}_0) = 0$

$\longrightarrow$ MONOPOLE at $\vec{x}_0$

ϕ winds, quantized magnetic charges

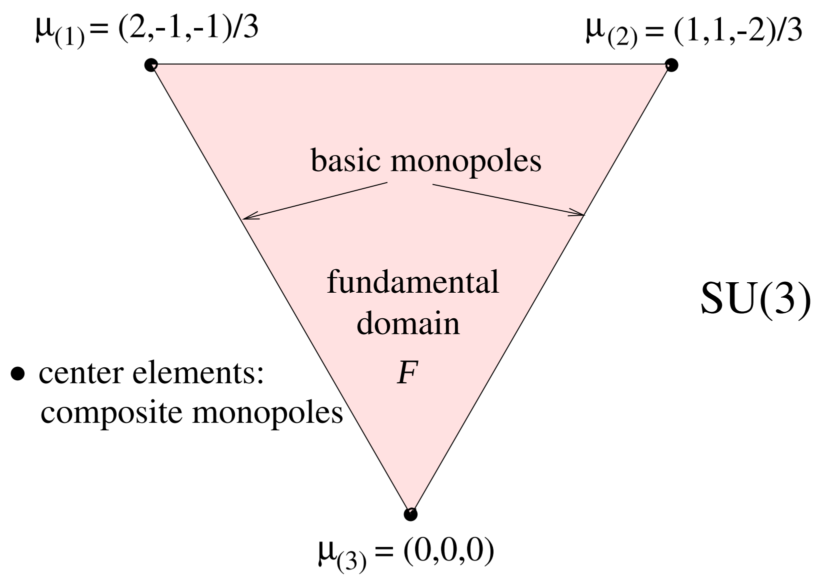
$$Q_{\rm mon} = \frac{1}{2\pi}\int_{S^2} F^{\|}, \quad e^{2\pi i Q_{\rm mon}} = \mathbb{1}$$

$$Q_{\rm mon} = \alpha^\vee \cdot H \quad \text{Goddard, Nuyts, Olive}$$

- fundamental domain for $A^{\|}$ known:

$$\mathcal{F} = \text{convex hull of } \left\{0, \frac{1}{n_1}\mu^\vee_{(1)}, \ldots, \frac{1}{n_r}\mu^\vee_{(r)}\right\}$$

$$0 = \alpha_{(0)} + \sum n_i \alpha_{(i)}, \quad n_i : \text{coxeter labels}$$



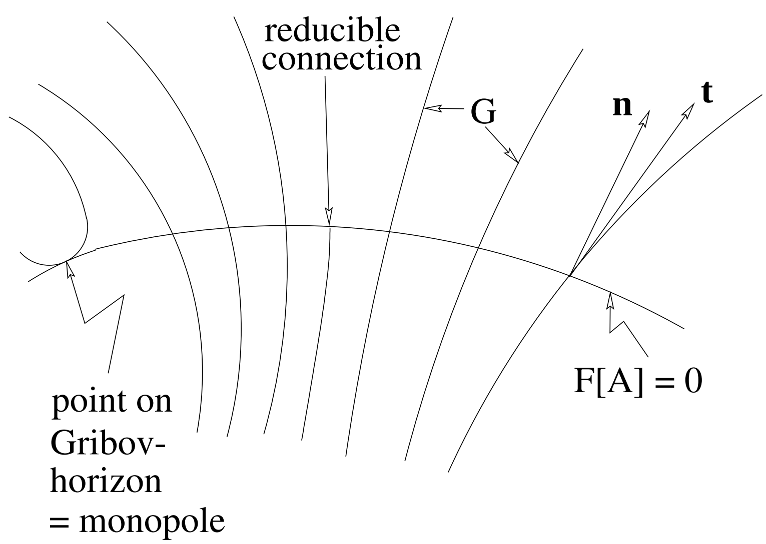
• instantons $\leftrightarrow$ monopoles

instanton number = total magnetic charge of *one* *type* of magnetic monopoles

• overall magnetic charge neutrality
• Fadeev-Popov determinant can be calculated

$$\det(FP) = d\mu_{\rm red}\big(\mathcal{P}(\vec{x})\big) \qquad \text{renorm.?}$$

• all monopoles sit on Gribov horizon!!



• kinematics very well understood: instantons vs. monopoles, fundamental regions, Gribov horizons, FP

maximal Abelian gauge (MAG)

popular in MC-simulations, only recently some analytic results, closely related to LAG

minimize $\quad F[A] = \int d^4x \, \text{Tr} \, A_\mu^\perp A_\mu^\perp, \quad \perp \text{Cartan}^\perp$

remaining gauge freedom $A \to V_D(A + id)V_D^{-1}$

LATTICE: maximize $\quad F[U] = \text{Tr}\left(\sigma_3 U_{n,\mu} \sigma_3 U_{n,\mu}^\dagger\right).$

GAUGE FIXING CONDITIONS, FADEEV-POPOV:

$$
\begin{aligned}
F'[A] &= D_\mu^\| A_\mu^\perp \equiv D_\mu A_\mu^\perp = 0 \\
F''[A^{\text{gf}}] &= \text{FP}_{SU(2)} = -Q(D_\mu^\| D_\mu^\| + \text{ad}^2 A_\mu^\perp)Q
\end{aligned}
$$

lattice:

- monopole density, string tension, form of flux tubes, Abelian dominance, monopole dominance. . .
- max($F[U]$) $\sim$ spin glas problem
- lattice Gribov copies

continuum:

- 't Hooft instantons are in MAG, sit on Gribov horizon
- instantons $\leftrightarrow$ monopoles (Brower et.al, Jahn, Jena)
- eff. monopole theory $(\ldots)$?
- variables of dual low energy theory

- Fadeev, Niemi; Chow; Di Giacomo; Chernodub, Polikarpov; Bornyakov; Diakonov; Lenz; Reinhardt; Schierholz; Müller-Preussker, Bali, . . .

Laplacian gauge fixing

$$D_\mu D^\mu \phi = \lambda \phi, \quad \phi \in \text{adjoint, ground state}$$
$$\phi \longrightarrow V\phi V^{-1} = \text{diagonal}$$
$$A \longrightarrow {}^V A \equiv A_{\rm LAG}$$

- similar results as for MAG
- no lattice gribov copies

remarks

results for all gauge group (center dominance for G_2?)
new formulae for general winding numbers

alternative description: n-field formulation

$\hat{A} = (A,n)n + i[n,dn], \quad \hat{D}n = 0, \quad A = \hat{A} + X$

$$\text{min}_{\{n\}} F[n] = \|A_\mu - \hat{A}_\mu(A,n)\|^2 \Rightarrow$$

$\hat{A}$: reducible $SU(N)$-potential, X matter field

diagonalize n: equivalent to Abelian gauges
unified view on Abelian gauges
related to results of Chow, Fadeev-Niemi (eff. theories)

instanton solutions on $\mathbb{T}^4$ (van Baal and Kraan; Ford, Tok, Pawlowki, Wipf)

relevance of *center of* G

approximations, eff. monopole theories

condensation of fermionic zero modes

confinement $\leftrightarrow$ chiral symmetry breaking