Introduction to Supersymmetry

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1 Introduction

Supersymmetric theories are highly symmetric and beautiful. They unify fermions (matter) with bosons (carrier of forces), either in flat space (supersymmetry) or in curved space-time (supergravity). Supergravity unifies the gravitational with other interactions. The energy at which gravity and quantum effects become of comparable strength can be estimated from the only expression with the dimension of energy that can be formed from the constants of nature $\hbar, c$ and G,

$$E_{Pl} = m_{Pl}c^2 = c^2\sqrt{\hbar c/G} \sim 10^{19}\text{GeV}.$$

The Schwarzschild radius of a point particle with Planck mass is just twice its Compton wavelength,

$$r_S = \frac{2G}{c^2}\,m_{Pl} = \frac{2G}{c^2}\sqrt{\hbar c/G} = \frac{2\hbar}{c}\sqrt{G/\hbar c} = \frac{2\hbar}{m_{Pl}\,c} = 2\lambda_c.$$

Supersymmetry transformations relate bosons to fermions,

$$Q|\text{Boson}\rangle \sim |\text{Fermion}\rangle \quad \text{and} \quad Q|\text{Fermion}\rangle \sim |\text{Boson}\rangle,$$

and hence relate particles with different spins. The particles fall into multiplets and the supersymmetry transforms different members of such a supermultiplet into each other. Each supermultiplet must contain at least one boson and one fermion whose spins differ by 1/2. All states in a multiplet (of unbroken supersymmetry) have the same mass.
So far no experimental observation has revealed particles or forces which manifestly show such a symmetry. Yet supersymmetry has excited great enthusiasm in large parts of the community and more recently in the context of superstring theories. It has even be said of the theory that it is SO BEAUTIFUL IT MUST BE TRUE.
Notation:

symbols	range	meaning
$i, j, k, \dots$	$1, 2, \dots, d-1$	space indices
$\mu, \nu, \rho, \sigma, \dots$	$0, 1, \dots, d-1$	spacetime indices
$\alpha, \beta, \gamma, \delta \dots$	$1, \dots 2^{[d/2]}$	Dirac-spinor indices
$\alpha, \beta, \dot\alpha, \dot\beta \dots$	$1, \dots 2^{d/2-1}$	Weyl-spinor-indices (d even)
$A^\dagger, A^*, A^T$	A matrix	adjoint, complex conjugate and transpose of A

The (anti)symmetrization of a tensor $A_{\mu_1\dots\mu_n}$ is

$$A_{(\mu_1\dots\mu_n)} = \frac{1}{n!}\sum_\sigma A_{\sigma(\mu_1)\dots\sigma(\mu_n)}, \quad A_{[\mu_1\dots\mu_n]} = \frac{1}{n!}\sum_\sigma \text{sign}(\sigma) A_{\sigma(\mu_1)\dots\sigma(\mu_n)}.$$

1.1 Reading

In the references you find a (of course incomplete) list of references. The references 1-13 are introductory books and review articles.