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Will be discussed: **19th week of year**

Problems in Supersymmetry

Sheet 5

Problem 17: Spinrotations in 2 dimensions

Take an irreducible representation for the γ-matrices in 2 dimensions and calculate

$$\Sigma^{\mu\nu} = \frac{1}{2\mathrm{i}}\gamma^{\mu\nu}$$

Also calculate the group elements $S = \exp(i\omega_{\mu\nu}\Sigma^{\mu\nu}/2)$ generated by Σ. Prove explicitly the identity

$$S^{-1}\gamma^\rho S = \Lambda^\rho{}_\sigma \gamma^\sigma, \quad \Lambda = e^\omega,$$

which was already introduced in problem 13. How does a spinor transform under spin transformation.
Take a chiral representation, for example $\gamma^0 = \sigma_1$ and $\gamma^1 = i\sigma_2$. Let ψ be a chiral spinor,

$$\gamma_*\psi = \pm\psi.$$

Are these constraints compatible with Lorentz invariance?
How does a chiral spinor transform under spin transformations?
Which lorentz-invariant tensor field can you build out of bilinears of ψ?

Problem 18: Spinrotations in 4 dimensions

We can choose the chiral representation

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \tilde{\sigma}_\mu & 0 \end{pmatrix}, \quad \sigma_\mu = (\sigma_0, -\sigma_i), \quad \tilde{\sigma}_\mu = (\sigma_0, \sigma_i)$$

for which the infinitesimal spin-rotations have the block-diagonal form

$$\gamma_{\mu\nu} = \begin{pmatrix} \sigma_{\mu\nu} & 0 \\ 0 & \tilde{\sigma}_{\mu\nu} \end{pmatrix}.$$

Calculate the matrices $\sigma_{\mu\nu}$ and $\tilde{\sigma}_{\mu\nu}$. What is $\gamma_* = -i\gamma_0\gamma_1\gamma_2\gamma_3$.