**Friedrich-Schiller-University Jena** Summer term 2016
Prof. Andreas Wipf
M.Sc. Tobias Hellwig Will be discussed: **17th week of year**

# Problems in Supersymmetry

## Sheet 3

**Problem 11: Pauli-Ljubanski vector**

The Pauli-Ljubanski vector is defined by

$$W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}, \quad \epsilon_{0123} = 1$$

where $\{P^\nu, M^{\rho\sigma}\}$ generate the Poincaré algebra. Prove that $W^2 = W_\mu W^\mu$ commutes with all generators of the Poincaré algebra. Compute $[W^\mu, M^{\rho\sigma}]$ for this end.

**Problem 12: Polyakov action and symmetries**

Consider the Polyakov action of a string moving in $D$-dimensional Minkowski space

$$\begin{aligned} S &= \frac{T}{2}\int_\Sigma d^2\sigma\sqrt{|h|}\, h^{\alpha\beta}(\sigma)\,\partial_\alpha X^\mu \partial_\beta X_\mu \\ \sigma^\alpha &= \{\tau,\sigma\}, \quad X^\mu = X^\mu(\sigma^\alpha), \quad \tau \in \mathbb{R},\ \sigma \in [0,\pi] \end{aligned}$$

We discuss the symmetries and corresponding conservation laws of the bosonic string. Show

- Invariance under parameterizations: $\sigma^\alpha \longrightarrow \tilde{\sigma}^\beta = \tilde{\sigma}^\beta(\sigma^\alpha)$
- Invariance under Weyl transformations: $h_{\alpha\beta} \longrightarrow \Omega^2(\sigma^\gamma)\, h_{\alpha\beta}$
- Compute the energy momentum tensor $T_{\alpha\beta}$
- Invariance under global Poincaré transformations $X^\mu \to \Lambda^\mu{}_\nu X^\nu + a^\mu$
- Compute the conserved Noether currents $J^\mu_\alpha$ and $J^{\mu\nu}_\alpha$ for the global Poincaré transformations,

$$\delta S = \int d^2\sigma\, \partial^\alpha \left(J^\mu_\alpha a_\mu + J^{\mu\nu}_\alpha \omega_{\mu\nu}\right),$$

  where the infinitesimal translations and Lorentz transformations are parametrized by $a_\mu$ and $\omega_{\mu\nu}$.

Hint: A symmetric and conserved energy-momentum tensor can be defined via

$$T_{\alpha\beta} = \frac{2}{\sqrt{|h|}}\frac{\delta S}{\delta h^{\alpha\beta}(\sigma)}.$$

It is useful to know

$$\delta\sqrt{|h|} = -\frac{1}{2}\sqrt{|h|}\, h_{\alpha\beta}\delta h^{\alpha\beta}.$$