Friedrich-Schiller-University Jena Summer term 2016
Prof. Andreas Wipf
M.Sc. Tobias Hellwig Will be discussed: **16th week of year**

Problems in Supersymmetry

Sheet 2

Problem 7: Fermionic vs. Clifford algebra

Show that the fermionic creation/annihilation algebra $\{b_i, b_j^\dagger\} = \delta_{ij}$ with $i, j = 1, \ldots, n$ is equivalent to the Clifford algebra in $D = 2n$ dimensions and that the Fock space representation provides a irreducible representation with $\gamma_{2i-1} = b_i + b_i^\dagger$ and $\gamma_{2i} = i(b_i - b_i^\dagger)$. Note that Lorentz-Transformations transform bosonic/fermionic states into themselves so that even/odd subspaces correspond to Weyl-spinors.

Problem 8: Lorentzalgebra III

Define the following 6 matrices $M_{\mu\nu}$:

$$(M_{\mu\nu})_{\alpha\beta} = -i\left(\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}\right).$$

and calculate the commutators

$$[M_{\mu\nu}, M_{\alpha\beta}].$$

Repeat the same calculations for

$$L_{\mu\nu} = \frac{1}{i}\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right) \text{and} S_{\mu\nu} = \frac{1}{4i}[\gamma_\mu, \gamma_\nu].$$

What do these results tell you?

Problem 9: Noether charges for Lorentz-Transformations

Calculate the Noether charges for Lorentz-Transformations,

$$\Phi(x) \longrightarrow S\Phi\left(\Lambda^{-1}x\right) = e^{\frac{i}{2}(\omega,S)}\Phi\left(e^{-\frac{i}{2}(\omega,M)}x\right).$$

Problem 10: Conserved current for Dirac field

The Lagrangian density for the Dirac field is $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$. Prove, that

- $\mathcal{L}$ is invariant under phase transformation of ψ. Calculate the Noether current.
- For $m = 0$ the density $\mathcal{L}$ is invariant under chiral rotations $\psi \to e^{i\alpha\gamma_5}\psi$. Calculate the Noether current.