Problems in Supersymmetry: sheet 6

Problem 8.1: Proof that the (on-shell) supersymmetry transformation

$$\delta A_\mu = i\bar{\alpha}\gamma_\mu\psi, \quad \delta\psi = iF^{\mu\nu}\Sigma_{\mu\nu}\alpha$$

commute with the infinetimal gauge transformations.

Problem 8.2 [superspace]: The superspace it obtained by adding four spinor degrees of freedom θ^α, $\bar{\theta}_{\dot{\alpha}}$. The spinor index is raised and lowered with the ε-tensor (see lecture). Under the supersymmetry transformation with $\mathcal{N}$ and transformation parameters ξ and $\bar{\xi}$ the superspace coordinates are taken to transform as

$$\begin{array}{ccl} x^\mu & \longrightarrow & x'^\mu = x^\mu + i\theta\sigma^\mu\bar{\xi} - i\xi\sigma^\mu\bar{\theta} \\ \theta & \longrightarrow & \theta' = \theta + \xi \\ \bar{\theta} & \longrightarrow & \bar{\theta}' = \bar{\theta} + \bar{\xi} \end{array}$$

These transformations are implemented by the operator $\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$.

- Show that a possible choice for the supercharges is

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \quad , \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu.$$

- Proof, that they satisfy the by now familier relations

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu.$$