Problems in Supersymmetry: sheet 5

Problem 7.1: The simplest supersymmetric gauge theory is consist of a Majorana (and hence uncharged) spinor and a $U(1)$ gauge field and has LAGRANEan density

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \tfrac{1}{2}\mathcal{G}^2, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \partial\!\!\!/ = \gamma^\mu\partial_\mu.$$

Check, that $\mathcal{L}$ is hermitean. The supersymmetry transformations are

$$\delta_\alpha A_\mu = i\bar{\alpha}\gamma_\mu\psi, \quad \delta_\alpha\psi = iF^{\mu\nu}\Sigma_{\mu\nu}\alpha + i\mathcal{G}\gamma_5\alpha, \quad \delta_\alpha\mathcal{G} = \bar{\alpha}\gamma_5\partial\!\!\!/\psi.$$

Check that $\mathcal{L}$ transforms into a divergence.

Problem 7.2: Verify via counting of degrees of freedom that (N=1) supersymmetric YANG-MILLS Theories can exist only in the following dimensions and with the following type of spinors

d	spinor
3	Majorana
4	Majorana (or Weyl)
6	Weyl
10	Majorana-Weyl

The fact that these dimensions are of the form $2+n$ with $n = 1, 2, 4, 8$ are the dimensions of the real division algebras is not coincidental. These are precisely the dimensions in which the classical superstring exists. Unlike superstring theory, which on a quantum level is only consistent in 10 dimensions, the supersymmetric YM-theories exists in each of these dimensions.
Hint: Count degrees of freedom on-shell and use that on-shell the $N = 1$ models only contain gauge bosons and fermions in the same representation.

Reading: I suggest you read the relevant chapters on classical gauge theories which you find in any reasonable book on field or gauge theories.