Problems in Supersymmetry: sheet 4

Problem 6.1: Let A and B be two scalar fields and ψ a Majorana spinor. The (on-shell) Lagrangian density has the form

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu A \partial^\mu A + \tfrac{1}{2}\partial_\mu B \partial^\mu B - \tfrac{1}{2}m^2(A^2 + B^2) + \tfrac{i}{2}\bar{\psi}\partial\!\!\!/\psi - \tfrac{1}{2}m\bar{\psi}\psi.$$

- Show that $\mathcal{L}$ is invariant under supersymmetry transformations

$$\begin{aligned} \delta_\alpha A &= \bar{\alpha}\psi, \qquad \delta_\alpha B = i\bar{\alpha}\gamma_5\psi \\ \delta_\alpha \psi &= -\big(i\partial\!\!\!/ + m\big)\big(A + i\gamma_5 B\big)\alpha, \end{aligned}$$

  up to a surface term. Here α is a constant Grassmann Majorana spinor anticommuting with itself and ψ and commuting with the bosonic (pseudo)scalar fields A and B.

- Calculate the surface term, the covariantly conserved Noether current and conserved Noether charge.

Problem 6.2: Now we shall investigate the so-called off-shell version of the same model.

- Show that the (off-shell) formulation containing two additional dummy fields $\mathcal{F}$ and $\mathcal{G}$,

$$\begin{aligned} \mathcal{L} &= \tfrac{1}{2}\partial_\mu A \partial^\mu A + \tfrac{1}{2}\partial_\mu B \partial^\mu B + \tfrac{i}{2}\bar{\psi}\partial\!\!\!/\psi + \tfrac{1}{2}\left(\mathcal{F}^2 + \mathcal{G}^2\right) \\ &\quad + m\big(\mathcal{F}A + \mathcal{G}B - \tfrac{1}{2}\bar{\psi}\psi\big) \end{aligned}$$

  is equivalent to the on-shell formulation by eliminating the dummy fields.

- Argue, that the supersymmetry transformations

$$\begin{aligned} \delta_\alpha A &= \bar{\alpha}\psi, \qquad \delta_\alpha B = i\bar{\alpha}\gamma_5\psi \\ \delta_\alpha \psi &= -i\partial\!\!\!/\big(A + i\gamma_5 B\big)\alpha + (\mathcal{F} + i\gamma_5\mathcal{G})\alpha \\ \delta_\alpha \mathcal{F} &= -i\bar{\alpha}\partial\!\!\!/\psi, \qquad \delta_\alpha \mathcal{G} = \bar{\alpha}\gamma_5\partial\!\!\!/\psi, \end{aligned}$$

  leave the corresponding action invariant.

- What are the mass-dimensions of the fields and the hermiticity properties of the various terms in $\mathcal{L}$ and in the transformation formulae. The off-shell transformation formulae are preferrable to the on-shell ones. Why?