Problems in Supersymmetry: sheet 1

Problem 3.1: Show from the that the sixteen matrices of the $4d$ CLIFFORD-algebra

$$M_i = \mathbb{1}, \quad \gamma^\mu, \quad \gamma^{\mu\nu} = \tfrac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \quad \gamma_*\gamma^\mu, \quad \gamma_* \tag{1}$$

are orthogonal with respect to the scalar product

$$(M_i, M_j) = \mathrm{Tr}\,(M_i M_j).$$

Problem 3.2: Show that the subspaces of tensors for which all pair traces are zero form an invariant subspace under LORENTZ transformations.

Problem 3.3: Show that if T is (anti)symmetric in two indexes, then the transformed tensor is also (anti)symmetric in the same indexes.

Problem 3.4: Show, that every tensor T of rank r can be decomposed uniquely into a pairwise traceless tensor $\hat{T}$ plus a sum of $\frac{1}{2}r(r-1)$ tensors,

$$T_{\mu_1\mu_2\ldots\mu_r} = \hat{T}_{\mu_1\mu_2\ldots\mu_2} + \sum_{i<j} \delta_{\mu_i\mu_j} G^{(ij)}_{\mu_1\ldots\check{\mu}_i\ldots\check{\mu}_j\ldots\mu_r}.$$