Strong Field (Q)ED

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Chris Engelbrecht Sommer School in Theoretical Physics, January 16-28, 2010

Stellenbosch

Introduction

Nonlinear Optics

- interaction (laser) radiation $\leftrightarrow$ matter (atoms)
- spectroscopy with extremely high resolution
- cross section/rates :-)

Nonlinear Electrodynamics

- interaction (laser) field $\leftrightarrow$ relativistic particles
- strong fields $\longrightarrow$ non-linear effects
- higher harmonics, non-linear Compton scattering
  no matter: vacuum-birefringence, pair creation
- cross sections/rates :-(
- requires high-intensity laser (PW)

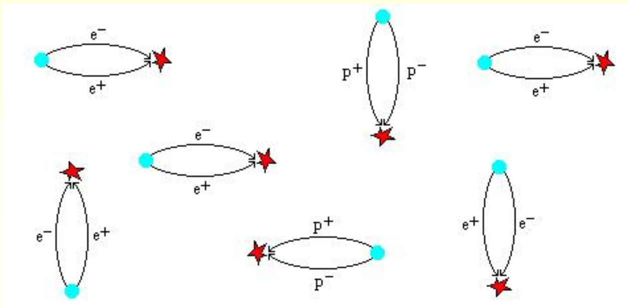
strong elm. field? quantum effects relevant?

Quantum vacuum

is complicated: flucutates and can be polarized

- Heisenberg: uncertainty principle $\Delta E \Delta t \geq \hbar/2$
- Dirac: every particle has anti-particle
- Einstein: $E = mc^2 \longrightarrow$ virtual particle-antiparticle pairs

Compton time/length $\lambda_c = h/mc \approx 2.43 \cdot 10^{-10}$ cm

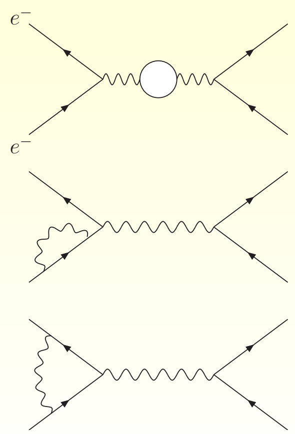


- fluctuating $e^- e^+$-field
- applied fields $\to$ polarization and pair production
- vacuum $=$ dispersive and absorptive medium
- light propagation modified ($\to$ birefringence $\to$ exp.)
- nonlinear medium $\to$ no superposition principle
- Maxwell equations modified
- photon splitting, higher harmonics, ...

Experimental tests

Lamb shift

- Dirac theory: $2s_{1/2}$ and $2p_{1/2}$ degenerate
- Exp: splitting $\Delta E = 4.37 \cdot 10^{-6}$ eV $\sim$ 1057 MHz



vacuum polarization, -27 MHz

fluctuating photon field, 1017 MHz

anom. magnetic moment, 68 MHz

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spontaneous decay

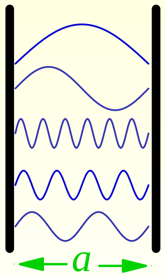
- atomic physics: ind. processes and spontaneous decay
- cavity: spont. decay inhibited, $f(\lambda, d)$ (Kleppner, Haroche)

Casimir effect

- vacuum energy density of elm field modified
- force between mirrors

$$\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}$$

(Lamoureaux, Mohideen)



accelerators, structure formation ...

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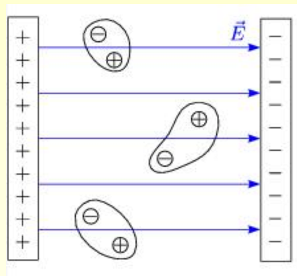
strong fields:

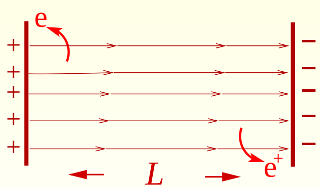
- vacuum polarization

$$\begin{aligned} \boldsymbol{D} &= \boldsymbol{E} + \boldsymbol{P} \\ \boldsymbol{B} &= \boldsymbol{H} + \boldsymbol{M} \end{aligned}$$

  - photon splitting
  - birefringence (PVLAS)
- pair production in $\boldsymbol{E}$-field

$$\lambda\!\!\!^{-}_e eE > m_e c^2 \Longrightarrow E_{\rm crit}$$





$$E > E_{\rm crit} \approx 1.32 \cdot 10^{16} \frac{\rm V}{\rm cm} \Longrightarrow \text{pair creation}$$

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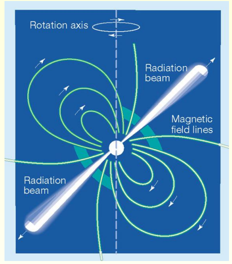
Strong fields in nature and lab

Magnetic Fields:

- of earth: 0.3 – 0.6 Gauss
- *laboratories*: $\lesssim 4 \cdot 10^5$ Gauss
- *pulsars* (1967): $\lesssim 10^{12}$ Gauss
  $\longrightarrow$ burst of interest
- *Magnetars*: rapidly rotating n^*
  $\lesssim 10^{15}$ Gauss.
- *pulsed lasers*:

$$B \approx \left(\frac{10^{10}}{300}\right) \text{Gauss} = 3 \cdot 10^7 \,\text{Gauss}$$



Electric fields:

- Coulomb field at 10^{-8} cm: $1.4 \cdot 10^9$ V/cm
- e^- accelerated by a laser field

$$m\Delta v = e\bar{E}\Delta t, \quad \bar{E} = \frac{2}{\pi}E_{\max}, \quad \Delta t = \frac{1}{2\nu}$$

- relativistic for

$$\frac{\Delta v}{c} \approx 1 \Longleftrightarrow a_0 \equiv \frac{eE}{m\omega c} > 1 \Longleftrightarrow E \gg \frac{\lambda_e}{\lambda} E_{\text{crit}}$$

numbers for relativistic parameter

$$a_0 = 0.31 \cdot E\left[10^{10}\frac{\text{V}}{\text{cm}}\right] \lambda\,[\mu\text{m}]$$

$a_0 > 1 \implies$ electrons relativistic during cycle

- PW-laser: $a_0 \approx 100$ (10^{10}V/cm $\sim 2.65 \cdot 10^{17}$ W/cm^2)
- Pomerantchuk: e^- with 10^{19} eV on earth 'sees' E_{crit}

Laser performance

characteristic	Vulkan Polaris	XFEL	XFEL ('goal')	ELI
$\hbar\omega_L$	1.2	$3.1 \cdot 10^3$	$8.3 \cdot 10^3$	1
focus	10^3	21	0.15	10^3
I	$3 \cdot 10^{22}$	$8 \cdot 10^{19}$	$7 \cdot 10^{27}$	10^{26}
E/E_c	10^{-4}	10^{-5}	10^{-1}	10^{-2}
η	50	$2 \cdot 10^{-3}$	10	$5 \cdot 10^3$

$\hbar\omega_L$ in eV, focus in nm, I in W/cm^2,

A free electron cannot absorb a photon

- 4-momentum of electron and photon

$$p^\mu = \begin{pmatrix} p^0 \\ \boldsymbol{p} \end{pmatrix}, \quad p^0 = \frac{E}{c}, \quad E_\gamma = \hbar\omega$$

Lorentz invariant

$$p^\mu p_\mu \equiv p \cdot p = \frac{E^2}{c^2} - \boldsymbol{p}^2 = m^2c^2$$

- p, p': incoming, outgoing free electrons $\rightarrow p^2 = p'^2$
  p_γ: absorbed photon $\rightarrow p_\gamma^2 = 0$
- energy and momentum conserved

$$p + p_\gamma = p' \implies 0 = pp_\gamma = \frac{EE_\gamma}{c^2} - \boldsymbol{p}\boldsymbol{p}_\gamma$$

- outgoing electron at rest: $\boldsymbol{p}' = 0$ or $\boldsymbol{p} + \boldsymbol{p}_\gamma = 0 \Longrightarrow$

$$\frac{EE_\gamma}{c^2} + \boldsymbol{p}^2 = 0 \Longrightarrow |\boldsymbol{p}| = |\boldsymbol{p}_\gamma| = 0$$

- $\Longrightarrow$ no absorption
- can plane wave field accelerate e^-
- possible for bound e^- or e^- in external field

Thomson-Scattering

- *elastic scattering* of radiation by free particle (*ED*)
- $\boldsymbol{E}$ accelerates particle $\to$ radiation emitted with $\nu = \nu_{\text{incident}}$
- particle non-relativistic $\to$ $\boldsymbol{B}$ negligible, el. dipol radiation
- radiation polarized along the direction of particle motion

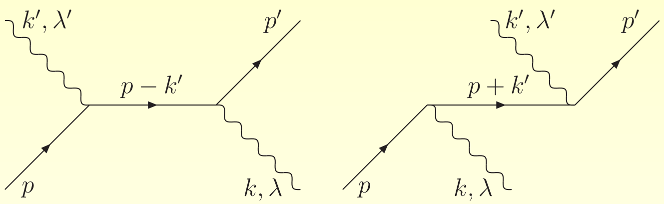
total cross section

$$\sigma_{\text{tot}} = \frac{8\pi}{3} r_e^2 = 6.65 \cdot 10^{-25}\,\text{cm}^2$$

Compton-Scattering

- *inelastic scattering* of energetic γ off e^-
- γ accelerates e^- and looses energy
- low-intensity $\Rightarrow$ light consists of particles

Feynman diagrams



- $p + \hbar k = p' + \hbar k' \Longrightarrow$

$$\lambda' - \lambda = \lambda_e(1 - \cos\theta), \quad \cos\theta = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}'$$

- cross section (unpolarized, incoherent): Klein-Nishina

$$\frac{d\sigma}{d\Omega}\Big|_{\text{KN}} = \frac{r_e^2}{2} P(E_\gamma,\theta)^2 \left(P(E_\gamma,\theta) + \frac{1}{P(E_\gamma,\theta)} - \sin^2\theta \right),$$

$$\text{with} \quad P(E_\gamma,\theta) = \frac{E'_\gamma}{E_\gamma} = \frac{\lambda}{\lambda'}$$

- total cross section: Set $E_\gamma/mc^2 = \eta \Longrightarrow$

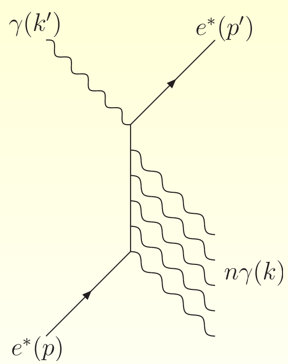
$$\sigma_{\text{KN}} = 2\pi r_e^2 \left(\frac{1+\eta}{\eta^2} \left[\frac{2+2\eta}{1+2\eta} - \frac{\ln(1+2\eta)}{\eta} \right] + \frac{\ln(1+2\eta)}{2\eta} - \frac{1+3\eta}{(1+2\eta)^2} \right)$$

$\lambda \gg \lambda_e \Longrightarrow \eta \ll 1$: Compton $\to$ Thomson scattering

inverse Compton-Scattering:

- $=$ Compton scattering in different inertial system
- soft γ + hard e^- $\Longrightarrow$ high energy photon
- CMB photons + hot e^- $\Longrightarrow$ *Sunyaev-Zel'dovich effect*
- laser light + relativistic e^- $\Longrightarrow$ backscattered γ (GeV)

nonlinear Compton scattering



Burke et al., Slac E-144

SI-units vs. Heaviside-Lorentz units

- SI-units: Coulomb law $\boldsymbol{F} = q_1 q_2/4\pi\epsilon_0 r^2$, fields $\boldsymbol{E}$, $\boldsymbol{B}$ and q in

$$\frac{\mathrm{V}}{\mathrm{m}}, \quad \mathrm{Tesla} = 10^4\,\mathrm{Gauss}, \quad \mathrm{Coulomb} = \mathrm{As}$$

- HL-units: cm, s and gramm; Coulomb law $\boldsymbol{F} = q_1 q_2/4\pi r^2$

$$\nabla\cdot\boldsymbol{B} = 0 \quad , \quad \nabla\wedge\boldsymbol{E} + \frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t} = 0$$
$$\nabla\cdot\boldsymbol{E} = \rho \quad , \quad \nabla\wedge\boldsymbol{B} - \frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t} = \frac{1}{c}\boldsymbol{J}$$

charges in electro-static units esu

$$[q] = \frac{\mathrm{cm}^{3/2}\mathrm{g}^{1/2}}{\mathrm{s}} = \mathrm{esu} \Longrightarrow e = 1.702\,691\cdot 10^{-9}\mathrm{esu}$$

- Coulomb and Maxwell $\Rightarrow [\boldsymbol{E}] = [\boldsymbol{B}] = \mathrm{esu/cm}^2$
- conversion factors SI $\longleftrightarrow$ HL

$$\frac{1}{\sqrt{\epsilon_0}} = 1.0627\cdot 10^{10}\frac{\mathrm{esu}}{\mathrm{As}}$$
$$\sqrt{\epsilon_0} = \frac{10^8}{\sqrt{4\pi}c}\frac{\mathrm{esu/cm}^2}{\mathrm{V/cm}} = 9.41\times 10^{-4}\frac{\mathrm{esu}}{\mathrm{Vcm}}$$
$$\mu_0 = \sqrt{4\pi}\frac{\mathrm{Gauss}}{\mathrm{esu/cm}^2}.$$

$E = B$ in Heaviside-Lorentz units $\Longrightarrow$

$$B_{\mathrm{SI}} = \frac{E_{\mathrm{SI}}}{10^{-8}\,c} \approx \frac{E_{\mathrm{SI}}}{300}$$

Fields and particles

- trajectory of charged particle in strong (laser) field
- accelerated particle radiates $\rightarrow$ higher harmonics
- radiation reacts back on charged particle
- use relativistically invariant formulation
- Lorentz-invariant distance

$$\xi^\mu\xi_\mu = \xi_0^2 - \boldsymbol{\xi}^2, \quad \xi^\mu = \begin{pmatrix}\xi^0\\ \boldsymbol{\xi}\end{pmatrix}, \quad \boldsymbol{\xi} = \begin{pmatrix}\xi^1\\ \xi^2\\ \xi^3\end{pmatrix}$$

- charge- and current density $\rightarrow$ 4-current

$$J^\mu = \begin{pmatrix}c\rho\\ \boldsymbol{J}\end{pmatrix}$$

$\Longrightarrow$ transformation under change of inertial system

- homogeneous Maxwell equations → potentials

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial}{\partial t}\boldsymbol{A} \quad \text{and} \quad \boldsymbol{B} = \nabla \wedge \boldsymbol{A}$$

- introduce 4-potential (in esu/cm)

$$A^\mu = \begin{pmatrix} \phi \\ \boldsymbol{A} \end{pmatrix} \equiv \begin{pmatrix} A^0 \\ \boldsymbol{A} \end{pmatrix}$$

- $\boldsymbol{E}, \boldsymbol{B} \Longrightarrow$ antisymmetric field strength tensor

$$(F_{\mu\nu}) = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- inhomogeneous Maxwell equations

$$\partial_\mu F^{\mu\nu} = \frac{1}{c} J^\nu \quad \stackrel{\partial_\mu A^\mu = 0}{\Longrightarrow} \quad \Box A^\mu = \frac{1}{c} J^\mu$$

energy-momentum tensor

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energy, momentum and stress of field

$$T^\mu{}_\nu = F^{\mu\alpha}F_{\alpha\nu} + \frac{1}{4}\delta^\mu{}_\nu F_{\alpha\beta}F^{\alpha\beta} \quad \text{in} \quad \frac{\mathrm{g}}{\mathrm{cm\ s}^2}$$

components

$$T_\mu{}^\nu = \begin{pmatrix} \frac{1}{2}\boldsymbol{E}\cdot\boldsymbol{E} + \frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{B} & (\boldsymbol{E}\wedge\boldsymbol{B})^t \\ \boldsymbol{B}\wedge\boldsymbol{E} & \boldsymbol{E}\boldsymbol{E}^t + \boldsymbol{B}\boldsymbol{B}^t - \frac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{E} + \boldsymbol{B}\cdot\boldsymbol{B})\mathbb{1} \end{pmatrix}$$

- T^{00}: energy-density
- T^{0i}: energy flux $\boldsymbol{S} = c\boldsymbol{E}\wedge\boldsymbol{B}$
- $\langle \boldsymbol{E}\wedge\boldsymbol{B}\rangle$: radiation pressure

relativistic particles

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- particle , charge e, mass m_0, trajectory $x^\mu(\tau)$
- proper time $\tau \longleftrightarrow$ coordinate time t

$$d\tau = \frac{1}{c}\sqrt{dx_\mu dx^\mu} = \sqrt{dt^2 - d\boldsymbol{x}^2/c^2} = dt\sqrt{1-\beta^2} = \frac{dt}{\gamma}$$

$$\Longrightarrow \partial_\tau = \gamma\partial_t, \quad \text{relativistic } \gamma\text{-factor}$$

- 4-velocity u^μ with $u^\mu u_\mu = 1$ (in units of c)

$$(u^\mu) = \frac{1}{c}\left(\frac{dx^\mu}{d\tau}\right) = \begin{pmatrix} \gamma \\ \gamma\boldsymbol{\beta} \end{pmatrix} \Longrightarrow dt = u^0(\tau)d\tau$$

- 4-momentum

$$p^\mu = mc\, u^\mu, \quad p_\mu p^\mu = m^2c^2$$

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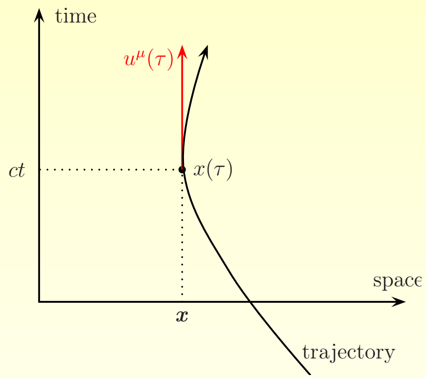

- charged particle in external field

$$\frac{d}{dt}(\gamma m_0 c^2) = e\boldsymbol{E}\cdot\boldsymbol{v} \qquad \text{change of } E$$

$$\frac{d}{dt}(\gamma m_0 \boldsymbol{v}) = e\boldsymbol{E} + e\boldsymbol{\beta}\wedge\boldsymbol{B} \qquad \text{change of } \boldsymbol{p}$$

$$\text{covariant} \qquad m_0\frac{du^\mu}{d\tau} = \frac{e}{c}F^\mu{}_\nu u^\nu$$

particles in constant fields

- ${F^\mu}_\nu$ constant $\Longrightarrow$ explicit solution

$$\frac{du^\mu}{d\tau} = \ell {F^\mu}_\nu u^\mu \Longrightarrow u^\mu(\tau) = {\left(e^{\ell\tau F}\right)^\mu}_\nu u^\nu(0), \quad \ell = \frac{e}{m_0 c}$$

- $e^{\ell F \tau}$ time dependent Lorentz transformation
- for $\boldsymbol{E} \perp \boldsymbol{B}$:
  - if $E > B$: system with $\boldsymbol{B} = 0$ (hyperbolic)
  - if $E < B$: system with $\boldsymbol{E} = 0$ (elliptic)
  - if $E = B$: (in all systems) (parabolic)
- for $\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$:
  - system with $\boldsymbol{E} \parallel \boldsymbol{B}$ (loxodromic = generic)

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hyperbolic case (boost)

- characteristic time scale

$$\tau_0 = \frac{m_0 c}{eE}, \quad \tau_0[\mathrm{fs}] = 1.7 \cdot \frac{1}{E[10^{10}\mathrm{V/cm}]}$$

- τ_0 = unit of times $\Longrightarrow$ $\tau = \operatorname{arcsinh}(t) \Longrightarrow$

$$\boldsymbol{\beta} = \frac{t}{\gamma}\,\hat{\boldsymbol{E}}, \quad \gamma = \sqrt{1 + t^2}$$

$t > \tau_0 \Longrightarrow$ electron relativistic

elliptic case (rotation)

- cyclotron frequency

$$\omega_c = \frac{eB}{m_0 c} = 1.76 \cdot 10^7 \cdot B[\mathrm{Gauss}]$$

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- associated energy scale $\hbar\omega_c = B/B_{\mathrm{crit}} \cdot mc^2$
- Particle circles in plane $\perp \boldsymbol{B}$, constant γ, e.g.

$$\boldsymbol{v}(t) = \begin{pmatrix} v_0 \cos \omega t \\ -v_0 \sin \omega t \\ 0 \end{pmatrix}, \quad \omega = \frac{\omega_c}{\gamma}$$

Particles in plane waves

- can a laser field (fixed ω) accelerate an electron?
- linear $\leftrightarrow$ circular polarization?
- plane wave approximation $\rightarrow$ motion exactly integrable
- acceleration at pulse front, decceleration at pulse tail

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- Lorentz gauge

$$\partial_\mu A^\mu = 0 \Longrightarrow \partial_\mu F^{\mu\nu} = \Box A^\nu = 0$$

- plane wave: $A_\mu(\eta)$ with $\eta = k_\mu x^\mu$, $(k_\mu k^\mu = 0)$

$$\partial_\mu \propto k_\mu \Longrightarrow k_\mu F^{\mu\nu} = 0$$

- conservation law

$$(*) \quad \frac{du^\mu}{d\tau} = \frac{e}{mc} {F^\mu}_\nu u^\nu \Longrightarrow k_\mu \frac{du^\mu}{d\tau} = 0$$

$\Longrightarrow$

$$k_\mu u^\mu = k_\mu u_0^\mu \equiv \frac{\Omega}{c} \Longrightarrow k_\mu x^\mu(\tau) = \Omega\tau$$

- Solving equations of motion:

$$F^{\mu}{}_{\nu}u^{\nu} = u^{\nu}\partial^{\mu}A_{\nu} - u^{\nu}\partial_{\nu}A^{\mu} = u^{\nu}\partial^{\mu}A_{\nu} - \frac{1}{c}\frac{dA^{\mu}}{d\tau}$$

- dimensionless potential $a^{\mu} = eA^{\mu}/mc^2$, (*) $\Longrightarrow$

$$\frac{d}{d\tau}\left(u^{\mu} + a^{\mu}\right) = c\,k^{\mu}u^{\nu}a'_{\nu}, \quad ' = \frac{d}{d\eta}$$

- polarization 4-vector ε_{μ}, $\varepsilon_{\mu}k^{\mu} = 0$
  $\Longrightarrow$ two conservation laws

$$\varepsilon_{\mu}v^{\mu} = \text{const.}, \qquad v^{\mu} = u^{\mu} + a^{\mu}$$

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- solving the equation for canonical velocity v^{μ}:

$$(**) \quad \frac{dv^{\mu}}{d\tau} = c\,k^{\mu}u^{\nu}a'_{\nu} = c\,k^{\mu}v^{\nu}a'_{\nu} - c\,k^{\mu}a^{\nu}a'_{\nu}$$

- along trajectory $d\eta = \Omega d\tau \Longrightarrow$

$$k^{\mu}a^{\nu}a'_{\nu} = \frac{k^{\mu}}{2\Omega}\frac{d}{d\tau}(a^{\nu}a_{\nu}) = k^{\mu}\frac{d}{cd\tau}\left(\frac{a_{\mu}a^{\nu}}{2k_{\nu}u_0^{\nu}}\right)$$

- derivative term to left in (**). Introduce

$$w^{\mu} = v^{\mu} + k^{\mu}\frac{a_{\nu}a^{\nu}}{2k_{\nu}u_0^{\nu}}$$

- Lorentz gauge $k^{\mu}a_{\mu} = 0 \Longrightarrow v^{\mu}a_{\mu} = w^{\mu}a_{\mu} \Longrightarrow$

$$\frac{dw^{\mu}}{d\tau} = c\,k^{\mu}v^{\nu}a'_{\nu} = c\,k^{\mu}a'_{\nu}\,w^{\nu}$$

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- time-dependent Schrödinger equation, nilpotent $H \propto k^{\mu}a'_{\nu}$

$$k^{\mu}a'_{\alpha}(\eta_1)\,k^{\alpha}a'_{\nu}(\eta_2) = 0$$

- solution by iteration

$$w^{\mu}(\tau) = w_0^{\mu} + \frac{k^{\mu}}{k_{\alpha}u_0^{\alpha}}\Delta a_{\nu}(\Omega\tau)\,w_0^{\nu}, \quad \Delta a^{\mu} = a^{\mu} - a_0^{\mu}.$$

- $\Longrightarrow$ 4-velocity at proper time τ:

$$u^{\mu}(\tau) = u_0^{\mu} - \left(\Delta a^{\mu} - k^{\mu}\frac{\Delta a_{\alpha}u_0^{\alpha}}{k_{\alpha}u_0^{\alpha}}\right) - k^{\mu}\frac{\Delta a_{\alpha}\Delta a^{\alpha}}{2k_{\alpha}u_0^{\alpha}}$$

- $u^{\mu}u_{\mu} = 1$ along trajectory, gauge invariant
- pulse: u^{μ}(before pulse) = u^{μ}(after pulse)
  $\Longrightarrow$ Lawson-Woodward theorem applies

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- plane wave in 3−direction

$$a_{\mu} = (0, a_1(\eta), a_2(\eta), 0)\,, \quad \eta = \omega(t - z/c)$$

- particle initially at rest $\Longrightarrow u_0 = 0 \Longrightarrow \Omega = \omega \Longrightarrow$

$$u^{\mu}_{\text{rest}} = \left(1 + u^3, \Delta a_1, \Delta a_2, \frac{1}{2}(\Delta a_1)^2 + \frac{1}{2}(\Delta a_2)^2\right)$$

- time-averaged relativistic factor

$$\bar{\gamma} = 1 + \frac{1}{2}\langle(\Delta a_1)^2\rangle + \frac{1}{2}\langle(\Delta a_2)^2\rangle$$

- particle without drift $\Longrightarrow$ choice of u_0 $\Longrightarrow$

$$u_{\rm cm} = \left(\frac{\Omega}{\omega} + u^3, g_1, g_2, \frac{\omega}{2\Omega}\sum\left(g_p^2 - \langle g_p^2\rangle\right)\right)$$
$$g_p = \Delta a_p - \langle \Delta a_p\rangle$$

- time-averaged relativistic factor from $u_\mu u^\mu = 1$:

$$\bar\gamma^2 = \frac{\Omega^2}{\omega^2} = 1 + \sum_p \langle g_p^2\rangle$$

periodic waves

- average over period: $\langle a_\mu\rangle = 0,\ p^\mu = m_0 c u^\mu \Longrightarrow$

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$\Rightarrow$ effective momentum

$$\langle p^\mu\rangle = p_0^\mu - \frac{mc}{2k_\alpha u_0^\alpha}\langle \Delta a_\beta \Delta a^\beta\rangle\, k^\mu$$

- harmonic wave $E = \omega A/c$ and

$$\langle \Delta a_\mu \Delta a^\mu\rangle = -\left(\frac{e}{\hbar\omega}\right)^2 \langle E^2\rangle,\quad a_0^2 = \left(\frac{e}{m\omega c}\right)^2 \langle E^2\rangle$$

- effective momentum

$$\langle p^\mu\rangle = p_0^\mu + \frac{c^2}{2k_\alpha p_0}(ma_0)^2\, k^\mu$$

- effective mass $m_* \gg m$

$$\langle p_\mu\rangle\langle p^\mu\rangle = m^2c^2(1+a_0^2) \Longrightarrow m_* = m\sqrt{1+a_0^2}$$

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harmonic, linear polarized wave

$$a_\mu = (0, \alpha\sin\eta, 0, 0),\quad \text{length in } \lambda\!\!\!^-$$

- particle initially at rest:

$$x^1 = \alpha(1-\cos\omega\tau),\quad x^2 = 0,\quad x^3 = \frac{\alpha^2}{8}(2\omega\tau - \sin 2\omega\tau)$$

- trajectory $\perp \boldsymbol{B}$, drift in direction $\boldsymbol{E}\wedge\boldsymbol{B}$
- cusps at which $\boldsymbol{v} = 0$
- particle without drift: $\bar\gamma^2 = 1 + \frac{1}{2}\alpha^2$

$$x^1 = -\frac{\alpha}{\bar\gamma}\cos\bar\gamma\omega\tau,\quad x^2 = 0,\quad x^3 = -\frac{\alpha^2}{8\bar\gamma^2}\sin 2\bar\gamma\omega\tau$$

- oscillations ω and 2ω, trajectory forms 'eight'

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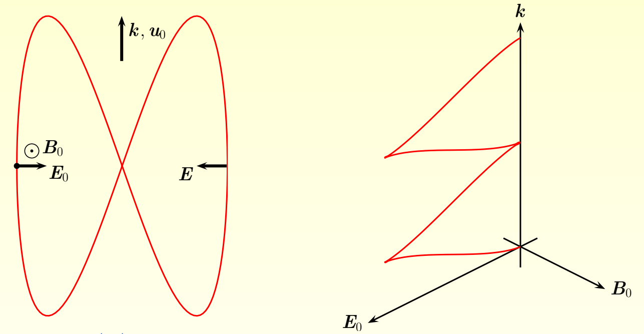
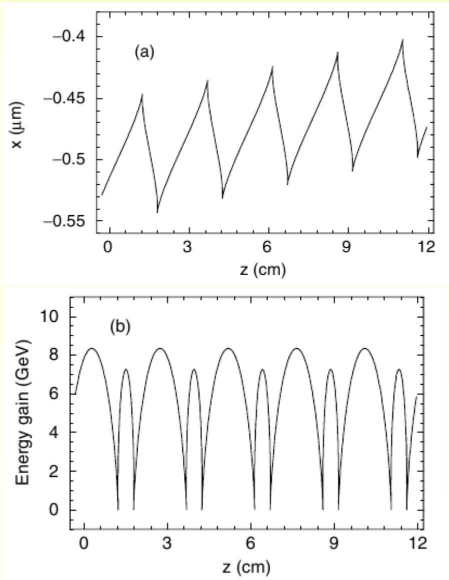


$\langle \boldsymbol{v}\rangle = 0$ $\qquad$ $v_0 = 0$

- $\boldsymbol{v}$ maximal at intersection point ($\boldsymbol{E} = \boldsymbol{B} = 0$)
- $\boldsymbol{v} = 0$ where field is maximal.



$\lambda = 1.056\,\mu$m, $\alpha = 100$, $I = 1.35 \cdot 10^{22}/\lambda^2$ W/cm^2

Salamin, Mocken, Keitl (2002)

harmonic, circular polarized wave

$$a_\mu = (0, \alpha \sin\eta, -\alpha\cos\eta, 0)\,, \quad \text{length in} \quad ƛ$$

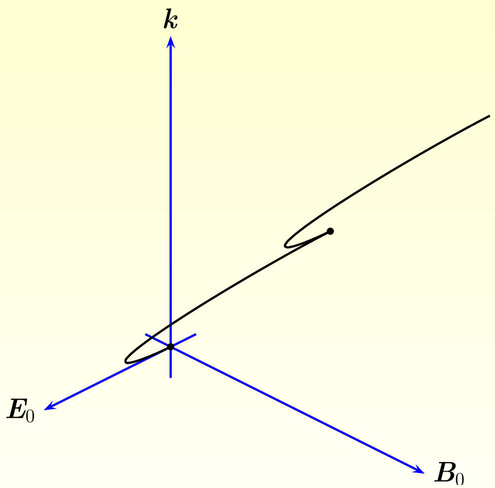
- no drift: average $\gamma = 1 + \alpha^2$

$$x^1 = -\frac{\alpha}{\gamma}\cos\gamma\omega\tau, \quad x^2 = -\frac{\alpha}{\gamma}\sin\gamma\omega\tau, \quad x^3 = 0$$

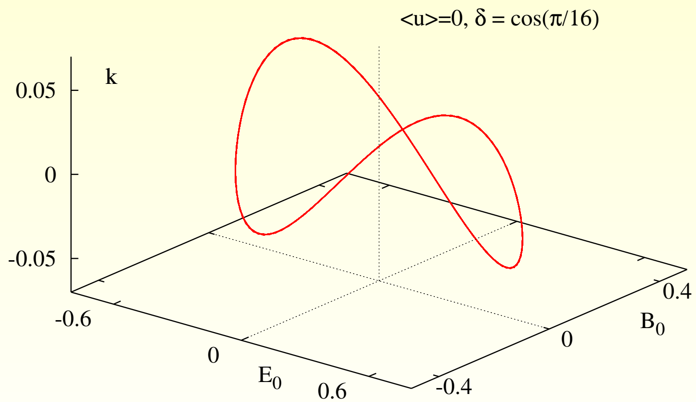
  trajectory = circle in $\boldsymbol{E},\boldsymbol{B}-$plane
- initially at rest:

$$x^1 = \alpha(1-\cos\omega\tau), \quad x^2 = \alpha(\omega\tau - \sin\omega\tau), \quad x^3 = \alpha x^2$$

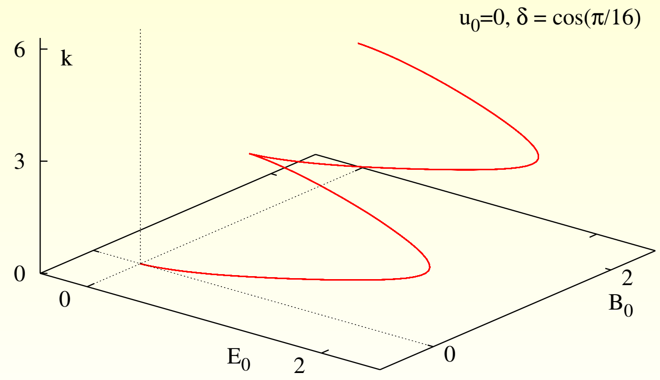
  cusps for $\omega\tau = 2\pi n$



Two cycles for particle initially at rest



elliptic polarization, no drift



elliptic polarization, particle initially at rest

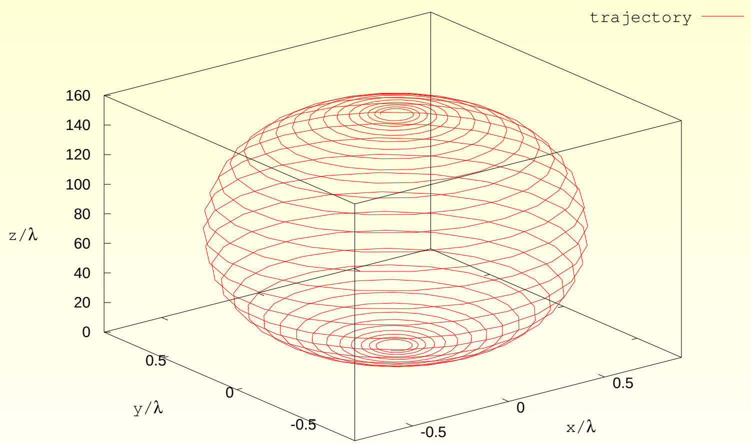
Modulated pulse

gauge potential

$$a_\mu = \alpha e^{-(\eta/\xi)^2}\,(0, \sin\eta, -\delta\cos\eta, 0)\,, \qquad \eta = k\cdot x$$

- ξ: pulse length
- δ: type of polarization
- initially at rest $\Longrightarrow \Omega = \omega,\ \eta = \omega\tau$
- time evolution from $d\boldsymbol{x} = \boldsymbol{u}cd\tau$ with

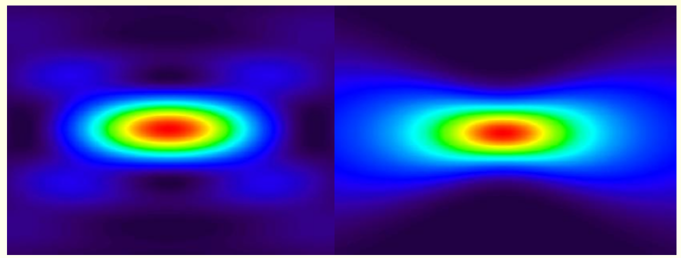
$$\begin{aligned} u_x &= \alpha e^{-\eta^2/\xi^2}\sin\eta \\ u_y &= -\alpha\delta e^{-\eta^2/\xi^2}\cos\eta \\ u_z &= \frac{\alpha^2}{2}e^{-2\eta^2/\xi^2}\left(\sin^2\eta + \delta^2\cos^2\eta\right) \end{aligned}$$



particle in modulates pulse: $\alpha = 5,\ \xi = 10\lambda$

- Lawson-Woodward theorem $\rightarrow$ violate adiabaticity
- possible in focal spot $w_0 \sim \lambda \Longrightarrow$ acceleration
- modelling focal spot of tightly focused laser pulse

$$\partial_\mu A^\mu = 0 \quad \text{and} \quad \Box \boldsymbol{A} = 0$$



exact solution — paraxial approximation, diffraction angle $w_0/z_r \ll 1$

- monochromatic wave $\Longrightarrow$ Helmholtz equation

$$\triangle \boldsymbol{A} + \boldsymbol{k}^2 \boldsymbol{A} = 0$$

- prescribe field on focal plane $z = 0$

$$\boldsymbol{A}(x, y, z = 0) = \boldsymbol{A}_0(x, y)$$

- Fourier transform on focal plane

$$\boldsymbol{A}_0(x, y) = \int \hat{\boldsymbol{A}}_0(\boldsymbol{\xi}) e^{ik_0(\xi_1 x + \xi_2 y)} d^2\xi$$

- away from focal plane

$$\boldsymbol{A}(x, y, z) = \int \hat{\boldsymbol{A}}(\boldsymbol{\xi}, z)\, e^{ik_0(\xi_1 x + \xi_2 y)} d^2\xi$$

- Helmholtz equation for $\boldsymbol{A}(\boldsymbol{x}) \Longrightarrow$

$$\frac{\partial^2 \hat{\boldsymbol{A}}}{\partial z^2} + k_0^2(1 - \xi^2)\, \hat{\boldsymbol{A}} = 0 \Longrightarrow \hat{\boldsymbol{A}}(z, \boldsymbol{\xi}) \Longrightarrow \boldsymbol{A}(\boldsymbol{x})$$

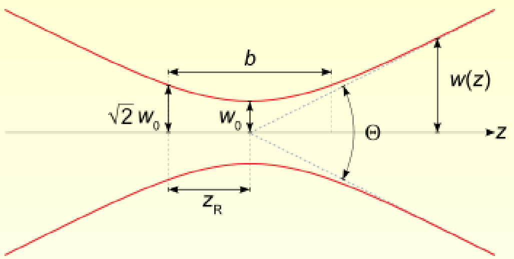
- solution

$$\boldsymbol{A}(\boldsymbol{x}) = \int \hat{\boldsymbol{A}}_0(\boldsymbol{\xi}) e^{imk_0 z} e^{ik_0(\xi_1 x + \xi_2 y)} d^2\xi, \quad m = \sqrt{1 - \xi^2}$$

- fix the field amplitude on the focal plane

$$\hat{\boldsymbol{A}}(x, y, z = 0) = \alpha_0\, \boldsymbol{e}_x \exp\left(-\frac{ik_0\rho^2}{4w_0 f}\right) \exp\left(-\frac{\rho^2}{2w_0^2}\right)$$

Bockarev, Bychenkov (2007)



- ρ: distance from z-axis
- $w_0 \propto$ beam radius at waist
- $f \Longrightarrow$ convergence angle of the wave front
- $\alpha_0 = |\alpha_0|\, e^{i\phi_0}$ constant, ϕ_0 phase on plane

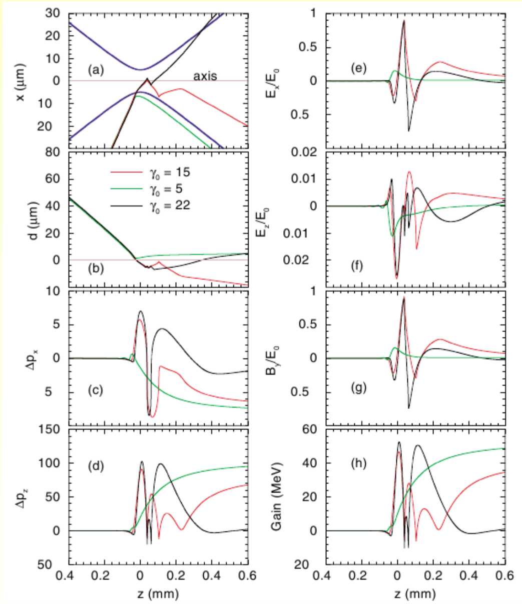
- inverse Fourier transform $\Longrightarrow$ exact solution

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{x}) &= e^{-i\omega t} \boldsymbol{e}_x \int_0^1 Q_1(z, \xi) J_0(k_0 \xi \rho) d\xi \\ &+ e^{-i\omega t} \boldsymbol{e}_x \int_1^\infty Q_2(z, \xi) J_0(k_0 \xi \rho) d\xi \end{aligned}$$

- coefficient functions

$$\begin{aligned} Q_1 &= \frac{\alpha_0 \xi}{\epsilon_0^2 \alpha} \exp\left(-\frac{\xi^2}{2\epsilon_0^2 \alpha}\right) \exp\left(ik_0 \sqrt{1 - \xi^2}\, z\right) \\ Q_2 &= \frac{\alpha_0 \xi}{\epsilon_0^2 \alpha} \exp\left(-\frac{\xi^2}{2\epsilon_0^2 \alpha}\right) \exp\left(-k_0 \sqrt{\xi^2 - 1}\, z\right) \end{aligned}$$

beam parameter $\quad \epsilon_0 = \dfrac{1}{k_0 w_0}, \quad \alpha = 1 + \dfrac{i}{2f\epsilon_0}$



reflection
capture
transmission
$z_0 = -5$mm
$\theta_i = 10^0$
$\alpha = 10$
$w_0 = 5\mu$m
$\lambda = 1\mu$m
gains:
43.2 MeV
34.5 MeV
4.0 MeV

(Salamin et al.)

- $\boldsymbol{A} \Longrightarrow \boldsymbol{E}, \boldsymbol{B} \Longrightarrow$ trajectories from Lorentz equation
- Δp_μ depends sensitively on initial conditions
- relativistic energies after few cycles possible
- escape at large angles to z-axis
- paraxial approximation overestimates $E_{\max}$

radiation of particles

external field → accelerated charges
accelerated charges → radiation

- no incoming radiation, solution of $\Box A^\mu = J^\mu$

$$A^\mu(x) = \frac{1}{c}\int d^4y\, D_{\rm ret}(x-y)J^\mu(y)$$

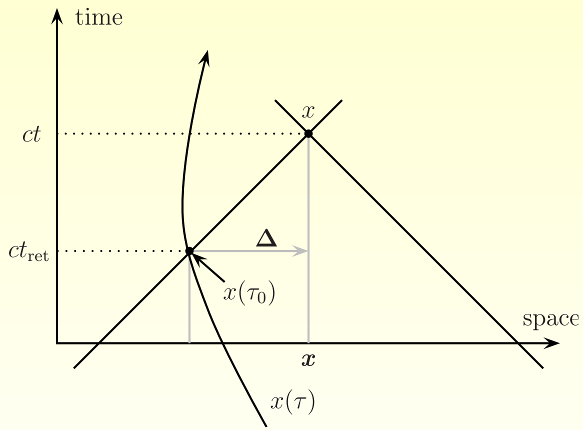
- retarded Greenfunction

$$D_{\rm ret}(x) = \frac{1}{2\pi}\theta(x^0)\delta(x^2) = \frac{1}{4\pi r}\delta(ct-r)$$

- point particle with world-line $x^\mu(\tau)$

$$J^\mu(x) = ec\int d\tau\, \dot{x}^\mu(\tau)\,\delta^4(x - x(\tau))$$

- insert, integration over d^4y and $d\tau \Longrightarrow$ only $x^\mu(\tau_0)$



- non-covariant: $\tau \to t$, $\boldsymbol{\Delta}(t) = \boldsymbol{x} - \boldsymbol{x}(t)$ and $\boldsymbol{\Delta} = R\boldsymbol{n} \Longrightarrow$

- magnetic field $\boldsymbol{B} = \boldsymbol{n}(t_{\rm ret}) \wedge \boldsymbol{E}$ with

$$\boldsymbol{E}(t,\boldsymbol{x}) = \frac{e}{4\pi}\left[\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\boldsymbol{n})^3 R^2}\right]_{\rm ret} + \frac{e}{4\pi c}\left[\frac{\boldsymbol{n}\wedge\{(\boldsymbol{n}-\boldsymbol{\beta})\wedge\dot{\boldsymbol{\beta}}\}}{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})^3 R}\right]_{\rm ret}$$

Lienard-Wiechert

- velocity field $\propto 1/R^2$: boosted Coulomb field
- acceleration field $\propto 1/R$: radiation

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Radiated power

- electrodynamics

$$P = \lim_{R\to\infty} R^2 \oint d\Omega\, \boldsymbol{n}\cdot\boldsymbol{S}, \quad \boldsymbol{S} = c\boldsymbol{E}\wedge\boldsymbol{B}$$

- only "acceleration fields" contribute
- power radiated in charge's own time per unit solid angle

$$\frac{dP(t')}{d\Omega} = R^2 \boldsymbol{S}\cdot\boldsymbol{n}\frac{dt}{dt'}$$

$$(*) \quad \frac{dP(t')}{d\Omega} = \frac{e^2}{16\pi^2 c}\left[\frac{\left|\boldsymbol{n}\wedge\left[(\boldsymbol{n}-\boldsymbol{\beta})\wedge\dot{\boldsymbol{\beta}}\right]\right|^2}{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})^5}\right]_{t'}$$

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- $\int d\Omega \Longrightarrow$ Lorentz invariant radiation power
- calculation: system with $\beta(t_{\rm ret}) = 0$, denominator $= 1$

$$\int d\Omega \left[(\boldsymbol{n}\wedge(\boldsymbol{\beta}\wedge\dot{\boldsymbol{\beta}}\right]^2 = \frac{8\pi}{3}\dot{\boldsymbol{\beta}}^2$$

- 4-acceleration

$$a^\mu = \frac{du^\mu}{d\tau}, \quad u^\mu u_\mu = 1 \Longrightarrow u^\mu a_\mu = 0$$

- in this system: $\gamma = 1$ and $d\gamma/dt = \gamma^3\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}} = 0 \Longrightarrow$

$$a_\mu a^\mu = -\dot{\boldsymbol{\beta}}^2$$

- total emitted power in charge's own time

$$W_{\rm rad} = -\frac{2}{3}\frac{e^2}{4\pi c}(a_\mu a^\mu)_{\rm ret}$$

- no other Lorentz invariant $f(u,a)$.

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- linear acceleration: rel. Lamour formula
- angular distribution is tilted forward

$$\cos\theta_{\rm max} = \frac{1}{3\beta}\left(\sqrt{1+15\beta^2}-1\right) \Longrightarrow \theta_{\rm max} = 1/2\gamma$$

constant $\boldsymbol{E}$ field (Unruh radiation)

$$\boldsymbol{\beta} = \frac{t}{\sqrt{1+t^2}}\boldsymbol{e}_\parallel \quad (t \text{ in units of } \tau_0)$$

- radiation formula $\Longrightarrow$

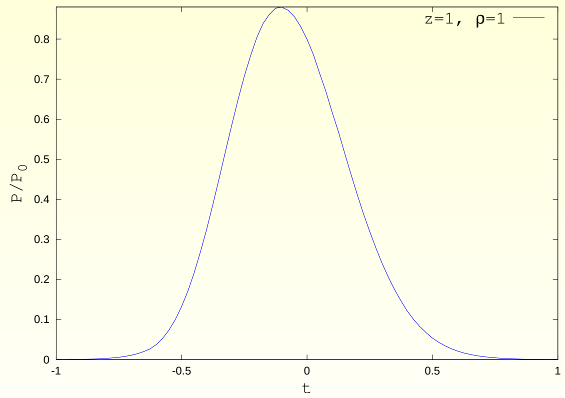
$$R^2\boldsymbol{n}\cdot\boldsymbol{S} = \frac{e^2}{16\pi^2 c}\frac{(1-n_\parallel^2)}{\left[(1+t^2)^{1/2}-n_\parallel t\right]^6}\Bigg|_{t_{\rm ret}}$$

- particular inertial system:

$$z^2(t) = 1 + t^2 \quad z \text{ in units of } c\tau_0$$

retarded time t_{ret} from

$$(ct - ct_{\text{ret}})^2 = x^2 + y^2 + (z - z(t_{\text{ret}}))^2$$



constant B-field

- constant $\boldsymbol{B} \Longrightarrow \boldsymbol{\beta} \cdot \dot{\boldsymbol{\beta}} = 0 \Longrightarrow$

$$W_{\text{rad}} = \frac{2}{3}\frac{e^2}{4\pi c}\gamma^4\left(\dot{\boldsymbol{\beta}}^2 + \gamma^2(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}})^2\right)$$

- times in units of $1/\omega = \gamma/\omega_c$, length in units of c/ω

$$\boldsymbol{x}(t) = \beta_0 \boldsymbol{e}_1 \sin t + \beta_0 \boldsymbol{e}_2 \cos t$$

- $\Longrightarrow$ retarded time t_{ret}
- radiation maximal for $\boldsymbol{n} \parallel \boldsymbol{\beta}$
- for $\boldsymbol{B} = B\boldsymbol{e}_3$, orbit in (x, y) plane

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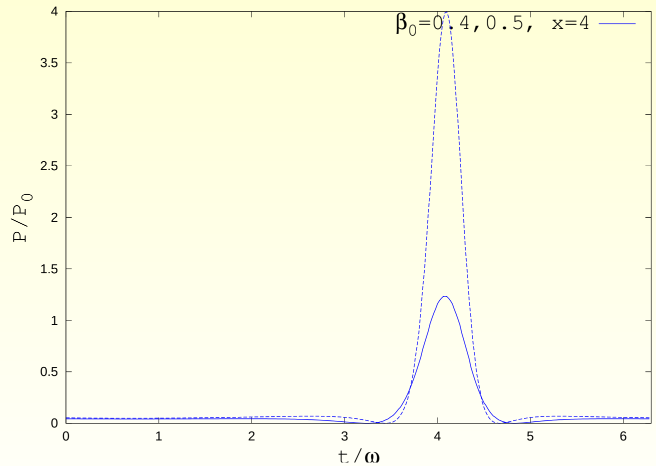
- initial position $\boldsymbol{x} = (0, \beta_0)$, initial velocity $\boldsymbol{\beta} = (\beta_0, 0)$



radiated energy near the x-axis for two values of β_0

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Radiation for J^μ, momentum space

- covariant gauge $\partial_\mu A^\mu \Longrightarrow$

$$\Box A^\mu = J^\mu \quad \text{with} \quad \partial_\mu J^\mu = 0 \qquad (c = 1)$$

- solution

$$A^\mu(x) = A^\mu_{\text{in}}(x) + \int d^4y\, D_{\text{ret}}(x - y) J^\mu(y)$$
$$A^\mu = A^\mu_{\text{in}} + D_{\text{ret}} * J^\mu, \quad \Box A^\mu_{\text{in}} = 0$$

- also possible

$$A^\mu = A^\mu_{\text{out}} + D_{\text{adv}} * J^\mu, \quad \Box A^\mu_{\text{out}} = 0$$

- assume $A^\mu_{\text{in}} = 0$

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- radiated energy

$$P^{\rm rad}_\mu = P_\mu(A^{\rm out}) - P_\mu(A^{\rm in}) = P_\mu(A^{\rm out})$$

- solve

$$A^\mu = D_{\rm ret} * J^\mu = A^\mu_{\rm out} + D_{\rm adv} * J^\mu$$

- for outgoing radiation

$$A^\mu_{\rm out} = D * J^\mu, \quad D = D_{\rm ret} - D_{\rm adv}$$

- $D(x)$ Pauli-Jordan function for photons

$$D(x) = \frac{1}{2\pi}\epsilon(x^0)\,\delta(x^2) = \frac{1}{4\pi r}\left(\delta(ct-r) - \delta(ct+r)\right)$$
$$D(k) = \frac{i}{(2\pi)^3}\epsilon(k^0)\delta(k^2).$$

- outgoing radiation in k-space

$$A^\mu_{\rm out}(k) = (2\pi)^4 D(k) J^\mu(k) = 2\pi i\epsilon(k^0)\delta(k^2) J^\mu(k)$$

- source free solution

$$A^\mu(x)_{\rm out} = \int \delta(k^2) A^\mu_{\rm out}(k) e^{-ikx} d^4k$$

- energy momentum tensor

$$T^\mu{}_\nu = F^{\mu\alpha}F_{\alpha\nu} + \frac{1}{4}\delta^\mu{}_\nu F_{\alpha\beta}F^{\alpha\beta}, \quad \partial_\mu T^\mu{}_\nu = J^\alpha F_{\alpha\nu} = 0$$

- source free field

$$\partial_\mu T^\mu{}_\nu = 0 \Longrightarrow P^\mu = \int d^3x\, T^\mu{}_0 \quad \text{conserved}$$

- result of several integrations

$$P^\mu = -4\pi^3 \int d^4k\,\delta(k^2)\epsilon(k^0)\, A^\alpha_{\rm out}(k) A^*_{{\rm out}\,\alpha}(k)$$

- insert $A^\mu_{\rm out}(k) \Longrightarrow$

$$P^\mu_{\rm rad} = 16\pi^5 \int d^4k\,\delta(k^2)\epsilon(k^0) k^\mu\, J^\alpha(k) J^*_\alpha(k)$$

- $k_\mu J^\mu = k^0 J^0 - \boldsymbol{k}\cdot\boldsymbol{J} = 0 \Longrightarrow$

$$J^\alpha J^*_\alpha = \boldsymbol{J}_\perp \cdot \boldsymbol{J}^*_\perp, \quad \boldsymbol{J}_\perp = \hat{\boldsymbol{k}} \wedge (\hat{\boldsymbol{k}} \wedge \boldsymbol{J})$$

$\Longrightarrow$ textbook expressions

- position space

$$P^{\rm rad}_\mu = \frac{1}{2}\int d^4x d^4y J^\alpha(x) J_\alpha(y) \partial_\mu D(x-y)$$

- $\boldsymbol{P^{\rm rad}_\mu}$: total energy-momentum transfer $J^\mu \to$ field
- total energy radiated

$$P_{\rm rad} = cP^{\rm rad}_0$$

- applications $\to$ differential spectral density

$$P_{\rm rad} = \int I(\omega,\Omega) d\omega d\Omega$$

- schematic form, also in quantum electrodynamics

$$\text{radiated power} \propto \int d^4k [\text{current}]^2 \times [\text{light cone}].$$

- point particles

$$J^\mu(x) = e\int d\tau \dot{x}^\mu \delta^4(x - x(\tau))$$
$$J^\mu(k) = \frac{e}{(2\pi)^4} e^{ik\cdot x(\tau)} \dot{x}^\mu(\tau)$$

Radiation reaction

- source of radiation affected by radiation reaction
- modification of Lorentz force law
- separate self-field and radiation field

Non-relativistic particle

- Lorentz and Planck

$$m\dot{\boldsymbol{v}} = \boldsymbol{F}_{\text{ext}} + \boldsymbol{F}_{\text{resist}}, \quad \boldsymbol{F}_{\text{resist}} = \frac{2}{3c^3}\frac{e^2}{4\pi}\ddot{\boldsymbol{v}} = \frac{2}{3}\frac{mr_0}{c}\ddot{\boldsymbol{v}}$$

- $\boldsymbol{F}_{\text{ext}} = 0 \Longrightarrow$ self-accelerating solution

$$\boldsymbol{v} = \boldsymbol{v}_0\, e^{t/\tau}, \quad \tau = \frac{2}{3}\frac{r_0}{c} \approx 6.26\cdot 10^{-24}\,\text{s}$$

- $F_{\text{resist}} \ll F_{\text{ext}} \Longrightarrow$ perturbation expansion
- leading order $m\dot{\boldsymbol{v}} \approx \boldsymbol{F}_{\text{ext}}$, next to leading

$$m\dot{\boldsymbol{v}} = \boldsymbol{F}_{\text{ext}} + \boldsymbol{f}^{(1)}, \quad \boldsymbol{f}^{(1)} \approx \frac{2}{3}\frac{r_0}{c}\dot{\boldsymbol{F}}_{\text{ext}}$$

- with $\boldsymbol{F}_{\text{ext}} = e\boldsymbol{E} + e\boldsymbol{\beta}\wedge\boldsymbol{B} \Longrightarrow$

$$\dot{\boldsymbol{F}}_{\text{ext}} = e\left(\dot{\boldsymbol{E}} + \dot{\boldsymbol{\beta}}\wedge\boldsymbol{B} + \boldsymbol{\beta}\wedge\dot{\boldsymbol{B}}\right).$$

- $\beta \ll 1$ and $\dot{\boldsymbol{v}} \approx e\boldsymbol{E}/m \Longrightarrow$

$$\dot{\boldsymbol{F}}_{\text{ext}} \approx e\dot{\boldsymbol{E}} + \frac{e^2}{mc}\boldsymbol{E}\wedge\boldsymbol{B}.$$

- radiation resistance in leading order

$$\boldsymbol{f}^{(1)} \approx \frac{2}{3c^3}\frac{e^2}{4\pi}\left(\frac{e\dot{\boldsymbol{E}}}{m} + \frac{e^2}{m^2c}\boldsymbol{E}\wedge\boldsymbol{B}\right).$$

- wave field $\dot{\boldsymbol{E}} \propto \omega\boldsymbol{E}$ and $\boldsymbol{E}\perp\boldsymbol{B}$ with $E = |\boldsymbol{E}| \approx |\boldsymbol{B}| \Longrightarrow$

$$f^{(1)} \approx F_{\text{ext}}\sqrt{\left(\frac{r_0}{\lambda\!\!\!^-}\right)^2 + \left(\frac{E}{E_0}\right)^2}$$

- need $\lambda\!\!\!^- \gg r_0$ and $E \ll E_0 =$ Coulomb field at r_0

Relativistic particle

- non-relativistic $\Longrightarrow$ relativistic generalization
  Abraham, Laue, Dirac

$$m\frac{du^\mu}{d\tau} = \frac{e}{c}F^\mu{}_\nu u^\nu + F^\mu_{\rm resist}$$

- radiation-reaction force

$$F^\mu_{\rm resist} = \frac{2}{3}\frac{mr_0}{c}\left(\frac{da^\mu}{d\tau} + \frac{1}{c^2}u^\mu\, a_\alpha a^\alpha\right), \qquad a^\mu = \frac{du^\mu}{d\tau}$$

- interpretation of terms
  - mu^μ renormalized 4 momentum
  - $\propto u^\mu a_\alpha a^\alpha$ radiated 4-momentum
  - Schott-term $\propto da^\mu/d\tau$ = 4-momentum of induction field.

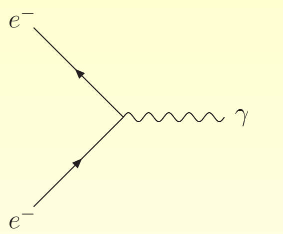
- alternatively

$$f^{(1)} \approx \alpha F_{\rm ext}\sqrt{\left(\frac{\lambda_c}{\lambda}\right)^2 + \left(\frac{E}{E_{\rm crit}}\right)^2}$$

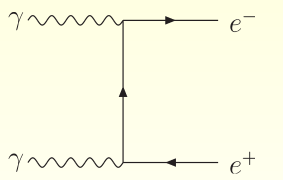
- quantum effects important for $\lambda < \lambda_c$ or $E > E_{\rm crit}$
- cp. with Compton scattering (e.g. $\lambda < \lambda_c$) $\Longrightarrow$
  initial photon $\sim$ external force field on electron
  final photon $\sim$ radiated wave
  radiation reaction in $P^\mu_{\rm in} = P^\mu_{\rm out}$
  radiation and backreaction important

QED

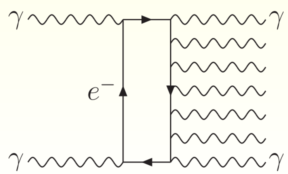
- QED - basic vertex



- pair production (absorptiv)
  perturbation theory

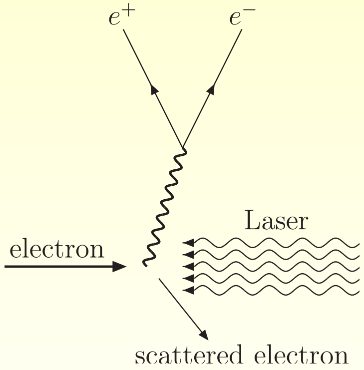


- $\gamma - \gamma$ scattering (dispersiv)



Experiment E-144 at SLAC: matter from light

- 46.6 GeV electrons ($\gamma = 9 \cdot 10^4$) $\longleftrightarrow$ laser field
- optical laser $\hbar\omega = 2.35\,$eV, $a_0 \approx 0.3$
- in system of incoming electrons: $E_\gamma = 428\,$KeV
- in laboratory $E = 2.7 \cdot 10^{10}$V/cm
  electrons 'see' $\approx 0.38 \cdot E_{\rm crit}$
- barrier for pair production: 5 laser photons
- experiment $\Longrightarrow$ $n_L = 5$ dominant
- low intensity ($a_0 < 1$) high energy quantum regime



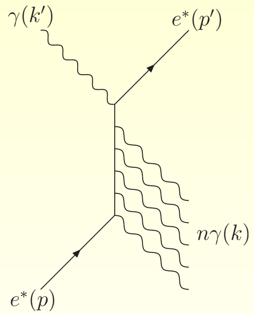
- NL-Compton:
  $e + n_L\gamma \to e' + \gamma$
- multi-photon Breit-Wheeler
  $\gamma + n'_L\gamma \to e^+e^-$
- γ from Terawatt laser
- $O(100)$ rate of e^+ production agreement with QED

strong field non-linear Compton effect

- large intensity laser $a_0 \gg 1$
- all-optical setup $\hbar\omega \ll mc^2$
- below Breit-Wigner pair creation threshold
- strong field scattering dominant

$$e^- + n\gamma_L \to e^- + \gamma$$

Nikishov, Ritus, Naroshnyi; Brown, Kibble; Goldman; Heinzl et al.



$$a_0^2 \sim \boldsymbol{E}^2 \sim n_\gamma, \quad a_0^2 = 4\pi\alpha\left(\frac{\hbar\omega}{mc^2}\right)^2 \lambda\!\!\!^{-3} n_\gamma$$

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- probability for

$$e^- + n\gamma_L \longrightarrow e^- + \gamma$$

$\sim a_0^{2n} \sim \rho_\gamma^n \Longrightarrow$ nonlinear for $n > 1$
- petawatt-laser

$$\lambda\!\!\!^{-3} n_\gamma \approx 10^{18}$$

- QED with classical background field

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QED with classical background fields

see Harvey, Heinzl, Ilderton, 2009

- in/out-state $=$ laser field with n laser photons
- n unknown, correspondence principle $\Longrightarrow$
  laser beam 'classical' $\Longrightarrow$ coherent state

$$|C\rangle = \exp\left(\sqrt{N}\int\frac{d^3k}{(2\pi)^3}C^\mu(\boldsymbol{k})a_\mu^\dagger(\boldsymbol{k})\right)|0\rangle$$

C_μ: momentum + polarization distribution
$N = \langle\hat{N}\rangle$ photon number

$$a_\mu(\boldsymbol{k})|C\rangle = \sqrt{N}C_\mu(\boldsymbol{k})|C\rangle$$

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- transition amplitude

$$\langle \text{out}; C|\hat{S}|\text{in}; C\rangle, \quad \hat{S} = T\exp\left(-\frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, \hat{H}_{\text{int}}(t)\right)$$

- interaction Hamiltonian

$$\hat{H}_{\text{int}} = e\int d^3x\, \underbrace{:\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x):}_{\hat{\jmath}^\mu(x)}\hat{A}_\mu(x)$$

- expand in 'Fourier modes' (σ : momentum, spin)

$$\hat{\psi}(x) = \hat{\psi}^{(-)}(x)+\hat{\psi}^{(+)}(x) = \sum_\sigma\left(\hat{b}_\sigma\psi_\sigma^{(-)}(x)+\hat{d}_\sigma^\dagger\psi_\sigma^{(+)}(x)\right)$$

annihilates electron, creates positron

$$\hat{\bar{\psi}}(x) = \hat{\bar{\psi}}^{(-)}(x)+\hat{\bar{\psi}}^{(+)}(x) = \sum_\sigma\left(\hat{d}_\sigma\bar{\psi}_\sigma^{(-)}(x)+\hat{b}_\sigma^\dagger\bar{\psi}_\sigma^{(+)}(x)\right)$$

annihilates positron, creates electron

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- current operator

$$\hat{\jmath}^\mu = (\gamma^\mu)_{\alpha\beta}\left(\hat{\bar{\psi}}^{(+)}_\alpha\hat{\psi}^{(+)}_\beta + \hat{\bar{\psi}}^{(-)}_\alpha\hat{\psi}^{(+)}_\beta - \hat{\psi}^{(-)}_\beta\hat{\bar{\psi}}^{(+)}_\alpha + \hat{\bar{\psi}}^{(-)}_\alpha\hat{\psi}^{(-)}_\beta\right)$$

- photon field ($\hat{a}_\sigma = \hat{a}_\lambda(\boldsymbol{k})$: momentum, polarisation)

$$\hat{A}_\mu(x) = \sum_\sigma\left(\hat{a}_\sigma A_\sigma(x) + \hat{a}_\sigma^\dagger A_\sigma^*(x)\right)$$

- infinitely many Feynman graphs (Fried, Eberly)
- coherent state = shifted vacuum state

$$|C\rangle = \hat{T}_C|0\rangle \Longrightarrow \langle\text{out}; C|\hat{S}|\text{in}; C\rangle = \langle\text{out}|\hat{T}_C^{-1}\hat{S}\hat{T}_C|\text{in}\rangle$$

- commutation relations $[\hat{a}_\mu(\boldsymbol{k}), \hat{T}_C] = C_\mu(\boldsymbol{k})\hat{T}_C \Longrightarrow$

$$\hat{T}_C^{-1}\hat{S}\hat{T}_C = \hat{S}[\mathcal{A}]$$

- shift $\hat{A}_\mu$ in $\hat{H}_{\text{int}}$ by classical $\mathcal{A}_\mu$= Fourier transform of C_μ

Kibble, Frantz

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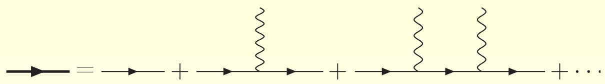
- shift in propagator

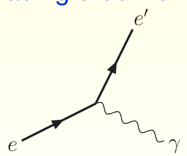
$$\hat{\bar{\psi}}(x)i\partial\!\!\!/\hat{\psi}(x) \to \hat{\bar{\psi}}(x)\left(i\partial\!\!\!/ - e\gamma^\mu\mathcal{A}_\mu(x)\right)\hat{\psi}(x)$$

- background $\to$ dressed e^- propagator



- Feynman diagrams otherwise unchanged
- nonlinear Compton scattering, leading order: one diagram

$$|\text{in}\rangle = |\boldsymbol{p}, \lambda\rangle$$
$$\langle\text{out}| = \langle\boldsymbol{p}', \lambda'; \boldsymbol{k}', \epsilon|$$



- intense laser field $\Longrightarrow$ no expansion in background field!

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Volkov solution

- exact solution of Dirac equation in plane wave
- $\sim$ dressed electron in laser field
- effective mass $m_* \gg m$ (oscillatory motion!)
- not seen in any finite order of QED perturbation theory
- Fried and Eberly: ∞ sum over class of Feynman diagrams
- $\Longrightarrow$ Volkov based calculation

- electrons relativistic $\longrightarrow$ Dirac theory

$$(i\not{D} - \mu)\,\psi = 0, \quad \not{D} = \gamma^\mu D_\mu, \quad \{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu}\mathbb{1}$$

- $\mu = mc/\hbar$ inverse Compton wave length
- covariant derivative $\longrightarrow$ coupling to elm. field

$$D_\mu = \partial_\mu + \frac{ie}{\hbar c} A_\mu$$

- conserved Noether current

$$J^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x), \quad \partial_\mu J^\mu = 0$$

- convert to second order equation

- Dirac equation $\Longrightarrow$

$$(i\not{D} + \mu)(i\not{D} - \mu)\psi = -(\not{D}^2 + \mu^2)\psi = 0$$

- second order Pauli-equations

$$\left(-D_\mu D^\mu + \frac{e}{\hbar c} F_{\mu\nu}\Sigma^{\mu\nu} - \mu^2\right)\psi = 0, \quad \Sigma^{\mu\nu} = \frac{1}{4i}[\gamma^\mu, \gamma^\nu]$$

- coupling electromagnetic field $\leftrightarrow$ moments of particle
- Volkov: exact solution in plane wave

$$\frac{e}{\hbar c} A_\mu = a_\mu(\eta), \quad \eta = k_\mu x^\mu, \quad k_\mu k^\mu = 0$$

Solution of Volkov

- Lorentz gauge $\Longrightarrow k_\mu a'^\mu = 0$
- field strength

$$\frac{e}{\hbar c} F_{\mu\nu}\Sigma^{\mu\nu} = \frac{1}{2i}\left(\not{k}\not{a}' - \not{a}'\not{k}\right) = -i\not{k}\not{a}' \qquad (\not{k} = \gamma^\mu k_\mu)$$

- Dirac equation $\Longrightarrow$

$$\left(-\Box - 2ia^\mu\partial_\mu + a^\mu a_\mu - i\not{k}\not{a}' - \mu^2\right)\psi = 0$$

- ansatz (Volkov)

$$\psi = e^{-ip_\mu x^\mu} F(\eta) \quad \text{with} \quad p_\mu p^\mu = \mu^2$$

p^μ wave vector (momentum) of electron

- ode for F. Solution contains one integral

- Hamilton-Jacobi-function

$$S = -p_\mu x^\mu - \frac{1}{k_\mu p^\mu}\int_0^{k_\mu x^\mu}\left(a_\mu(\eta)p^\mu - \frac{a_\mu(\eta)a^\mu(\eta)}{2}\right)d\eta$$

$=$ classical action of particle motion

- Volkov solution — Volkov 1935

$$\psi_{p,s}^{(-)} = \left(1 + \frac{1}{2}\frac{\not{k}\not{a}}{k_\mu p^\mu}\right)e^{iS}\,\frac{1}{\sqrt{2\omega}}\,u_{p,s}$$

- $u_{p,s}$ constant spinor, momentum p_μ, spin-projection s

$$(\not{p} - \mu)u_{p,s} = 0, \quad \bar{u}_p u_p = 2\mu \Longrightarrow \bar{\psi}_{p,s}\psi_{p,s} = \frac{\mu}{\omega}$$

- $A_\mu = 0 \Longrightarrow \psi^{(-)}$ solution of free Dirac equation

- $\psi^{(-)}_{p,s}$ negatively charged electron $\Rightarrow$ positron

$$\psi^{(+)}_{p,s} = \psi^{(-)}_{p,s}\left(p_\mu \to -p_\mu, u_{p,s} \to v_{p,s}\right)$$

- current density

$$J^\mu = \bar\psi^{(-)}_{p,s}\gamma^\mu\psi^{(-)}_{p,s} = \frac{1}{\omega}\left(p^\mu - a^\mu + k^\mu\left(\frac{a_\alpha p^\alpha}{k_\alpha p^\alpha} - \frac{a_\alpha a^\alpha}{2k_\alpha p^\alpha}\right)\right)$$

- compare with classical result

$$u^\mu(\tau) = u^\mu_0 - \Delta a^\mu + k^\mu\left(\frac{\Delta a_\alpha u^\alpha_0}{k_\alpha u^\alpha_0} - \frac{\Delta a_\alpha \Delta a^\alpha}{2k_\alpha u^\alpha_0}\right)$$

- average over one cycle

$$\bar J^\mu = \frac{1}{\omega}\left(p^\mu - \frac{1}{2k_\alpha p^\alpha}\langle a_\alpha a^\alpha\rangle k^\mu\right)$$

- particle moves with effective momentum

$$q^\mu = p^\mu - \frac{1}{2k_\alpha p^\alpha}\langle a_\alpha a^\alpha\rangle k^\mu$$

- harmonic wave $E = \omega A/c$ and

$$\langle a_\alpha a^\alpha\rangle = -\left(\frac{e}{\hbar\omega}\right)^2\langle E^2\rangle, \quad a_0^2 = \left(\frac{e}{m\omega c}\right)^2\langle E^2\rangle$$

- effective momentum

$$q^\mu = p^\mu + \frac{1}{2k_\alpha p^\alpha}(\mu a_0)^2 k^\mu$$

- effective mass

$$q_\mu q^\mu = \mu^2(1+a_0^2) \Longrightarrow m_* = m\sqrt{1+a_0^2}$$

- complete and 'orthonormal' basis

$$\int \psi^{(\pm)\dagger}_{\boldsymbol p}\psi^{(\pm)}_{\boldsymbol p'} \propto (2\pi)^3\delta(\boldsymbol p - \boldsymbol p')$$

- not stationary, no fixed energy
- wave in z-direction $\Longrightarrow i\partial_x, i\partial_y$ and $i(\partial_0 - \partial_3)$ commute
- good quantum numbers $p_1, p_2, p_0 - p_3$

- linearly polarized light

$$A_\mu = a\left(\varepsilon^\mu_1 \cos k_\mu x^\mu + \varepsilon^\mu_2 \sin k_\mu x^\mu\right)$$

- transition amplitude in first order

$$\langle \boldsymbol p'; \boldsymbol k', \varepsilon' | \hat S[\mathcal A] | \boldsymbol p\rangle \approx -ie\int d^4x\,\bar\psi_{\boldsymbol p'}(x)\frac{e^{ik'_\mu x^\mu}}{\sqrt{2\omega'}}\not\varepsilon'\psi_{\boldsymbol p}(x)$$

- summing over polarization and spin states
- calculation $\Longrightarrow$ energy-momentum conservation

$$q^\mu + nk^\mu = q'^\mu, \quad n = 1, 2, \ldots,$$

- consider head-on collisions

- frequency of scattered photon

$$\omega'_n = n\omega\left\{1+\left(\frac{1-\beta}{1+\beta}\right)^{1/2}\kappa_n(a_0)(1+\cos\theta)\right\}^{-1}$$

$$\kappa_n = n\frac{\hbar\omega}{mc^2} - \sinh\zeta + \frac{a_0^2}{2}e^{-\zeta}, \quad e^{-\zeta}=\gamma(1-\beta)$$

- $\theta =$ scattering $\gamma \to \gamma'$
- $\kappa_n > 0$: Compton scattering
- $\kappa_n < 0$: inverse Compton scattering
- maximal scattered frequency: backscattering $\theta = 0$

$$\omega'_{n,\max} = \frac{n\omega e^{2\zeta}}{1+2ne^{\zeta}\hbar\omega/mc^2 + a_0^2}$$

- $a_0 > 0$: red shift compared to linear Compton scattering

- amplitude $\Longrightarrow$ differential cross sections

$$\frac{d\sigma_n}{d\omega'}, \quad n = 1, 2, \ldots$$

Heinzl, Seipt, Kämpfer, 2009

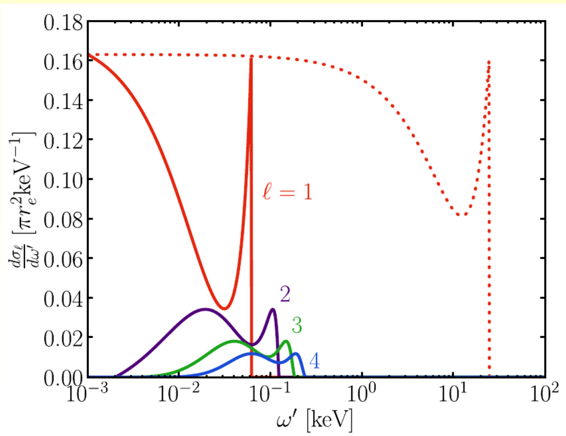
- frequency domain

$$n\omega \le \omega'_n \le \omega'_{n,\max}$$

- Compton-edge red-shifted by factor a_0^2 (exp. test!)
- higher harmonics $\Longrightarrow$ additional peaks in

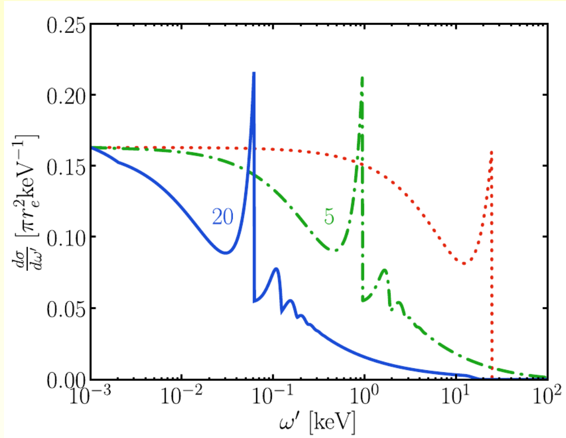
$$\frac{d\sigma}{d\omega'} = \sum_n \frac{d\sigma_n}{d\omega'}$$

- large $a_0 \Longrightarrow$ high harmonics visible
- energies: $E_e = 40\,\text{MeV}$ ($\gamma = 80$), 100 TW laser



partial differential cross sections, $a_0 = 20$
dotted line: linear Compton scattering

from Heinzl, Seipt, Kämpfer



summed cross sections, for $a_0 = 20$ and $a_0 = 5$
dotted line: linear Compton scattering

from Heinzl, Seipt, Kämpfer

Birefringence and pair production

- central object: effective action
  $\Longrightarrow$ vacuum polarization tensor
  $\Longrightarrow$ light propagation and particle production
- special BGs $\Longrightarrow$ exact one-loop results available
- low-energy limit (slowly varying fields)

$$\nu = \frac{\hbar\omega}{m_e c^2} \ll 1$$

  $\Rightarrow$ Heisenberg-Euler regime
- 'small fields'

$$\epsilon = \frac{E}{E_c} \ll 1, \quad E_{\text{crit}} \approx 1.3 \cdot 10^{16}\,\frac{\text{V}}{\text{cm}}$$

  e.g. X-probe ($\approx$ 5 KeV), uh-power laser (10^{26} W/cm^2):

$$\nu \approx \epsilon \approx 10^{-2}$$

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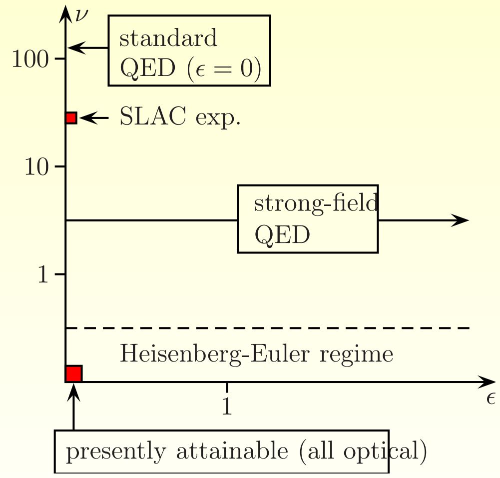

effective action

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- classical Maxwell theory: $S[A] = \int d^4x\, \mathcal{L}_{\text{M}}[A]$

$$\mathcal{L}_{\text{M}}[A] = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2)$$

- QED-corrections due to electron-positrons

$$\Gamma[A] = \int d^4x\, \mathcal{L}_{\text{eff}}[A] = \int d^4x\, \mathcal{L}_M[A] + \int d^4x\, \Delta\mathcal{L}[A]$$

- quantized electron-positron field $\Delta\mathcal{L} = O(\hbar)$,

$$\int d^4x\, \Delta\mathcal{L} = -i \log\det(i\not{D} + m)$$

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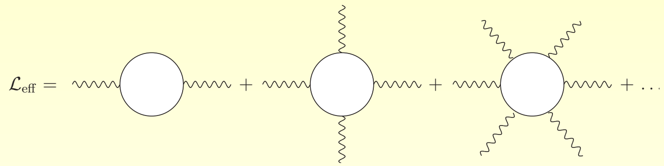
- expansion in powers of field strength $\rightarrow \infty$ diagrams



- soft-photon limit

$$A_\mu(\omega, \boldsymbol{k}), \quad \omega, |\boldsymbol{k}| \ll m, \quad (c = \hbar = 1)$$

- slowly varying background fields ($\nu \ll 1$)
- constant magnetic field (renormalized $e, \boldsymbol{B}$)

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\boldsymbol{B}^2 - \frac{1}{8\pi^2}\int_0^\infty \frac{dT}{T^3}\, e^{-m^2 T} \times \left(eBT\coth(eBT) - \frac{e^2T^2}{3}B^2 - 1\right)$$

- general constant $\boldsymbol{E},\boldsymbol{B}\Longrightarrow$ only invariants

$$\boldsymbol{E}^2-\boldsymbol{B}^2=-2\mathcal{F}\quad,\quad \boldsymbol{E}\cdot\boldsymbol{B}=-\mathcal{G}$$

- $\mathcal{F}$ scalar, $\mathcal{G}$ pseudoscalar

$$\mathcal{L}_{\text{eff}}\equiv\mathcal{L}_{EH}=-\mathcal{F}+\Delta\mathcal{L}(\mathcal{F},\mathcal{G})$$

- even in $\mathcal{G}$, spectrum of matrix $F^{\mu}{}_{\nu}\Longrightarrow$

$$a=\sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}+\mathcal{F}},\quad b=\sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}-\mathcal{F}}$$

- effective Lagrangian

$$\begin{aligned} eBT\coth(eBT) &\longrightarrow eaT\coth(eaT)\,ebT\cot(ebT)\\ \boldsymbol{B}^2 &\longrightarrow \boldsymbol{B}^2-\boldsymbol{E}^2\end{aligned}$$

Euler, Heisenberg; Schwinger; Weisskopf, ...

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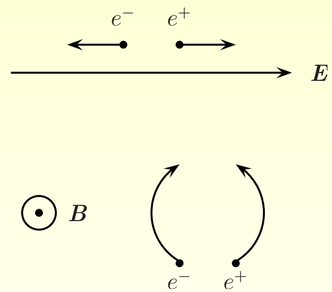
vacuum pair creation



- external $\boldsymbol{E}$-field: can produce pairs
- virtual particles may become real
- external $\boldsymbol{B}$-field: no asymptotic states
- constant $\boldsymbol{E}$: energy density $\varepsilon\propto\Delta L_{EH}(\boldsymbol{E})$

$$\varepsilon=\frac{m^4}{8\pi^2}\left(\frac{E}{E_{\text{crit}}}\right)^2\int\frac{ds}{s^3}e^{-E_{\text{crit}}/E\cdot s}\left(s\cot s+\frac{s^2}{3}-1\right)$$

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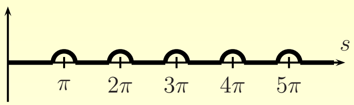
- poles $\pi,\,2\pi,\,3\pi,\dots$
- principal parts



- $\Im(\varepsilon)<0\Longrightarrow$ decay, decay rate $\Gamma\propto\Im(\varepsilon)$
- particle production rate is very small

$$\Gamma=\frac{e^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{e^{-n\pi E_{\text{crit}}/E}}{n^2}$$

- non-perturbative terms $e^{-m^2\pi n/eE}$
- $\Im(\varepsilon)<0\Longrightarrow$ loss of energy
- $\boldsymbol{E}$-field: modification of energy of Dirac sea
  Stark effect + pair production (cf ionization)
- $\boldsymbol{B}$-field: mod. of energy of Dirac sea, Zeeman effect

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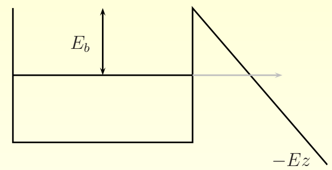
- Literature:
G. Dunne: hep-th/0406216
P. Milonni: "The Quantum Vacuum", 1994
W. Dittrich, H. Gies: "Probing the Quantum Vacuum", Springer 2000
S. Blau, M. Visser, A. W.: "Analytic Results for the Effective Action", 1991
- atomic ionization
  constant $\boldsymbol{E}$-field: $V(x)=-eEz$
  atomic tunneling



- tunnelling rate

$$R=\exp\left(-\frac{2}{\hbar}\int_0^{E_b/eE}\sqrt{2m(E_b-Ez)}dz\right)$$

- hydrogen atom

$$E_b=\frac{me^4}{2\hbar^2}$$

- ionization → good approximation

$$R_H = \exp\left(-\frac{2}{3}\frac{m^2e^5}{E\hbar^4}\right)$$

- compare to EH

$$\frac{\pi m^2c^3}{eE\hbar} = \frac{3\pi}{2}\frac{1}{\alpha^3}\cdot\frac{2}{3}\frac{m^2e^5}{E\hbar^4} \approx 10^7\cdot\frac{2}{3}\frac{m^2e^5}{E\hbar^4}$$

$$R_{EH} \approx (R_H)^{10^7}$$

- pair production = Dirac hole tunnelling process
- time-dependent $\boldsymbol{E}$-field: adiabatic approximation fails (exp, examples)

Keldish, JETP 20 (1965) 1307

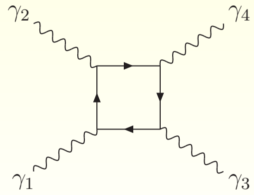
weak fields

- $\mathcal{L}_{EH}$: leading order in derivate expansion, valid for $\nu \ll 1$
- weak fields $\epsilon \ll 1 \Longrightarrow$ power series expansion

$$\begin{aligned}\Delta\mathcal{L} &= \frac{8}{45}\frac{\alpha^2}{m^4}\mathcal{F}^2 + \frac{14}{45}\frac{\alpha^2}{m^4}\mathcal{G}^2 + \dots \\ &= \frac{2\alpha^2}{45m_e^4}\left((\boldsymbol{E}^2-\boldsymbol{B}^2)^2 + 7(\boldsymbol{E}\cdot\boldsymbol{B})^2\right) + \dots\end{aligned}$$

- light-light scattering

$$\mathcal{M} \sim \frac{\partial^4\Delta\mathcal{L}}{\partial F\partial F\partial F\partial F}$$



- cross sections (cm system, $\hbar\omega \ll mc^2$)

$$\frac{d\sigma}{d\Omega} = \frac{139}{8100}\left(\frac{\alpha}{2\pi}\right)^2 r_0^2\left(\frac{\hbar\omega}{mc^2}\right)^6(3+\cos^2\theta)$$

$$\sigma_{\rm tot} = \frac{973}{10125}\frac{\alpha^2}{\pi}r_0^2\left(\frac{\hbar\omega}{mc^2}\right)^6$$

$$\sigma_{\rm tot}[\mathrm{cm}^2] = 7.3\times 10^{-66}\ (\hbar\omega[\mathrm{eV}])^2$$

Euler; Karplus and Neumann

- cp. Thomson scattering

$$\sigma_{\rm tot} = \frac{8\pi}{3}r_0^2$$

- upper limits (3 laser beams) $\approx 10^{-50}$ cm^2 — Bernard et al.
- photon-splitting, ... — S. Adler

- propagation of light in background fields
- quantum Maxwell equation for a 'light probe' $f^{\mu\nu}$:
- strong background + propagating probe field

$$F_{\mu\nu} \to F_{\mu\nu} + f_{\mu\nu}, \quad f_{\mu\nu} \ll F_{\mu\nu}$$

- linearize field equations to $\mathcal{L}_M + \Delta\mathcal{L}$
- with $F_{\mu\nu}$ quasi-constant

$$0 = \partial_\mu f^{\mu\nu} - \frac{8}{45}\frac{\alpha^2}{m^4}F_{\alpha\beta}F^{\mu\nu}\partial_\mu f^{\alpha\beta} - \frac{14}{45}\tilde{F}_{\alpha\beta}\tilde{F}^{\mu\nu}\partial_\mu f^{\alpha\beta}$$

Toll '54, Baier, Breitenlohner '67, Narozhniy '69
Bialynicka-Birula '70, Adler '71

- effective $\epsilon(\boldsymbol{E},\boldsymbol{B}), \mu(\boldsymbol{B},\boldsymbol{E})$ observable?
- nonlinear optics:
  probe plane wave $k = (\omega, \omega\boldsymbol{n}) \Longrightarrow f(\boldsymbol{n},\boldsymbol{E},\boldsymbol{B}) = 0$

$$n_\pm = |\boldsymbol{n}_\pm| = 1 + \Delta n_\pm$$

- similar to uniaxial crystal:

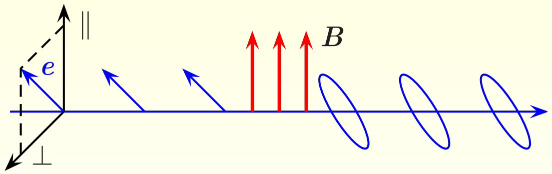
$$\Delta n_\pm = \frac{\eta_\pm}{2}\frac{\alpha}{45\pi^2 E_c^2}\Big(\boldsymbol{E}^2 + \boldsymbol{B}^2 - 2\boldsymbol{S}\cdot\boldsymbol{k} - (\boldsymbol{E}\cdot\boldsymbol{k})^2 - (\boldsymbol{B}\cdot\boldsymbol{k})^2\Big)$$

  $\boldsymbol{S}$: Poynting, QED: $\eta_+ = 7, \eta_- = 4$, BI: $\eta_+ = \eta_-$
- quantum vacuum induces birefringence
  detection schemes: PVLAS, BMV, Photon-collider, ...

Light propagation in B field

- phase velocities depend on polarisation

$$V_\parallel \approx 1 - \frac{14}{45}\frac{\alpha^2}{m^4}\boldsymbol{B}^2\sin^2\theta_B$$
$$V_\perp \approx 1 - \frac{8}{45}\frac{\alpha^2}{m^4}\boldsymbol{B}^2\sin^2\theta_B$$

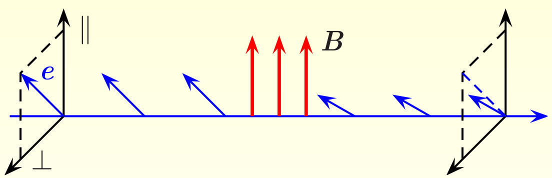


linear polarisation → elliptic polarisation

$$\psi_\ell = \pi\frac{L}{\lambda}\Delta v \sin 2\theta, \quad \Delta v(5.5\mathrm{T}) \approx 10^{-22}$$

- above threshold (QED: $\omega > 2m_e$)

$$\text{damping} \quad \kappa_{\parallel,\perp} = -\frac{1}{\omega}\mathrm{Im}\,\Pi_{\parallel,\perp}$$



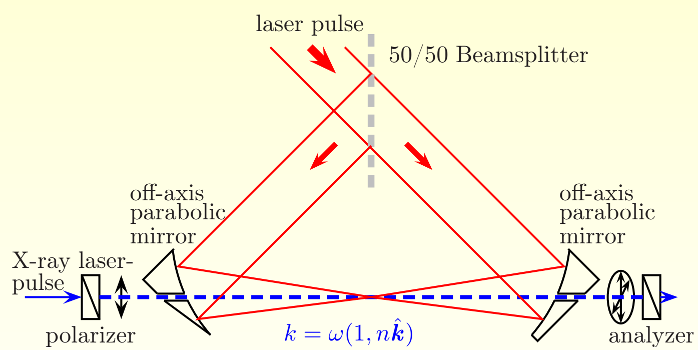
- dichroism induces rotation:

$$|\Delta\theta| \approx \frac{1}{4}\Delta\kappa\, L \sin 2\theta$$

Birefringence at photon collider

experimental setup

Heinzl, Liesfeld, Amthor, Schwoerer, Sauerbrey, Wipf; Koch



birefringence maximal for counter-propagating probe beam

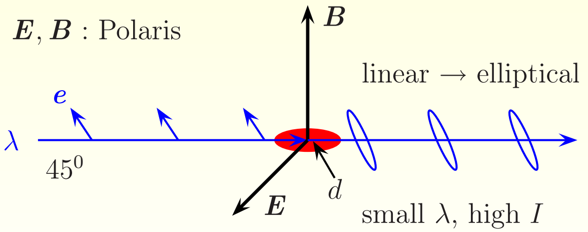
$$n_\pm = 1 + \frac{\alpha}{45\pi}\begin{Bmatrix}14\\8\end{Bmatrix}\frac{I}{I_c}$$

- relative phase shift: focus length d, probe λ:

$$\Delta\phi = \frac{2\pi d}{\lambda}(n_+ - n_-) = \frac{4\alpha}{15}\frac{d}{\lambda}\frac{I}{I_c}$$

- Gaussian beam: $d \to \kappa z_0$

  z_0 Rayleigh length, κ intensity integral



- Polaris:

$$\hbar\omega \approx 2\times 10^{-3} mc^2, \quad I \approx 2\times 10^{-8} I_c$$

  (backscattered Thomson photons)

- parameters (ω in KeV, λ in nm, z_0 in μm)

	ω	λ	z_0	$\triangle\phi$ (rad)	ellipticity δ^2
Jena	12	0.1	10	1.2×10^{-6}	4.9×10^{-11}
XFEL	15	0.08	25	4.4×10^{-5}	4.8×10^{-10}

- $\delta^2 = (\frac{1}{2}\Delta\phi)^2 \approx 10^{-11}$ conceivable

  (E. Alp et.al, Hyperfine Interactions **125** (2000) 45)

- ELI: $\delta^2 \approx 10^{-7}\dots10^{-4}$
- with Jena cuts (I. Uschmann et al.)

  transmission ratio of 2×10^{-9} at 6.44 keV

  aim: 12 KeV with improved brilliance

- previous best value (Toellner, APS Argonne, 1996):

  transmission ratio of 4.4×10^{-7} at 15.4 keV

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pair creation, birefringence and experiments

First polarization purity measurement

Experimental campaign ad ID 06 at ESRF: 1.-7.12. 2009

Uschmann, Marx, Höfer, Lötsch, Wehrhan, Marschner, Förster, Kaluza, Paulus

Ephoton / keV source	Silizium Reflection	Transmission ratio	Peak ratio
8.05 X-ray tube	333	3.9×10^{-4}	3.5×10^{-4}
6.44 ESRF	400	2.2×10^{-9}	1.2×10^{-9}
11.16 ESRF	444	2.3×10^{-8}	3.9×10^{-9}
12.88 ESRF	800	1.4×10^{-8}	1.3×10^{-9}
15.4 APS Argonne	840	4.4×10^{-7}	

FSU Jena, GSI, DESY, Helmholtz Association

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further improvement of polarization purity by:

tilted channel cuts, asymmetric reflection, crystal with lower Z

nonlinear quantum induced vacuum effects

- Im $\mathcal{L}_{EH}$: pair production, vacuum dichroism
- Re $\mathcal{L}_{EH}$: vacuum birefringence
- static $\boldsymbol{B}$-field

  $\longrightarrow$ testing physics beyond standard model

- light-by-light scattering :-(
- photon splitting
- birefringence :-)

challenging but feasable

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