Introduction to Wess-Zumino-Models and $\mathcal{N} = 1$ Super-Yang-Mills Theory on spacetime lattices

Andreas Wipf

Theoretisch-Physikalisches Institut
Friedrich-Schiller-University Jena

seit 1558

International Centre for Theoretical Sciences
ICTS-TIFR, March 14, 2018

in collaboration with:
Daniel August, Andre Sternbeck, Marc Steinhauser, Sebastian Uhlmann, Bjoern Wellegehausen, Christian Wozar, . . .

- only extension of Poincaré invariance $\{P_\mu, M_{\mu\nu}\}$
- $\mathcal{Q}^i_\alpha$ with $i = 1, \dots, \mathcal{N}$ femionic charges

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}^\beta\} = 2(\gamma^\mu)_\alpha^{\ \beta} P_\mu$$
$$[\mathcal{Q}_\alpha, P_\mu] = 0$$
$$[J_{\mu\nu}, \mathcal{Q}_\alpha] = (\Sigma_{\mu\nu}\mathcal{Q})_\alpha$$

- Physics beyond Standard Model:
  hierarchy problem, gauge coupling unification
  dark matter candidates, string theory, AdS/CFT
- insights into strongly coupled gauge theories
- susy inspired approaches/approximations (fermions)
- mathematical physics, integrable systems, ...

- $\mathcal{Q}_\alpha$ realized on local fields $\phi, \psi, A_\mu, \dots$
- conserved Noether-(super)currents and (super)charges

charge	$\mathcal{Q}$	$\mathcal{Q}_\alpha$	P^μ
current	J^μ	J^μ_α	$T^{\mu\nu}$
spin	1	$3/2$	2
source field	A_μ	ψ^α_μ	$g_{\mu\nu}$
particle	photon	gravitino	graviton

- nonperturbative ab initio definition of QFT
- access to susy breaking, phase diagrams, particle masses, ...
- check/confirm conjectured/existing results
  - effective low energy theory (e.g. $\mathcal{N}=1$ SYM)
  - calculate phase transition lines
  - find particle spectrum
  - matching weak to strong coupling (extended SYM)
- important: treatment of lattice fermions (non-trivial fixed points!)
- results can be stepwise improved
- comparison with functional renormalization group, ...

- PIONEERING WORK:

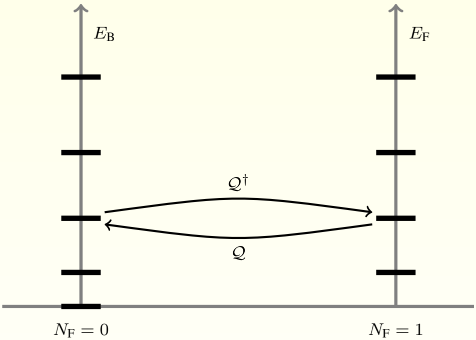
  Dondi, Nicolai; Curcio, Veneziano; Elitzur, Schwimmer; Sakai, Sakamoto; Beccaria, Curci, D'Ambrosio, ...

  structural investigations, exact susy on lattice, $\mathcal{N}=(2,2)$ Wess-Zumino in two dimensions, Nicolai mapping

- FURTHER RESULTS:

  Bergner, Brower, Bonini, Catterall, Cohen, Damgaard, Farchiano, Feo, Giedt, Hanada, Harada, Joseph, Kanamori, Kaplan, Katz, Lemos, Montvay, Münster, Poppitz, Schaich, Sugino, Suzuki, Unsal, Wellegehausen, Wozar, ...

  susy breaking, precise numerical investigations: phases, symmetries, mass spectra; various fermion species: Wilson, overlap, SLAC; $\mathcal{Q}$-exact formulations, susy-gauge theories in various dimensions and various $\mathcal{N}$



isospectral H_B, H_F

- $H = \{\mathcal{Q}, \mathcal{Q}^\dagger\}$, nilpotent $\mathcal{Q}^2 = 0 \Rightarrow \mathcal{H} = \mathcal{H}_B \oplus \mathcal{H}_F$
- $\mathcal{Q}, \mathcal{Q}^\dagger$: eigenstate$_E \rightarrow$ {0,eigenstate$_E$}

- particular realization of super-Hamiltonian $\mathcal{Q} = \sigma_+\left(\frac{\mathrm{d}}{\mathrm{d}\phi} + P\right)$

$$H = -\frac{\mathrm{d}^2}{\mathrm{d}\phi^2} + P^2(\phi) - \sigma_3 P'(\phi) = \begin{pmatrix} H_\mathrm{B} & 0 \\ 0 & H_\mathrm{F} \end{pmatrix}$$

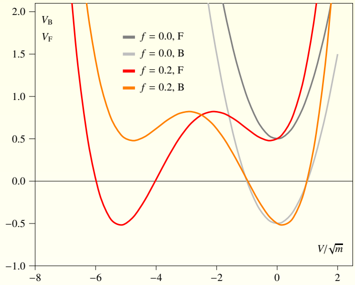
- isospectral up to possible zero-modes

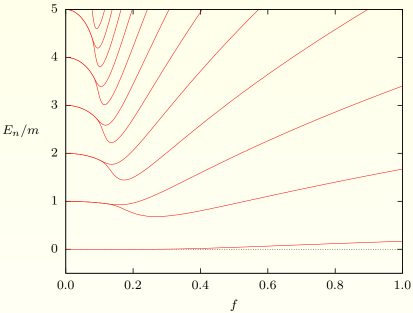
$$\text{Witten-Index:} \quad \Delta = n_\mathrm{B} - n_\mathrm{F} = \mathrm{tr}\left(\mathrm{e}^{-\beta H_\mathrm{B}} - \mathrm{e}^{-\beta H_\mathrm{F}}\right)$$

- $\Delta \neq 0 \Rightarrow$ susy unbroken, susy broken $\Rightarrow \Delta = 0$
- susy anharmonic oscillator with broken susy

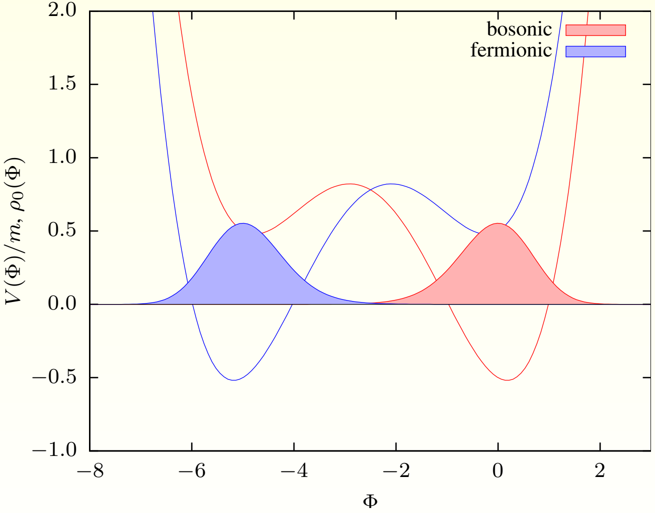
$$P(\phi) = m\phi + h\phi^2 \Rightarrow \Delta = 0$$

- $f = h/m^{1.5}$ dimensionless coupling





- no ground state with zero energy
- every energy double degenerate
- $f \lessapprox 0.1$: almost four-fold degeneracy (splitting $\mathrm{e}^{-S/\hbar}$)
- dimensional parameters m and h, m sets scale
- cp. Gerald Dunne last talk


bosonic
fermionic
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−8
−6
−4
−2
0
2
Φ
V(Φ)/m, ρ0(Φ)

- partition function and Witten index

$$\int \mathcal{D}\phi\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,\mathrm{e}^{-S[\phi,\psi,\bar{\psi}]} = \begin{cases} \mathcal{Z} & \text{anti-periodic bc} \\ \Delta & \text{periodic bc} \end{cases}$$

Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}P^2(\phi) + \bar{\psi}M(\phi)\psi, \quad M(\phi) = \partial + P'(\phi)$$

- supersymmetries

$$\begin{aligned} &\delta^{(1)}\phi = \bar{\varepsilon}\psi, \quad \delta^{(1)}\bar{\psi} = -\bar{\varepsilon}(\dot{\phi} + P(\phi)), \quad \delta^{(1)}\psi = 0 \\ &\delta^{(2)}\phi = \bar{\psi}\varepsilon, \quad \delta^{(2)}\psi = (\dot{\phi} - P(\phi))\varepsilon, \quad \delta^{(2)}\bar{\psi} = 0 \end{aligned}$$

- expectation values

$$\langle\mathcal{O}\rangle = \mathcal{Z}^{-1}\int \mathcal{D}\phi\,\mathcal{O}[\phi]\,\det M(\phi)\,\mathrm{e}^{-S_{\mathrm{B}}[\phi]}$$

- discretization, dimensionless lattice-fields and couplings

$$S = -\sum_{x,y} \frac{1}{2}\phi_x(\partial^2)_{xy}\phi_y + \frac{1}{2}\sum_x P(\phi_x)^2 + \sum_{x,y}\bar{\psi}_x\left(\partial_{xy} + P'(\phi_x)\delta_{xy}\right)\psi_y$$

- fermion determinant

$$\mathcal{Z} = \int \mathcal{D}\phi \, \det M[\phi] \, \mathrm{e}^{-S_\mathrm{B}[\phi]}, \qquad M = \partial^{\mathrm{slac}} + P'(\phi)$$

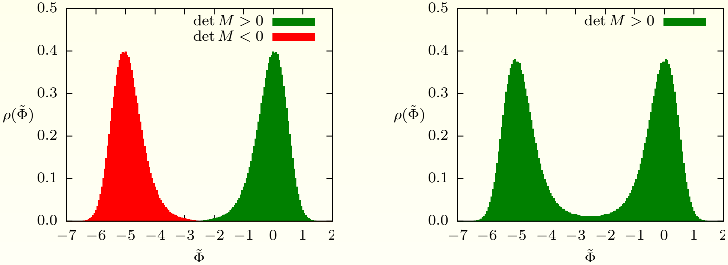
- (bosonic) expectation values

$$\langle \mathcal{O}[\phi] \rangle = \mathcal{Z}^{-1} \int \mathcal{D}\phi \, \mathcal{O}[\phi] \, \det M[\phi] \, \mathrm{e}^{-S_\mathrm{B}[\phi]}$$

- sign-quenched distribution

$$\rho[\phi] = \mathrm{e}^{-S_\mathrm{B}[\phi] + \ln|\det M[\phi]|}.$$

- sign quenched ensemble, 10^6 configurations, $m\beta = 4$, $f = 0.2$
- sign(det) $\leftrightarrow \tilde{\phi} =$ lattice average of field $\leftrightarrow$ bc



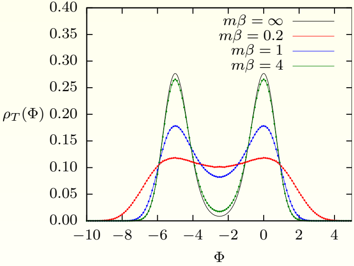
- periodic (left, $N = 101$) and antiperiodic (rights, $N = 100$)

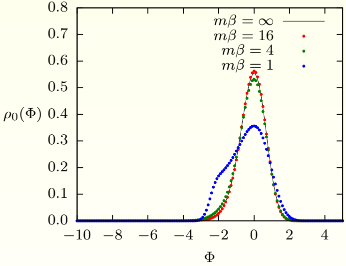
can show: for ∂^{slac} and even $P(\phi)$: Wozar, AW

$$\det_{\mathrm{ap}}(M[-\phi]) = \det_{\mathrm{ap}}(M[\phi]), \quad \det_{\mathrm{p}}(M[-\phi]) = -\det_{\mathrm{p}}(M[\phi]) \Rightarrow \Delta = 0$$

- thermal probability distribution

$$\rho_T(\phi) = Z^{-1} \sum_n \mathrm{e}^{-E_n/T} |\langle \phi | \psi_n \rangle|^2$$

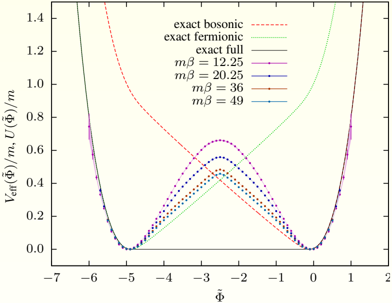




- exact results (lines) vs. simulation results (points) ($f = 0.2$)
- left: thermal distributions
- right: exact distribution for $|\psi_\mathrm{B}\rangle$ and simulations with $\tilde{\phi} > -\frac{1}{2f}$.

- constraint effective potential (CEP)

$$U(\phi_0) = -\frac{1}{\beta} \ln \left(\int \mathcal{D}\phi \; \det M[\phi] \, e^{-S_B[\phi]} \, \delta(\tilde{\phi} - \phi_0) \right),$$



- exact V_{eff} for $T = 0$
  exact: high precision numerics
- finite temperature:
  CEP U from simulations
- not convex at low temperature
- $f = 0.2$, $N = 300$, $\approx 10^7$ conf

$$\min_{m\beta \to \infty} U(\phi_0) = \min \left\{ U_B(\phi_0), U_F(\phi_0) \right\}$$

- determination of energies (masses)

see C. Wozar, AW (2011)

on-shell formulation

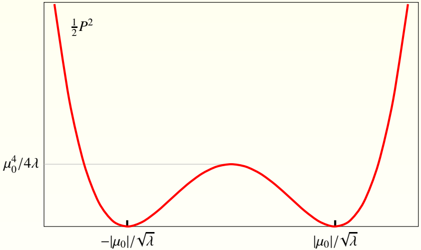
$$S = \int \mathrm{d}^2 x \, \frac{1}{2}\Big((\partial_\mu \phi)^2 + \bar{\psi}(\not{\partial} + P'(\phi))\psi + P(\phi)^2\Big).$$

- minimal field content: real ϕ, Majorana ψ (real F)
- not a finite QFT, superpotential renormalized
- not obtained from dimensional reduction
- no $\mathcal{Q}$-exact formulation, no Nicolai map
- $\Delta = 0$ possible $\Rightarrow$ susy may be broken — Witten
- expect phase transition: susy vs. $\mathbb{Z}_2$ (cp. FRG results)
- laboratory for tensor networks
- most accurate: fermion loop representation — Wenger, Steinhauer

Beccaria, Campostini, Feo; Wozar, AW; Steinhauer, Wenger; Synatschke, Wipf, AW

- quadratic prepotential $\Rightarrow \Delta = 0$

$$P(\phi) = \frac{1}{\sqrt{2\lambda}}(\mu_0^2 + \lambda\phi^2) \Longrightarrow \frac{1}{2}P(\phi)^2 = \frac{\mu_0^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \text{const}$$

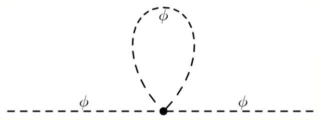


- $\mathbb{Z}_2$-symmetric
- $\mathbb{Z}_2$ spontaneously broken?
- susy broken?
- $\mu_0^2 \ll 0$: system cannot tunnel

  $\Rightarrow$ susy not broken
- $\mu_0^2 > 0$: ground state energies lifted

  $\Rightarrow$ susy broken

- FRG-analysis
  - second order PT
  - susy breaking $\Leftrightarrow \mathbb{Z}_2$ restoration
  - massive susy phase for weak coupling
  - broken phase with goldstino and light scalar ($m \propto 1/L$)
- continuum: divergent diagram (susy: fermion loop)



- λ not renormalized $\Rightarrow$ scale
- $\hat{\lambda} = \lambda a^2 \to 0$ determines a
- $\hat{\mu} = \mu a \to 0$
- $\hat{\mu}_0^2 = \hat{\mu}^2 - \#\hat{\lambda} A_{\hat{\mu}}$ counter term
- $A_{\hat{\mu}} = N^{-2} \sum \left(\hat{p}^2 + \hat{\mu}^2\right)^{-1}$ logarithmically divergent

$$\hat{V} = \frac{\hat{\mu}_0^2}{2}\phi^2 + \frac{\hat{\lambda}}{4}\phi^4$$

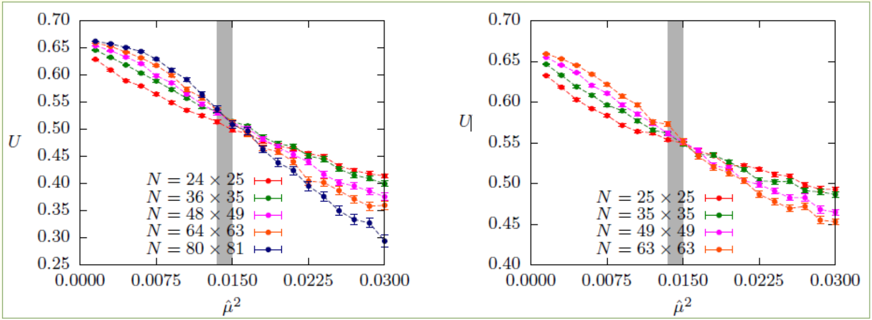
- fixed $\hat{\lambda} \to$ always 2nd order $\mathbb{Z}_2$ breaking PT at critical $\hat{\mu}_c^2$
- renormalized critical coupling

$$f_c = \left[\frac{\lambda}{\mu^2}\right]_{\text{crit}} = \lim_{\hat{\lambda}\to 0} \hat{f}_c, \quad \hat{f}_c = \frac{\hat{\lambda}}{\hat{\mu}_c^2}$$

- algorithm:
  - fix $\hat{\lambda} \Rightarrow$ determine $\hat{\mu}_c^2$ from Binder cummulant $\Rightarrow \hat{f}_c$
  - extrapolate $\hat{f}_c \approx f_c + \alpha\hat{\lambda} + \beta\hat{\lambda}\log\hat{\lambda}$ to small $\hat{\lambda}$ — Schaich, Loinaz

results for bosonic and susy model

model/method	f_c	authors
bosonic, MC Cluster (2009) on 1200^2	10.81(7)	Schaich, Loinaz.
bosonic, ferm. loop MC (2015)	11.10(2)	U. Wenger, privat comm.
bosonic, MC worm (2015) on 512^2	11.15(9)	Bosetti et al.
bosonic, matrix product states (2013)	11.064(20)	Milsted et al.
Hamiltonian truncation (2017):	11.04(12)	Miro, Rychkov, Vitale
bosonic, SLAC derivative on 256^2:	10.92(13)	Wozar, AW
susy model (SLAC) on 80×81:	21.1(1.1)	Wozar, AW



$$U = 1 - \frac{\langle \phi^4 \rangle}{3\langle \phi^2 \rangle^2}$$

- Binder cummulant for thermal (left) and susy bc (right)
- peridoic bc $=$ anti-periodic bc
- $\mathbb{Z}_2$ breaking PT at $f > f_c \approx 21$; recall $\hat{\mu}_0^2 = \hat{\mu}^2(1 - \#\hat{f}A_{\hat{\mu}})$

Marorana fermions: $\mathcal{D}\psi$ yields Pfaffian

$$\mathcal{Z} = \int \mathcal{D}\phi\,\mathcal{D}\psi\,\mathrm{e}^{-S_B[\psi]-\psi^t M[\phi]\psi} = \int \mathcal{D}\phi\,\mathrm{Pf}(M[\phi])\,\mathrm{e}^{-S_B[\phi]}$$

- antisymmetric fermion matrix $M = \mathcal{C}\left(\partial\!\!\!/^{\,\mathrm{slac}} + P'\right)$:

$$M = \begin{pmatrix} \partial_1^{\mathrm{slac}} & \partial_0^{\mathrm{slac}} - P' \\ \partial_0^{\mathrm{slac}} + P' & -\partial_1^{\mathrm{slac}} \end{pmatrix}.$$

- "square root" of determinant, $(\mathrm{Pf}\,M)^2 = \det M$
- expansion and identities

$$\mathrm{Pf}(M) = \frac{1}{2^N N!}\sum_{\sigma\in S_{2N}} \mathrm{sign}(\sigma)\prod_{i=1}^{N} M_{\sigma_{2i-1},\sigma_{2i}}$$

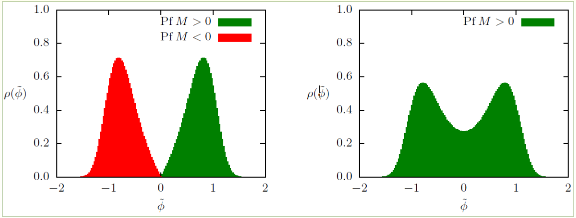
$$\mathrm{Pf}(A) = (-)^n\mathrm{Pf}(A^t), \quad \mathrm{Pf}(BAB^t) = \det(B)\mathrm{Pf}(A)$$

- can show (transpose, $\mathcal{C}$): $\partial\!\!\!/^{\text{slac}}$ and odd $P'(\phi) \Rightarrow$

$$\text{Pf}(M[-\phi]) = \begin{cases} +\text{Pf}(M[\phi]) & \text{thermal anti-periodic bc} \\ -\text{Pf}(M[\phi]) & \text{susy periodic bc} \end{cases}$$

- $\Delta = 0$ & continuums $\mathbb{Z}_2$-symmetry for thermal bc

- left: 9×9 periodic bc
- right: 8×9 anti-periodic bc
- $\hat{f} = 100 > \hat{f}_c$
- $\hat{\lambda} = 0.1$
- 10^6 config.



- not true for Wilson-fermions!

sign of Pfaffian

$\mathbb{Z}_2$-broken phase: $\tilde{\phi} \cdot \mathrm{sign}\,\mathrm{Pf}(M) > 0 \Rightarrow$ positive measure for thermal bc

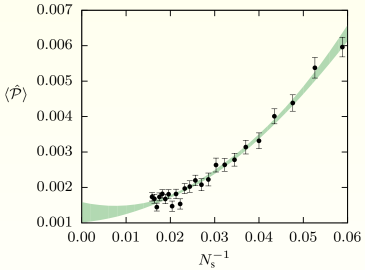
- $\mathbb{Z}_2$-broken phase: system in one ground state $\Rightarrow$ fixed sign of Pf
  physics insensitive to boundary conditions (Binder cummulant)
- approach phase transition from broken phase $f > f_c$
- to see susy-breaking: must control $T > 0$ and $a > 0$ effects
  $\Rightarrow$ careful extrapolations required!

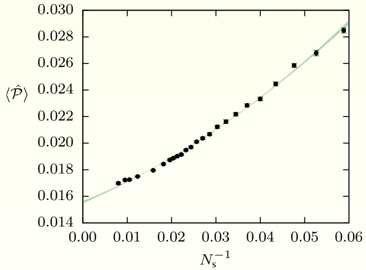
Breaking of supersymmetry

Ward identity $\langle \delta\psi \rangle = 0 \Longrightarrow \langle \mathcal{P} \rangle = 0, \qquad \mathcal{P} = \frac{1}{V} \sum P(\phi_x)$

- infinite volume extrapolations, $N_t = N_s + 1$

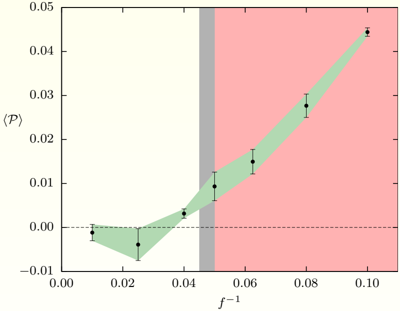
$$\langle \mathcal{P} \rangle = A + BN_s^{-1} + CN_s^{-2}$$





- $\mathbb{Z}_2$ broken phase $\hat{f} = 100$
- $\mathbb{Z}_2$ symmetric phase $\hat{f} = 10$

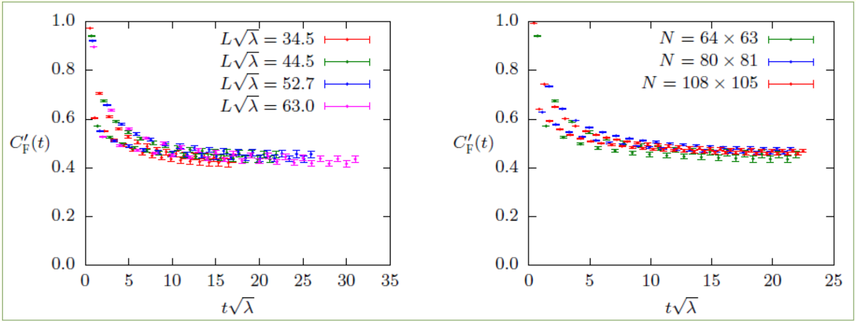
- for each $f \in \{10, 12.5, 16, 20, 25, 40, 100\}$:
  extrapolate $\langle \mathcal{P} \rangle$ to its continuum value at $\hat{\lambda} \to 0$.



- $\mathbb{Z}_2$ and susy breaking at same f_c
- red: $\mathbb{Z}_2$ restored, susy broken
- gray: $\mathbb{Z}_2$ phase transition
- PT at $f_c \approx 21.1$

- Goldstino: massless mode in susy broken phase:

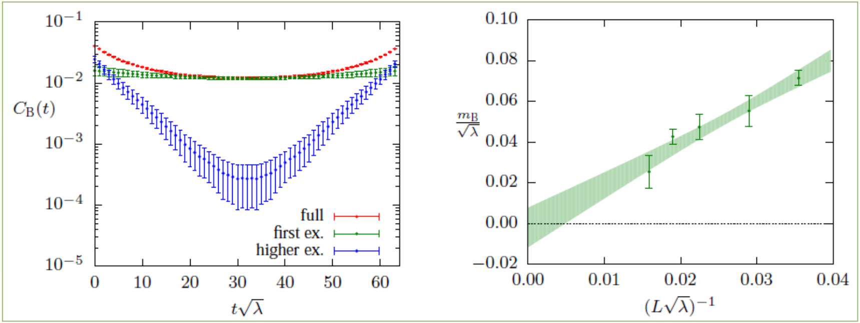
$$C'_F(t) = N_s^{-2} \sum_{\alpha,x,x'} \langle \bar{\psi}_{1,(t,x)} \psi_{1,(0,x')} - \bar{\psi}_{2,(t,x)} \psi_{2,(0,x')} \rangle$$



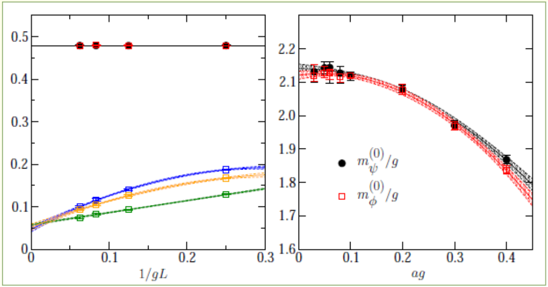
- left: different physical volumes
- right: fixed physical volume, different N
- $\rightarrow$ massless Goldstino

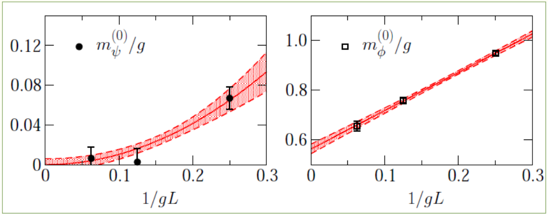
- FRG $\rightarrow$ massless boson — Gies, Synatschke, AW

$$C_B(t) = N_s^{-2} \sum_{x,x'} \langle \phi_{(t,x)} \phi_{(0,x')} \rangle$$



- bosonic states, first and second excited $\rightarrow$ massless boson
- discretization errors $<$ statistical errors ($64 \times 63, 80 \times 81, 108 \times 105$)





- left: infinite volume extrapolations

  m_ϕ, m_ψ in susy phase

  m_ϕ for $am = 0.3, 0.28,$ 0.02 in susy broken phase
- right: $ag \to 0$ extrapolation in susy phase

- infinite volume extrapolation

  susy broken phase

K. Steinhauer, U. Wenger (2014)

$\mathcal{N} = 2$ WZ-model in two dimensions = dimensional reduction of 4d WZ-model

4 dimensions	2 dimensions
Majorana spinor	two Majorana spinors $\equiv$ one Dirac spinor
two real scalar fields	two real scalar fields $\equiv$ one complex field

- action

$$S_{\text{cont}} = \int \mathrm{d}^2 x \left(2\bar{\partial}\bar{\varphi}\partial\varphi + \frac{1}{2}|W'(\varphi)|^2 + \bar{\psi}M\psi \right).$$

- fermion matrix

$$M = \gamma^z \partial + \gamma^{\bar{z}} \bar{\partial} + W'' P_+ + \overline{W}'' P_- .$$

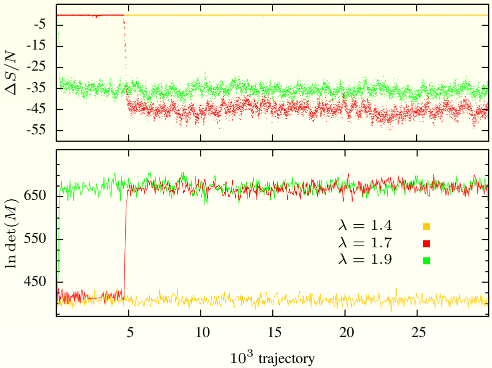
- holomorphic prepotential W not renormalized
- quantum corrections $\to$ Kähler metric $Z(\phi)(\bar{\partial}\bar{\varphi}\partial\varphi)$

- supersymmetries (ϵ complex)

$$\delta\varphi_x = \bar{\varepsilon}\psi_{1,x}, \qquad \delta\bar{\psi}_{1,x} = -\frac{1}{2}\bar{\xi}_x\bar{\varepsilon}, \qquad \delta\psi_{1,x} = 0,$$
$$\delta\bar{\varphi}_x = \bar{\varepsilon}\psi_{2,x}, \qquad \delta\bar{\psi}_{2,x} = -\frac{1}{2}\xi_x\bar{\varepsilon}, \qquad \delta\psi_{2,x} = 0.$$

- susy never broken, no Goldstino
- exist $\mathcal{Q}$-exact formulation ($\mathcal{Q}^2 = 0$)
- equivalent to Nicolai map-construction (Parisi-Sourlas)
- no divergences (besides vacuum energy)
- detailed investigations of various discretization, fermions species
- measured: Ward identities, sign-problem, finite size effects, masses

- $\mathcal{Q}$ exact formulation may become instable for strong coupling
- $\mathcal{N} = (2,2)$ WZ-model does:



- MC history of improvement term and fermion determinant $\hat{m} = 0.6$
- sudden jump into artifact phase with large gradients

Kästner, Bergner, Uhlmann, AW, Wozar

Gauge theory with one supersymmetry

- $\mathcal{N} = 1$ vector multiplet $(A_\mu, \lambda = \lambda_c)$ in adjoint
- for gauge group $SU(3)$: super-QCD
- theory dynamical (not narrowed down by symmetries)
- on-shell susy transformations

$$\delta_\epsilon A_\mu = \mathrm{i}\bar{\epsilon}\gamma_\mu\lambda, \quad \delta_\epsilon\lambda = \mathrm{i}\Sigma_{\mu\nu}F^{\mu\nu}\epsilon$$

- field strength tensor $F_{\mu\nu}$
- inf. spin rotations $\Sigma_{\mu\nu} \equiv \frac{\mathrm{i}}{4}[\gamma_\mu, \gamma_\nu]$
- inf. anti-commuting constant Majorana spinor ϵ

Continuum on-shell Lagrange density

$$\mathcal{L}_{\mathrm{SYM}} = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\not{D}\lambda - \frac{m}{2}\bar{\lambda}\lambda\right) .$$

- relevant mass-term breaks susy softly
- needed for susy continuum limit
- fine-tuning to reach 'correct' continuum limit (vanishing gluino mass)
- expect confinement of color charge at low energies
- only color-neutral bound states admitted: Mesons, Glueballs, ...
- bound states arranged in super-multiplets
- effective field theory (symmetries plus anomaly matching)

- a-f_0 means adjoint f_0
- characterization of states J^{pc}
- multiplet of Veneziano and Yankielowicz

1 bosonic scalar	0^{++} gluinoball	a-$f_0 \sim \bar{\lambda}\lambda$
1 bosonic pseudoscalar	0^{-+} gluinoball	a-$\eta' \sim \bar{\lambda}\gamma_5\lambda$
Majorana-type spin $\frac{1}{2}$	gluino-glueball	$\chi \sim F_{\mu\nu}\Sigma^{\mu\nu}\lambda$

- multiplet of Later, Farrar, Gabadadze and Schwetz

1 bosonic scalar	0^{++} glueball	$0^{++} \sim F_{\mu\nu}F^{\mu\nu}$
1 bosonic pseudoscalar	0^{-+} glueball	$0^{-+} \sim \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$
1 Majorana-type spin $\frac{1}{2}$	gluino-glueball	$\chi \sim F_{\mu\nu}\Sigma^{\mu\nu}\lambda$

- $m = 0 \Rightarrow$ global chiral $\mathrm{U}(1)_\mathrm{A}$ symmetry

$$\lambda \mapsto \mathrm{e}^{\mathrm{i}\alpha\gamma_5}\lambda$$

- anomalous breaking $\mathrm{U}(1)_\mathrm{A} \Longrightarrow \mathbb{Z}_{2N_\mathrm{c}}$

$$\lambda \mapsto \mathrm{e}^{\mathrm{i}\alpha_n\gamma_5}\lambda \quad \text{with} \quad \alpha_n = \pi\frac{n}{N_\mathrm{c}}, \quad n \in \{1, \dots, 2N_\mathrm{c}\}$$

- further spontaneous breaking by gluino condensate $\langle\bar{\lambda}\lambda\rangle \neq 0$
- breaking pattern $\mathbb{Z}_{2N_\mathrm{c}} \Rightarrow \mathbb{Z}_2$
- N_c physically equivalent vacua
- strongly interacting theory $\rightarrow$ lattice, FRG, ...
- no $\mathcal{Q}$ exact formulation
- conventional approach

- recover chiral symmetry in continuum $\rightarrow$ susy restoration
- Ginsparg-Wilson fermions $\rightarrow$ susy guaranteed in continuum

  first benchmarks: gaugino condensate, static potential, spectrum of Dirac operator Endress; Giedt, Brower, Catterall, Fleming, Vranas; JLQCD collaboration
- Wilson fermions $\rightarrow$ fine-tuning of m

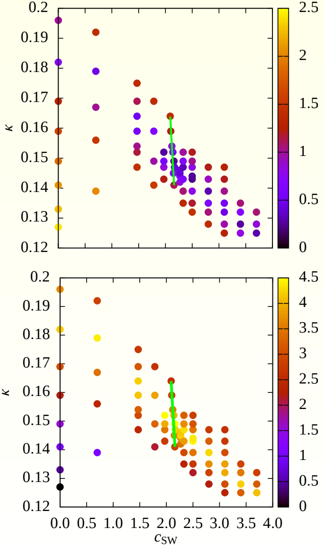
  mass-spectrum for SU(2), first results for SU(3), thermodynamics, Ward-Identities

  SU(2), SU(3): DESY-Münster collaboration: Montvay, Münster, Bergner, ...
  SU(3): Jena-group: Steinhauser, Sternberg, Wellegehausen, AW

- find m with minimal mass of gluino and (adjoint) pion
- thermodynamic limit to avoid finite size effects
- continuum limit
- reduce lattice artifacts
  - Symanzik improved Lüscher-Weisz gauge action $O(a^2)$
  - Clover fermion operator

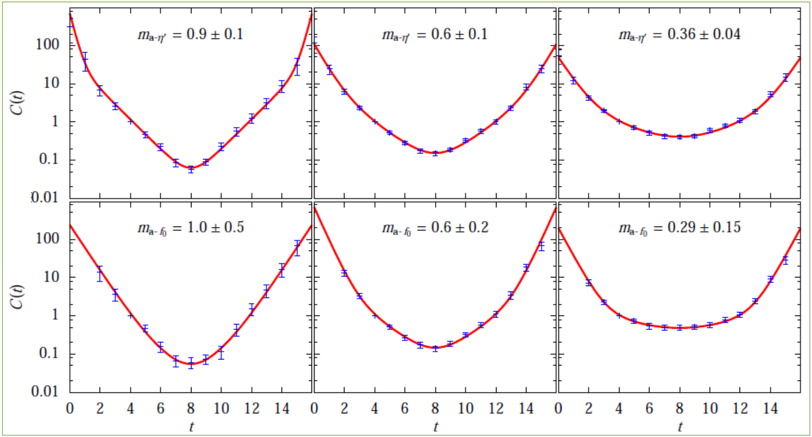
$$D_{\mathrm{C}}(x,y) = D_{\mathrm{W}}(x,y) - c_{\mathrm{SW}}\,\frac{\kappa}{2}\Sigma_{\mu\nu}F^{\mu\nu}\delta_{x,y}$$

  - proper choice for Sheikholeslami-Wohler coefficient $c_{\mathrm{SW}} \Rightarrow O(a^2)$
- from 1-loop PT, Tadpole-improvement, Schrödinger functional,...
  Jena group: minimize $m_{\mathrm{a}-\pi}$



- top-left: adjoint pion-mass
- top-right: $\langle S_B \rangle / V = 12$
- bottom: (subtr.) chiral condensate
- green: tadpole improved

- $c_{SW} = 2.27$, $\beta = 5.2$, $\kappa \in \{0.142, 0.144, 1.45\}$
- on the way: larger lattices,

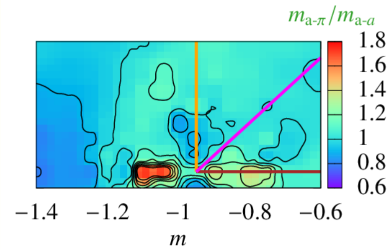


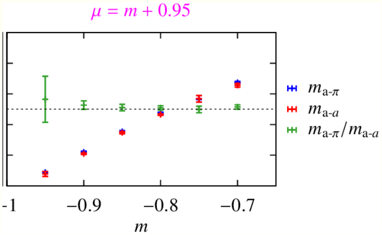
- $m_{a-\pi}$ too large: numerical experiments with 'twisted mass'

$$D_W^{tw}(x,y) = (4 + m + i\mu\gamma_5)\delta_{x,y} - \frac{1}{2}\sum_{\mu=\pm 1}^{\pm 4}(1-\gamma_\mu)\mathcal{V}_\mu\delta_{x+\mu,y}$$

- along diagonal to critical point $(m_c, \mu_c) = (0.95, 0)$

  $\Rightarrow m_{a-\pi} \approx m_{a-a}$





- much more about $\mathcal{N} = 1$ SYM: talk by G. Bergner

- susy-breaking under control in simple models
- $\mathbb{Z}_2$ breaking $\leftrightarrow$ susy breaking
- sign problem $\leftrightarrow$ order parameter
- Yukawa-type modely: SLAC fermions useful (symmetries)
  cp. recent results on $4-$ Fermi theories
- goldstino and massless boson in $\mathcal{N}=1$ WZM
- deceneracy of masses in susy phase (high accuracy)
- $\mathcal{N}=1$ SYM: optimization of Clover term
- numerical experiments with 'twisted' mass terms
- masses for SU(3): see DESY-Münster collaboration
- $\mathcal{N}=1$ SYM in four dimensions $\rightarrow$ $\mathcal{N}=(2,2)$ in two dimensions
- convincings results for $(2,2)$ (talk): moduli, masses, ...