Friedrich-Schiller-Universität Jena Winter term 2017/18
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Problems in Advanced Quantum Mechanics

Problem Sheet 8

Problem 18: Scattering at a hard sphere 2+3+1 = 6 points

We study the scattering of particles at a hard sphere with radius a.

1. Determine the total scattering cross section $\sigma = \sum_\ell \sigma_\ell$ for the elastic scattering of particles at a hard sphere with radius a (the de Broglie wave length satisfies $\lambda \ll a$).
2. Determine the dimensionless ratio $\sigma/(2\pi a^2)$ with your favorite computer program (matlab, octave, mathematica,..) for $ka = 1, 2, 3, \ldots, 50$ and plot the result.
3. Compare the result for fast particles ($ka >> 1$) with the classical cross section.

Hint: You may probably need

$$\mathrm{i}\tan\delta_\ell = \frac{\mathrm{e}^{2\mathrm{i}\delta_\ell} - 1}{\mathrm{e}^{2\mathrm{i}\delta_\ell} + 1} \quad \text{and} \quad \sin^2\delta_\ell = \frac{\tan^2\delta_\ell}{1 + \tan^2\delta_\ell}$$

Problem 19: Gaussian integral 3 points

Let A be a real and symmetric matrix. Prove the formula

$$\int \prod_{i=1}^{N} \mathrm{d}x_i \, \mathrm{e}^{\frac{\mathrm{i}}{2\hbar}\boldsymbol{x}^T A \boldsymbol{x}} = (2\mathrm{i}\pi\hbar)^{N/2} \frac{1}{\sqrt{\det A}} \, .$$

Hint: transform first to coordinates for which A is diagonal.

Problem 20: Action of one-dimensional harmonic oscillator 4+1 = 5 points

Show that the action along the classical path from (t_1, x_1) to (t_2, x_2) is

$$S[x_2, t_2; x_1, t_1] = \frac{m\omega}{2\sin(\omega T)} \left((x_1^2 + x_2^2)\cos(\omega T) - 2x_1 x_2 \right) , \quad T = t_2 - t_1 \, ,$$

where ω is the circular frequency of the oscillator. The propagator is given by

$$K(x_1, t_2; x_1; t_1) = \left(\frac{m\omega}{2\pi\mathrm{i}\hbar \sin\omega T} \right)^{1/2} \mathrm{e}^{\mathrm{i}S(x_2,t_2;x_1,t_1)/\hbar}$$

Show that this propagator fulfills $\lim_{T\to 0} K(x_2, t_2; x_1, t_1) = \delta(x_2 - x_1)$.

Hint: Concerning the last question, recall what is the propagator for a free particle.

Submission date: Thursday, 15. December 2017, before the lecture begins.