Prof. Andreas Wipf, MSc. Marc Steinhauser December 4, 2019

Problem sheet „Advanced Quantum Mechanics“

winter term 2019/20

Sheet 8

Problem 18: Spherical potential well and potential step 2+1+2 = 5 points

Investigate the Born approximation for a spherical potential

$$V(r) = \begin{cases} V_0 & r < a \\ 0 & r > a\,. \end{cases}$$

Determine

1. the scattering amplitude,
2. differential scattering cross section and
3. total cross section.

For the potential well V_0 is negative and for the potential step V_0 is positive. Discuss the results for $a\Delta k \ll 1$.
Hint: When calculating the total cross section you may convert the integral over ϑ into an integral over q ($q = 2k\sin\vartheta/2$).

Problem 19: Scattering phase 5 points

Determine the phase shifts δ_ℓ for the scattering at the potential $V = A/r^2$ and calculate the differential cross section for $0 \le \mu A/\hbar^2 \ll 1$.
Hint: Set the function $u_{E\ell} = r f_{E\ell}$ in the corresponding radial Schrödinger equation equal to $u_{E\ell} = \sqrt{r} g_{E\ell}$. The differential equation for $g_{E\ell}$ should be known to you (spherical Bessel functions). The resulting sum over Legendre polynomials simplifies with the identity

$$\sum_{\ell=0}^{\infty} P_\ell(\cos\theta) = \frac{1}{2\sin(\theta/2)}.$$

Submission date: Thursday, 12.12.2019, before the lecture