Friedrich-Schiller-Universität Jena
Theoretisch-Physikalisches-Institut

Prof. Dr. Andreas Wipf
Dr. Luca Zambelli

seit 1558

Klausur: Quantenmechanik II, Wintersemester 2017/18

name: **Matrikel number:**

time: 10:15 – 12:45; place: Helmholtzweg 3, Hörsaal 3
permitted tools: **at most one written sheet of paper**
Hint: Please mark every sheet of paper with your name

Aufgabe	1	2	3	4	5	6	Σ	Note
Punkte								
max. Punkte	7	4	4	2	4	3	24	

problem 1: Comprehension questions 1+1+1+1+1+1+1 = 7 points

Please give short and precise answers to the following questions:

1. Explain, why a totally symmetric or totally anti-symmetric wave function describing identical bosons or identical fermions remains symmetric or anti-symmetric under the time evolution.
2. What needs to be taken into account when one describes the scattering of identical fermions or bosons?

3. A system with angular momentum j_1 and a system with angular momentum j_2 are coupled to a total system. What are the allowed angular momenta of the total system (only the result)?
4. What are the Clebsch-Gordan coefficients (in words)?
5. When considering space-rotations in non-relativistic quantum mechanics: why do we need SU(2) instead of SO(3)? When considering with Lorentz transformations in relativistic quantum mechanics: why do we need SL(2,$\mathbb{C}$) instead of SO(1,3)?
6. Why can the Schrödinger equation not be relativistically covariant (look the same in all inertial systems)?
7. What is the principle of minimal coupling?

problem 2: Scattering 3 points

Calculate in the first Born approximation the neutron scattering cross-section in a three-dimensional potential

$$V(r) = \begin{cases} U_0 & r \leq a \\ 0 & r > a \end{cases}$$

problem 3: Time-dependent perturbation theory 3 points

The Hamiltonian $H(t) = H_0 + V(t)$ contains a time-independent part H_0 and a time-dependent perturbation $V(t)$. In the interaction picture the solution of the time-dependent Schrödinger equation is given by the Dyson series

$$|\psi_W(t)\rangle = \left(\mathbb{1} + \frac{1}{\mathrm{i}\hbar}\int_0^t V_W(t_1)\mathrm{d}t_1 + \frac{1}{(\mathrm{i}\hbar)^2}\int_0^t V_W(t_1)\mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 V_W(t_1) + \dots\right)|\psi(0)\rangle$$

with

$$V_W(t) = \mathrm{e}^{\mathrm{i}H_0 t/\hbar} V(t) \mathrm{e}^{-\mathrm{i}H_0 t/\hbar}$$

Show that in first order perturbation theory the expectation value of an observable A is given by

$$\langle\psi(t)|A|\psi(t)\rangle = \langle\psi(0)|A_W(t)|\psi(0)\rangle + \frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' \, \langle\psi_0(t')|[V_W(t'), A_W(t)]|\psi 0(t)\rangle$$

problem 4: Many electron system 1+2 = 3 points

Consider N non-interacting electrons in a one-dimensional infinitely high potential well of width L. What is the smallest value of the total energy for large N?
Hints: Recall that at most two electrons can occupy the same energy level (the must have

different s_z). For large N it does not matter whether the highest level is occupied with one or two electrons. Finally you may need

$$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6} \approx \frac{k^3}{3}$$

problem 5: Spin and magnetic moment of the deuteron 3 points

Assume that the electron cloud is in a state with energy E_J and total angular momentum $J(J+1)\hbar^2$ and the nucleus is in a state with energy E_I and total angular momentum $I(I+1)\hbar^2$. The respective magnetic moments are $\boldsymbol{\mu} = g_J \mu_B \boldsymbol{J}/\hbar$ and $\boldsymbol{\mu} = g_I \mu_N \boldsymbol{I}/\hbar$, where g_J and g_I are dimensionless factors. The magnetic interaction Hamiltonian of the electron cloud with the nucleus is of the form $W = a\boldsymbol{\mu}_J \cdot \boldsymbol{\mu}_I$, where a is a constant which depends on the electron distribution around the nucleus.

1. What are the possible values $K(K+1)\hbar^2$ of the total angular momentum $\boldsymbol{K} = \boldsymbol{J} + \boldsymbol{I}$ of the atom?
2. Express W in terms of $\boldsymbol{I}^2$, $\boldsymbol{J}^2$ and $\boldsymbol{K}^2$. Express the hyperfine energy levels of the atom in terms of I, J and K (without interaction between the electron-cloud and nucleus the energy is $E_J + E_I$).
3. Calculate the splitting between two consecutive hyperfine levels.

problem 6: Klein-Gordon equation 2 points

Let the scalar function $\phi(x) = \phi(t, \boldsymbol{x})$ be a solution of the Klein-Gordon equation

$$\Box\phi + \mu^2\phi = 0$$

Show, that the charge density and 3-current density obey the continuity equation

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\,\boldsymbol{j} = 0\,,$$

where the densities are

$$\rho = \frac{\mathrm{i}\hbar}{2mc^2}\left(\phi^*\frac{\partial\phi}{\partial t} - \frac{\partial\phi^*}{\partial t}\phi\right) \quad \text{und} \quad \boldsymbol{j} = \frac{\hbar}{2\mathrm{i}m}\left(\phi^*\nabla\phi - \phi\nabla\phi^*\right).$$

problem 6: Chiral symmetry 1+2 = 3 Punkte

Consider the following transformation of a Dirac spinor

$$\psi \to \psi' = \exp(\mathrm{i}\alpha\gamma_5)\psi$$

with constant real parameter α and hermitean γ_5, which anti-commutes with all γ^μ.

- Determine the transformation of the Dirac-conjugate Spinor $\bar{\psi} = \psi^\dagger \gamma^0$.
- When is the Lagrangian density $\mathcal{L} = \bar{\psi}\left(\mathrm{i}\not{\partial} - m\right)\psi$ invariant under above transformation?

Hint:

$$\gamma_5\gamma_5 = \mathbb{1}, \qquad \exp(\mathrm{i}\alpha\gamma_5) = \mathbb{1}\cos\alpha + \mathrm{i}\gamma_5 \sin\alpha.$$

Viel Erfolg!