Friedrich-Schiller-Universität Jena Wintersemester 2016/17
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Übungen zu Symmetrien in der Physik

Blatt 3

Problem 9: Conjugacy classes of S_6

Consider the permutation group (symmetric group) of 6 elements $\mathcal{S}_6$ containing $6! = 720$ elements. How many conjugacy classes are in this group? Compute the table of classes (see lecture).

Problem 10: Galilei-Group

The Galilei-transformations

$$t' = t + \tau, \quad \boldsymbol{x}' = R\boldsymbol{x} + \boldsymbol{u}t + \boldsymbol{a}, \quad \text{mit} \quad \boldsymbol{u},\, \boldsymbol{a} \in \mathbb{R}^3, \quad R^T R = \mathbb{1}\,,$$

depend on the 10 parameters $(\tau, \boldsymbol{a}, \boldsymbol{u}, R)$. Now perform a second transformation $(t', \boldsymbol{x}') \to (t'', \boldsymbol{x}'')$ with parameters $(\tau', \boldsymbol{a}', \boldsymbol{u}', R')$.

- Show, that the composition of two Galilei-transformations is again a Galilei-transformation.
- What is the inverse Galilei-transformation to $(\tau, \boldsymbol{a}, \boldsymbol{u}, R)$?
- This then proves that the transformations form a group, it is the Galilei-group. Find the invariant subgroups of the Galilei-group.
- Can you write the Galilei-group as semi-direct product of a subgroup and invariant subgroup?

Problem 11: Lorentz-Group

The set of d-dimensional matrices

$$\mathcal{L} = \big\{\Lambda \in \mathrm{GL}(d, \mathbb{R}) \big| \Lambda^T G \Lambda = G, \quad G = \mathrm{diag}\big(1, -1, \ldots, -1\big)\big\}$$

defines the Lorentz group in d dimensions. The metric tensor G defines the line element in d dimensions: $ds^2 = (dx^0)^2 - \sum (dx^i)^2 = dx^\mu G_{\mu\nu} dx^\nu$.

- show, that $\mathcal{L}$ is a subgroup of $GL(d, \mathbb{R})$.
- what is the dimension of the group Lorentzgroup $\mathcal{L}$?
- prove that for $\Lambda \in \mathcal{L}$ the determinant is $\det \Lambda = \pm 1$.
- prove, that the set $\mathcal{L}^+ = \{\Lambda \in \mathcal{L} | \det \Lambda = 1\}$ is a normal subgroup of $\mathcal{L}$. It consists of the proper Lorentz-Transformations.

Problem 12: Dimensions of matrix groups

What are the dimensions of SO(m) and SU(n)? For which n and m have the groups the same dimension? Explain the latter result (recall the discussion of the spin in quantum mechanics).