Friedrich-Schiller-Universität Jena Wintersemester 2016/17
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Übungen zu Symmetrien in der Physik

Blatt 2

Aufgabe 5: Group of automorphisms of a group

Let G be a group. Show, that the set of maps

$$\mathrm{Aut}(G) = \{\varphi : G \to G | \, \varphi \text{ is isomorphism}\}$$

defines a group. It is the group of automorphisms of G introduced in the lecture.

Aufgabe 6: Presentations

Which two groups are defined by the presentations

$$\begin{aligned} &a^2 = b^2 = (ab)^3 = e, \quad \text{and} \\ &a^5 = b^2 = e, \quad bab^{-1} = a^{-1} \end{aligned}$$

Aufgabe 7: Conjugated subgroups

Let $H \leq G$ be a subgroup of G. Proof, that for every $g \in G$ the set gHg^{-1} defines a subgroup of G. Show that the so-defined conjugated subgroups are all isomorphic to H.

Aufgabe 8: Normal subgroups

Let S_n be the permutations group of n elements. The order of this group is $n!$. Let A_n be the alternate subgroup consisting of the even permutations in S_n (an permutation is even, if it is a product of an even number of transpositions).

1. Which permutations of S_3 are even and which are odd?
2. Show that A_n is a subgroup of S_n.
3. What is the order of A_n?
4. Show that A_n is a normal subgroup of S_n.
5. Which group is S_n/A_n?