Friedrich-Schiller-Universität Jena Summer term 2018
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Exercises to „Symmetries in Physics“

Sheet 7

Problem 25: Campbell-Hausdorff Formula

Assume that two matrices (operators) A, B obey the relations

$$[A,[A,B]] = [B,[A,B]] = 0.$$

Show that in this situation the following identity holds true:

$$\exp(A)\exp(B) = \exp\left(A + B + \frac{1}{2}[A,B]\right).$$

Hint: you may find a differential equation for $D(t) \equiv \mathrm{e}^{tA}\,\mathrm{e}^{tB}\,\mathrm{e}^{-t(A+B)}$.

Problem 26: Conjugacy classes of SU(3)

Characterize the conjugacy classes of SU(3).
Hint: consider the eigenvalues of a matrix in SU(3)

Problem 27: Representations

Let $g \mapsto D(g)$ be any representation of a group. Represent the mappings $g \mapsto D^\dagger(g)$ or $g \mapsto D^{-1}(g)$ or $g \mapsto D^*(g)$ the group as well?

Let $g \mapsto D(g)$ be a irreducible representation of a group which is equivalent to the complex conjugated representation. This means that there is an invertible S with $D^*(g) = SD(g)S^{-1}$. Show that $SS^* = \lambda\mathbb{1}$.

Show that for an unitary D the matrix S is symmetric or antisymmetric and that $SS^\dagger = \lambda'\mathbb{1}$.

Hint: Use that any any matrix that commutes with all $D(g)$ of an irreducible representation is proportional to the identity (Lemma of Schlur)

Problem 28: Rotations of wave functions

We consider the wave functions of a particle $\psi(\boldsymbol{x}) \in L_2(\mathbb{R}^3)$ in position space. Such a wave function transform under rotations according to

$$(U(R)\psi)(\boldsymbol{x}) = \psi(R^{-1}\boldsymbol{x})$$

Prove, that $R \to U(R)$ is a (infinite-dimensional) unitary representation of SO(3) on the Hilbert space $L_2(\mathbb{R}^3)$. Can you guess which subspaces support the one-, the three- and the five-dimensional irreducible representations?