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Exercises to „Symmetrien in der Physik“

Sheet 1

Problem 1: Cayley-Table

Work out the multiplication table for the dihedral group, generated by the elements a and b with $a^2 = b^3 = (ab)^2 = e$. Do you recognize the group.
Hint: Such representations of a group are discussed in the lecture notes (appendix to chapter 2).

Problem 2: Permutation Group

Work out the multiplication table for the permutation group S_3 of three elements.

Problem 3: Isomorphic Groups

State which of the following groups are isomorphic to each other, giving the explicit correspondence where an isomorphism exists:

1. the complex numbers $\{1, i, -1, -i\}$ with respect to multiplication;
2. the integers $\{2, 4, 6, 8\}$ with respect to multiplication modulo 10;
3. the permutations

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix};$$

4. the permutations

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix};$$

Problem 4: Center of group

The center of a group G consists of all elements z in G with $zg = gz$ for all elements g of the group. Show, that the center forms an Abelian subgroup of the group.