Theoretisch-Physikalisches Institut
Friedrich-Schiller Universität Jena

Particles and Fields

Winter term 2018/19

Examples sheet 8

Exercise 15

In the following we consider the action for a free massive Dirac Fermion with mass m, given by the Lagrangian density

$$\mathcal{L} = \bar{\Psi}(x)\,(i\gamma^\mu\partial_\mu - m)\,\Psi(x)\,, \quad \text{with} \quad \bar{\Psi} \equiv \Psi^\dagger\,\gamma^0\,. \tag{1}$$

(a) Discuss the symmetries of the action.

(b) Compute the Noether current associated with spacetime translations and Lorentz transformations. Is the canonical energy-momentum tensor obtained from the Noether procedure symmetric?

(c) Show that the Lagrangian is invariant under

$$\Psi(x) \mapsto \tilde{\Psi}(x) = e^{-i\alpha}\Psi(x).$$

Derive the associated Noether current J_V^μ, the so-called vector current. Show explicitly that it is conserved and determine the corresponding conserved charge.

(d) In the case of a massless Dirac fermion, i.e. $m = 0$, consider the transformation

$$\Psi(x) \mapsto \tilde{\Psi}(x) = e^{-i\gamma_5\alpha}\Psi(x).$$

How does $\bar{\Psi}$ transform? Show that the Lagrangian is invariant under this transformation. Derive the associated Noether current J_A^μ, the so-called axial current. Show explicitly that J_A^μ conserved is only conserved if $m = 0$.

Exercise 16

In the following we consider the Lagrangian of a Dirac fermion with mass m coupled to a real scalar field ϕ in four dimensions,

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\Psi}\,(i\gamma^\mu\partial_\mu - m)\,\Psi - \lambda\phi\bar{\Psi}\Psi \tag{2}$$

(a) Determine the canonical dimension of $\phi\bar{\Psi}\Psi$. Is the operator classically relevant/irrelevant or marginal?

(b) Derive the equations of motion for the Dirac fermion Ψ and the scalar field ϕ.