Theoretisch-Physikalisches Institut
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Particles and Fields

Winter term 2018/19

Examples sheet 5

Exercise 9

We consider a real scalar field in two spacetime dimensions with Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\,\eta^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\,, \qquad V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{3m^4}{2\lambda}\,. \tag{1}$$

The constant contribution to the potential $V(\phi)$ does not change the equations of motion and is chosen for later convenience.

(a) Discuss the shape of the potential $V(\phi)$ for the cases:

- case 1: $m^2 > 0$ and $\lambda > 0$,
- case 2: $\mu^2 \equiv -m^2 > 0$ and $\lambda > 0$.

(b) We consider from now on case 2. Determine the minima $\pm v$ (with $v > 0$) of the potential $V(\phi)$. Expand the field around the vacuum solution, i.e. consider $\phi(x) = v + \delta\phi(x)$. What is the effective mass of the fluctuation $\delta\phi(x)$?

Hint: Substitute the ansatz $\phi(x) = v+\delta\phi(x)$ into the Lagrangian density and expand to second order in $\delta\phi(x)$.

(c) Construct a static solution to the equations of motion which interpolates between the vacua v and $-v$, i.e. which satisfies

$$\lim_{x\to\infty}\phi(x) = v \qquad \lim_{x\to-\infty}\phi(x) = -v\,. \tag{2}$$

This so-called *kink* solution is an example of a (non)-perturbative topological object.

Hint: Multiply the time-independent equations of motion with $\mathrm{d}\phi/\mathrm{d}x$ and integrate once (c.f. effective potential in classical mechanics).

Exercise 10

We consider a N component real scalar field $\phi(x) = (\phi_1(x), \phi_2(x), \dots, \phi_N(x))$ with $O(N)$ symmetric Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\,\eta^{\mu\nu}\,\partial_\mu\phi_a(x)\,\partial_\nu\phi_a(x) + \frac{1}{2}\mu^2\phi_a(x)\phi_a(x) - \frac{\lambda}{4!}(\phi_a(x)\phi_a(x))^2\,, \tag{3}$$

where $\mu^2 > 0$ and $\lambda > 0$.

(a) Show that the true vacuum is given by the constant solution $\phi_a(x) = v_a$ with $v_a\,v_a = 6\mu^2/\lambda$.

(b) We choose the vacuum to be $\phi_1(x) = v$ and $\phi_i(x) = 0$ with $i = 2, \dots, N$ and $v = +\sqrt{6\mu^2/\lambda}$. Argue that the $O(N)$ symmetry is spontaneously broken and determine the residual symmetry of the system.

Bitte wenden!

(c) In the following we consider fluctuations $\sigma(x)$ and $\boldsymbol{\pi}(x)$ (where $\boldsymbol{\pi}(x)$ is a $N-1$ component scalar field) given by $\phi(x) = (v + \sigma(x), \boldsymbol{\pi}(x))$. Determine the Lagrangian for $\sigma(x)$ and $\boldsymbol{\pi}(x)$ by substituting the ansatz for $\phi(x)$ into the Lagrangian and expanding it to fourth order in the fluctuations. What is the mass for the σ-fluctuations and for the $\boldsymbol{\pi}$-fluctuations?